Combined maintenance activities in flow shop scheduling

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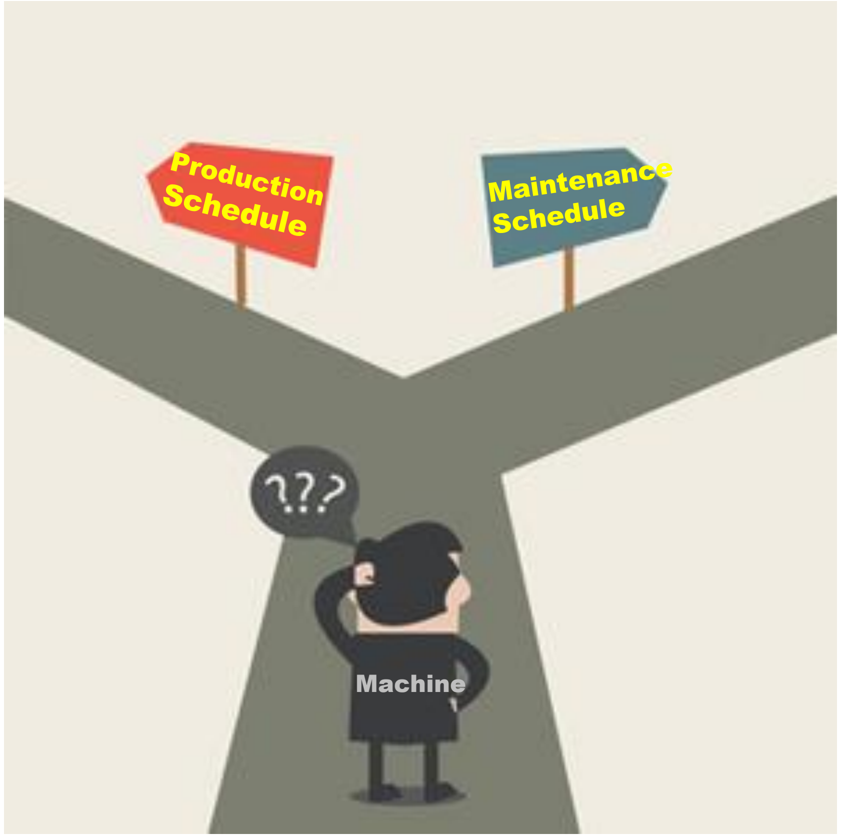
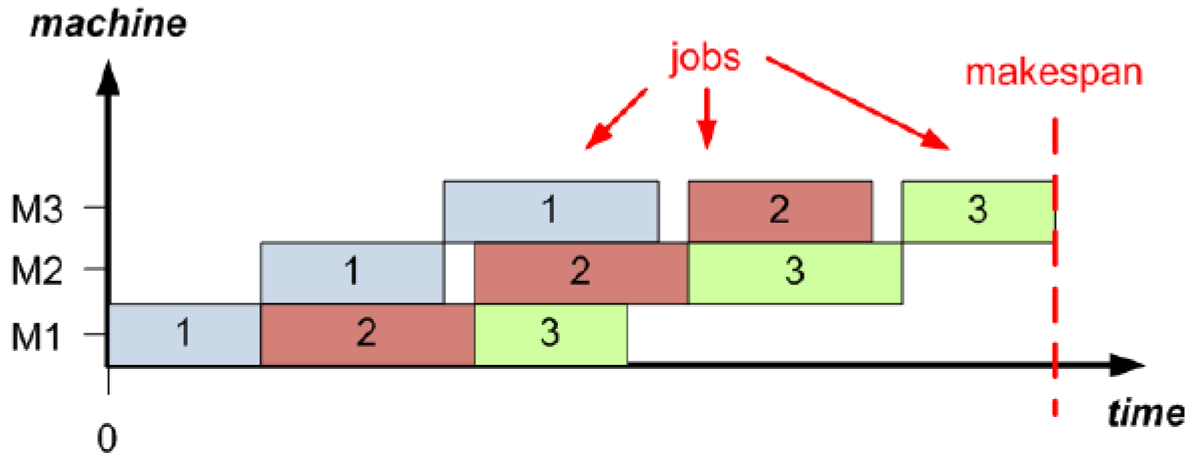
A Common Problem

Client Expectations •
$$$ •



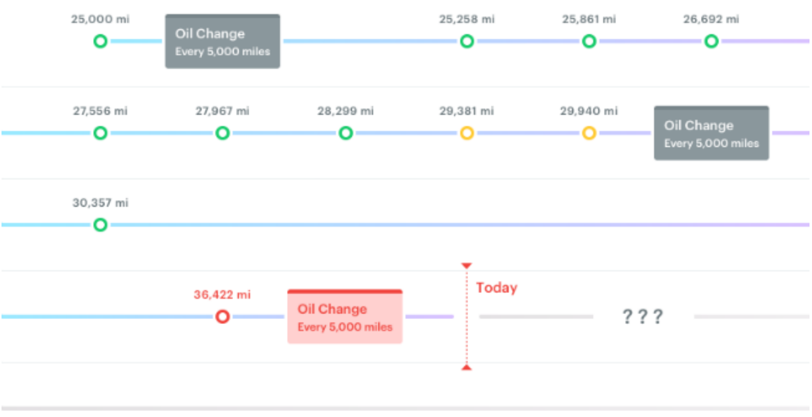
- PM System
- Uninterrupted Production

Flow shop scheduling problem



Maintenance scheduling

- Time-Based Preventative Maintenance
- Meter-based Preventative Maintenance



Maintenance scheduling

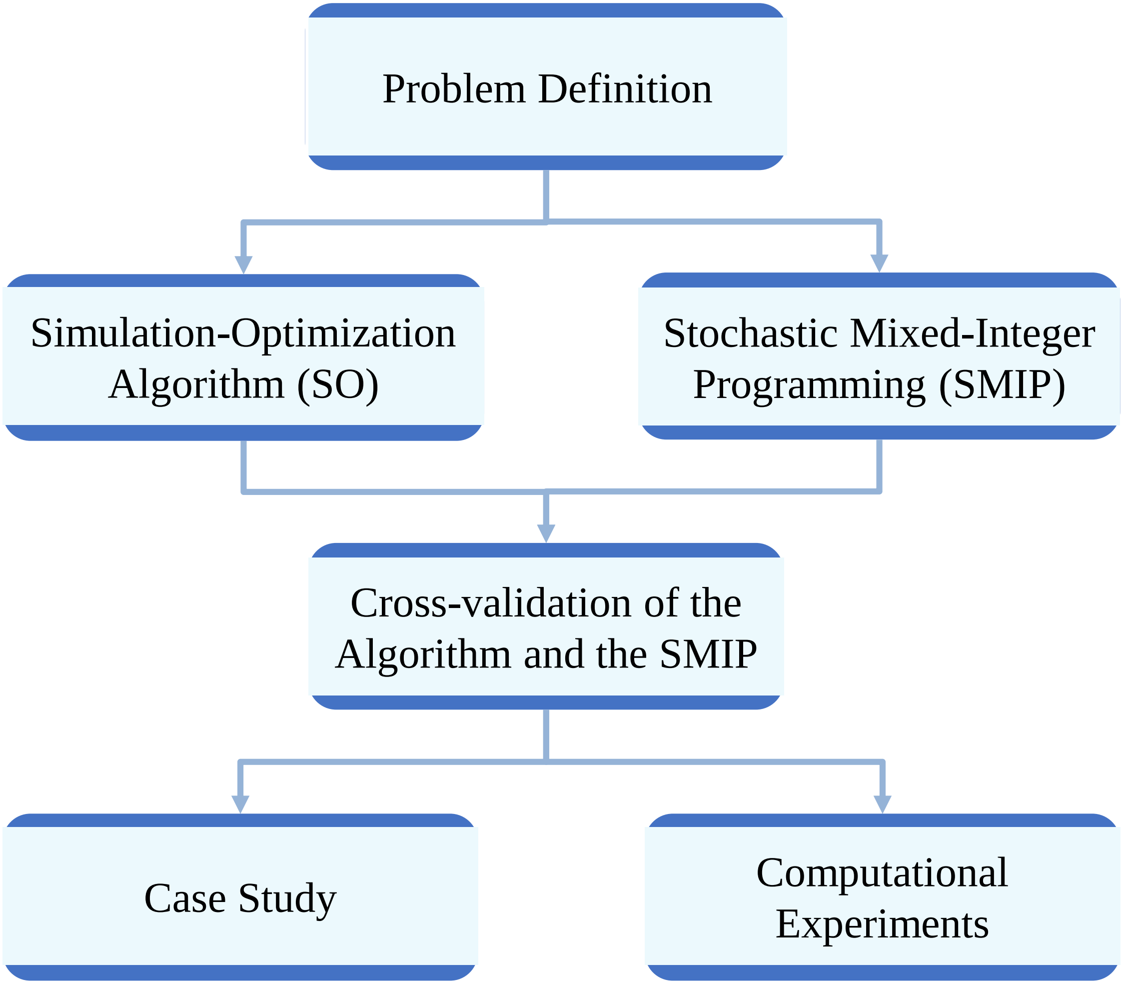
- Time-Based Preventative Maintenance
  - \+ Simple and easy
  - \+ Easy to predict
  - \- Not based on utilization
  - \- over-maintenance or under-maintenance
- Meter-based Preventative Maintenance
  - \+ Based on actual utilization
  - \+ Only when needed, not by an arbitrary date
  - \- Getting timely meter readings is difficult, leading to late or missed (and costly) PMs
  - \- Hard to predict the date of future occurrences

https://www.geotab.com/blog/usage-based-preventative-maintenance/

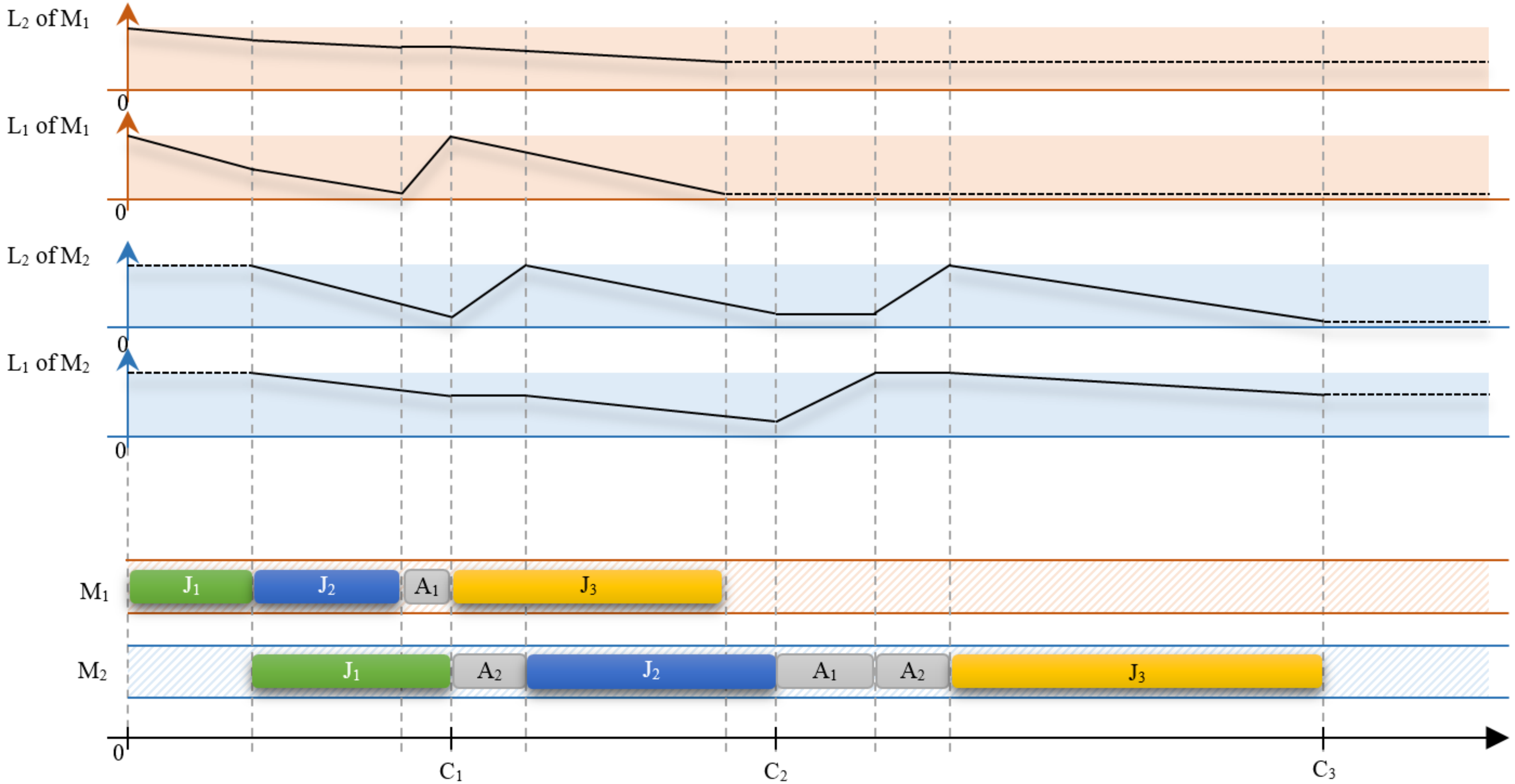
Challenges in the Integration of Production and Maintenance

- Modeling challenges
- Complexity challenges
- Uncertainty in:
  - MTTR
  - Processing times
- Technical nuances:
  - Combining MAs
  - How MAs affect processing times?

Methodology

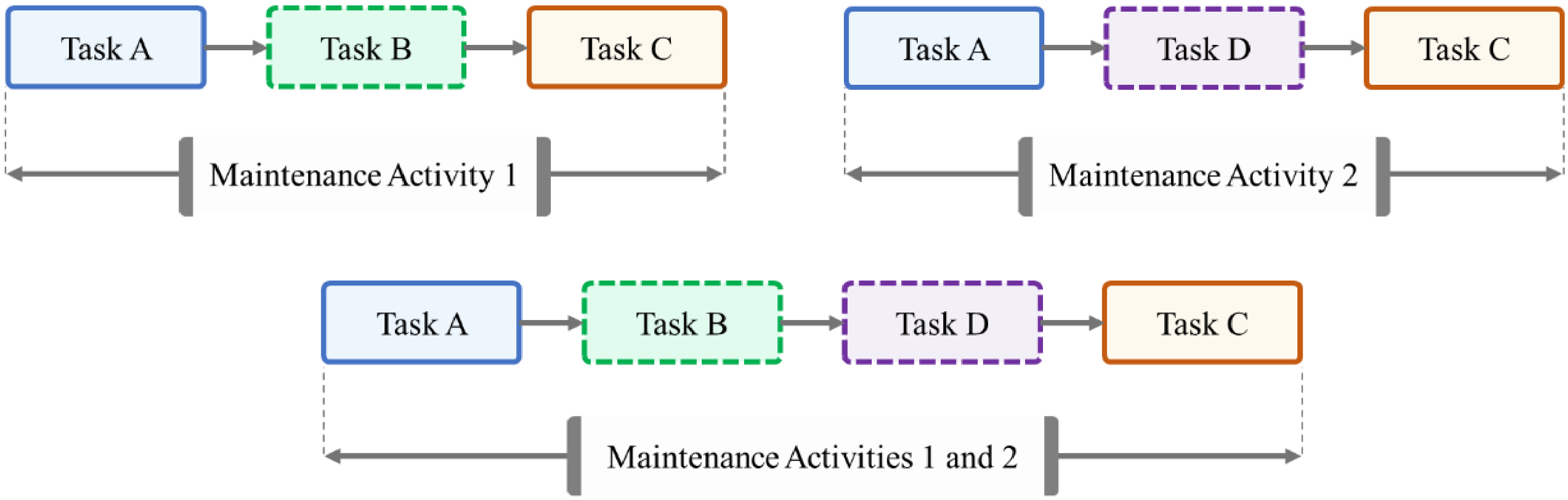


Integrated Flow Shop and Maintenance Scheduling



M: Machine – **J**: Job – **L**: Maintenance Level –**A**: Maintenance Activity – **C**: Completion Time

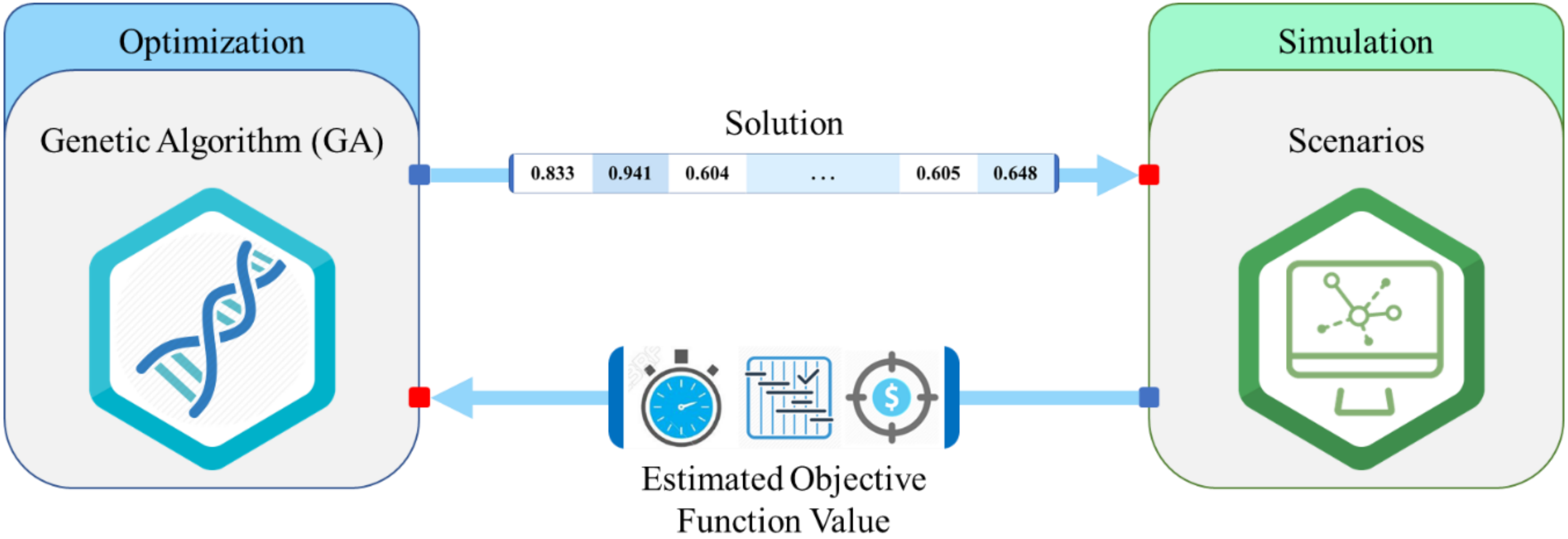
Maintenance Combination



Stochastic Programming

- Two-stage
  - First stage decisions: sequence of the jobs, position of maintenance activities
  - Second stage decisions: start/finish time, tardiness, …
- Minimize the expected cost
  - Penalty cost for tardiness
  - Maintenance cost
- 37 sets of constraints
- 13 sets of variables

Simulation-Optimization



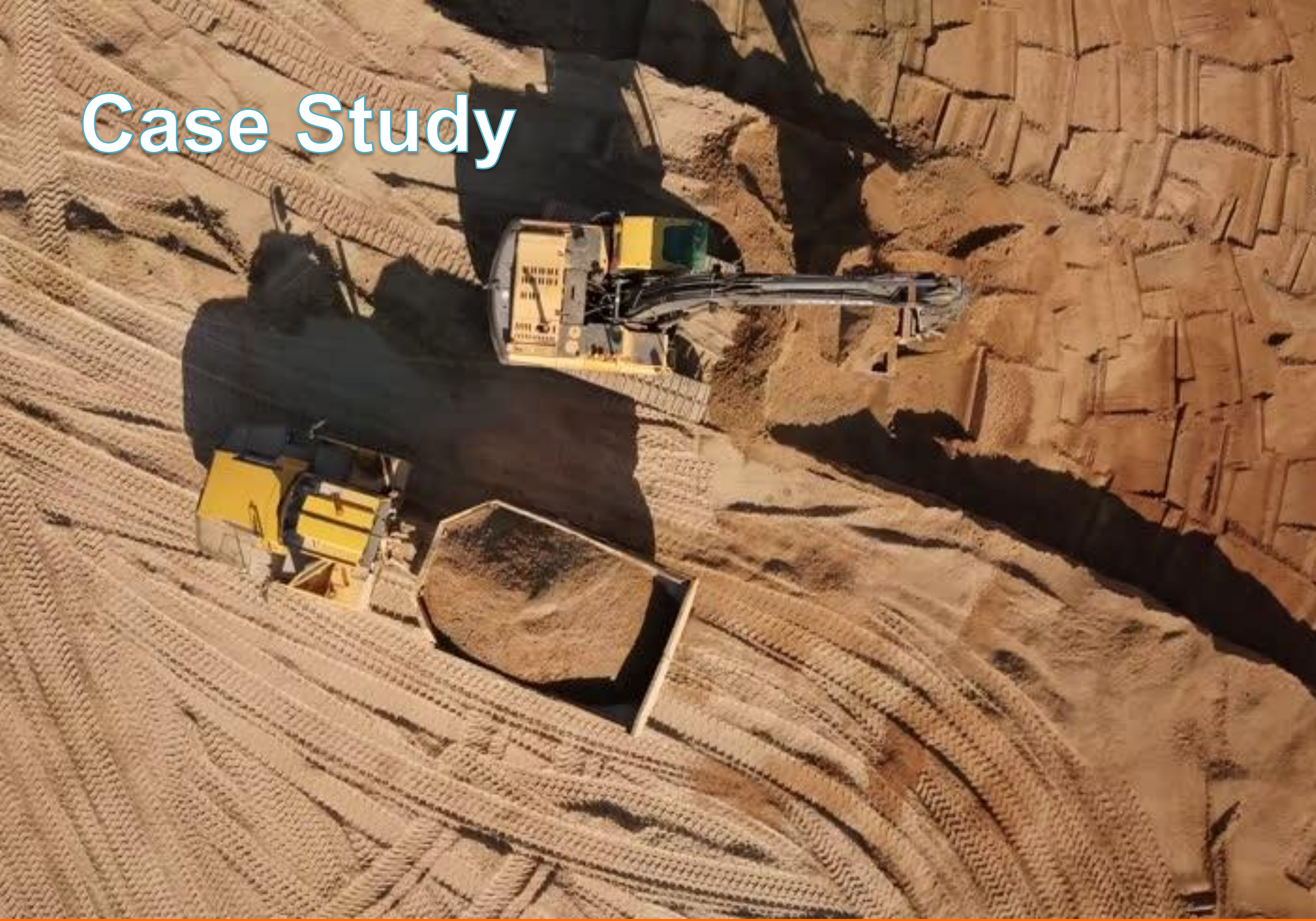
Computational Experiments

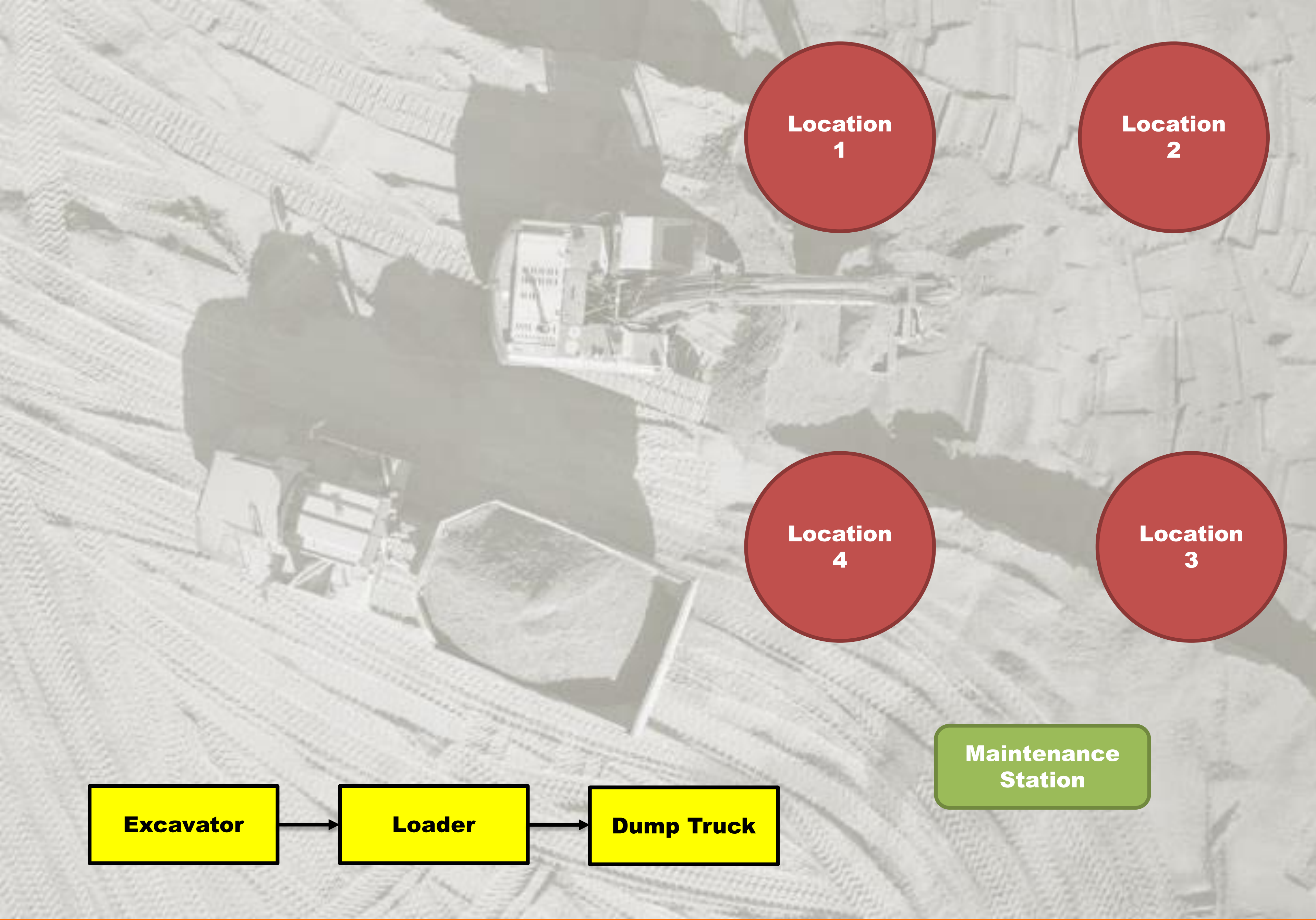
- Test Problem Generation
- Find the best population size
- Performance Evaluation
- Sensitivity to the input data

Computational Experiments

- for **small-sized problems** we recommend the use of commercial/exact solvers
- for **medium to large-scale instances** and under **a limited time frame**, the presented solution method outperforms these computationally and financially expensive solvers
- not sensitive to the input data, the **robustness** of the method
- CPLEX’s performance depends on the input data

Case Study





Location 1

Location 2

Location 4

Location 3

Maintenance Station

Excavator → Loader → Dump Truck

Demand

Location (Jobs)	Processing Times (no. of days × hours/day)			Due Date (days)	Penalty/Day
	Excavator	Wheel Loader	Truck		
$\boldsymbol{L_1}$	$\sim TRI(20, 22, 25) \times 8$	$\sim TRI(25, 28, 30) \times 8$	$\sim TRI(5, 7,9) \times 16$	90	\$211
$\boldsymbol{L_2}$	$\boldsymbol{\sim TRI(15, 20, 25) \times 8}$	$\boldsymbol{\sim TRI(20, 25, 30) \times 8}$	$\boldsymbol{\sim TRI(7, 10, 14) \times 16}$	**100**	**\$118**
$\boldsymbol{L_3}$	$\sim TRI(10, 15, 20) \times 8$	$\sim TRI(15, 20, 25) \times 8$	$\sim TRI(3, 9, 14) \times 16$	80	\$118
$\boldsymbol{L_4}$	$\sim TRI(18, 23, 28) \times 8$	$\sim TRI(15, 20, 25) \times 8$	$\sim TRI(9, 11, 13) \times 16$	70	\$346

Maintenance Schedule

Machine	10	50	100	250	500	1000
Excavators	✓	✓	✓	✓	✓	✓
Wheel Loaders	✓	✓	✓	✓	✓	✓
(Haul) Trucks	✓	✓			✓	✓

$\sim TRI(1,2,5)$

Solution

Sequence of Jobs	Scheduled Location	Scheduled Maintenance Combinations		
		Excavator	Loader	Truck
1	L_3	0	0	0
2	L_4	1	**1** (5)	0
3	L_1	4	**7** (4)	2
4	L_2	1	1	0

Total Expected Cost:	**Expected Maintenance Cost:**	**Expected Penalty Cost:**
\$4,078	**\$3,076**	**\$1,002**
(\$4,188)	(\$3,170)	(\$1,018)

Conclusions

- The conflict between production and maintenance can be solved by **integrating** maintenance decisions with production scheduling
- **Uncertainty** and **technical nuances** should be considered for better implementation
- **Simulation Optimization** for large-scale problems
- Exact methods/**Commercial solvers** for validation and small problems, using stochastic mixed-integer programs
- Future research: considering **potential random failures**.

Reference

Seif, J., M. Dehghanimohammadabadi, and A. Yu, **Incorporating combined maintenance activities in flow shop scheduling: applying stochastic programming and simulation optimization**. *European Journal of Operational Research*, 2017. Manuscript submitted for publication.

Thanks to my co-authors Dr. Mohammad Dehghani and Dr. Andrew Yu

Thank you!

Q&A

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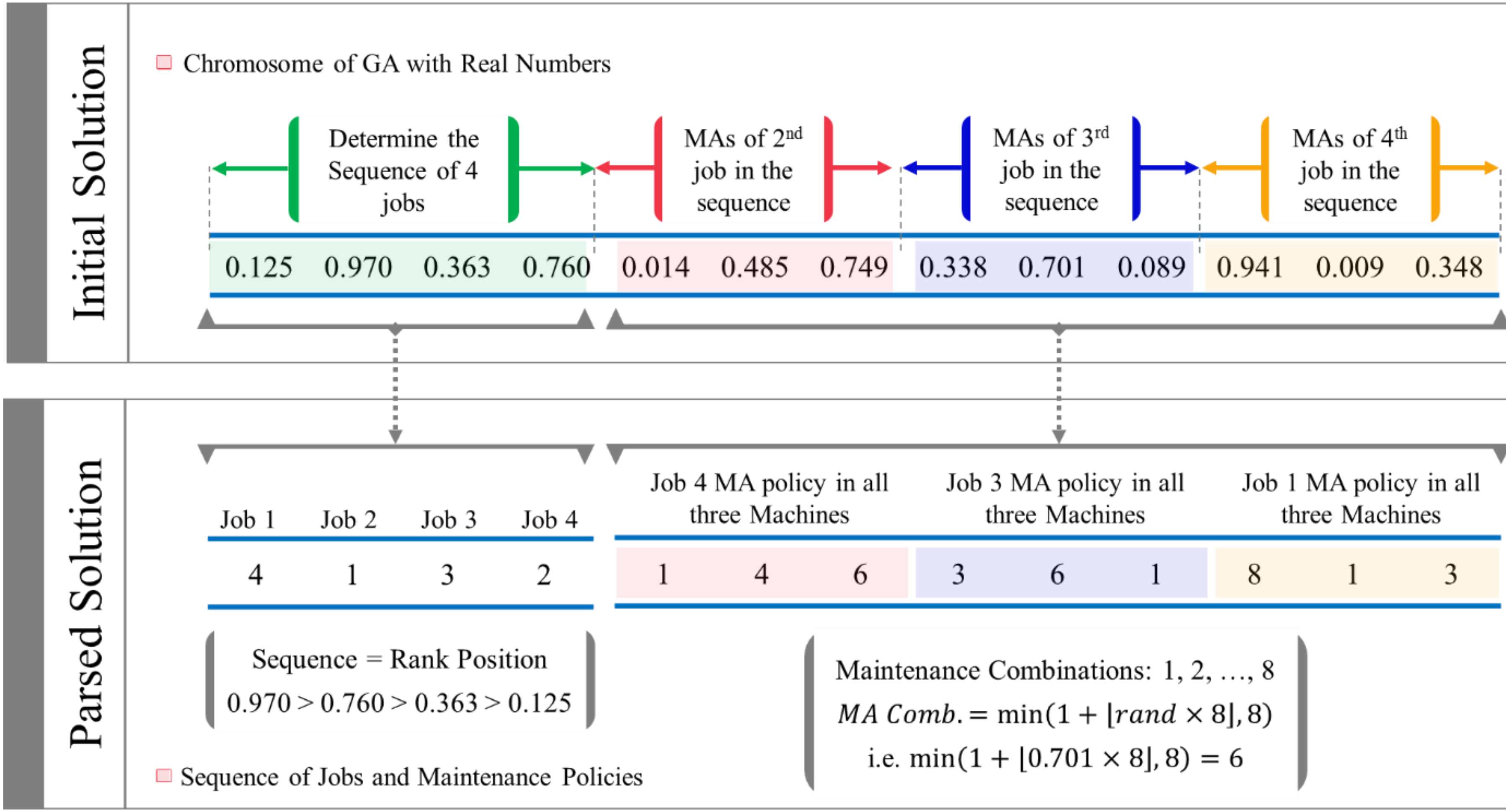
Complementary Slides: GA



Complementary Slides: Combining maintenance activities

Combination	y_1	y_2	y_3	Number of MAs	Nominal Duration	Actual Duration
1	0	0	0	0	0	$e_1' = 0$
2	1	0	0	1	e_1	$e_2' = e_1$
3	0	1	0	1	e_2	$e_3' = e_2$
4	0	0	1	1	e_3	$e_4' = e_3$
5	1	1	0	2	$e_1 + e_2$	$e_5' = 0.85(e_1 + e_2)$
6	1	0	1	2	$e_1 + e_3$	$e_6' = 0.85(e_1 + e_3)$
7	0	1	1	2	$e_2 + e_3$	$e_7' = 0.85(e_2 + e_3)$
8	1	1	1	3	$e_1 + e_2 + e_3$	$e_8' = 0.75(e_1 + e_2 + e_3)$

$$\sum_{r=1}^{o=2^l-1} b_r \phi_r = \sum_{k=1}^{l} a_k y_k \,, \qquad \phi_r, y_k \in \{0,1\}, \forall k = 1, \dots, l$$

Complementary Slides: Combining maintenance activities

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1	0	0	0	0	0	$e_1' = 0$
2	1	0	0	1	e_1	$e_2' = e_1$
3	0	1	0	1	e_2	$e_3' = e_2$
4	0	0	1	1	e_3	$e_4' = e_3$
5	1	1	0	2	$e_1 + e_2$	$e_5' = 0.85(e_1 + e_2)$
6	1	0	1	2	$e_1 + e_3$	$e_6' = 0.85(e_1 + e_3)$
7	0	1	1	2	$e_2 + e_3$	$e_7' = 0.85(e_2 + e_3)$
8	1	1	1	3	$e_1 + e_2 + e_3$	$e_8' = 0.75(e_1 + e_2 + e_3)$

$$\sum_{r=1}^{o=2^l-1} b_r \phi_r = \sum_{k=1}^{l} a_k y_k \,, \qquad \phi_r, y_k \in \{0,1\}, \forall k = 1, \ldots, l$$

Complementary Slides: Increased Processing Times

$$\rho = \begin{cases} \lambda^1 p, & A < \mathrm{F} \leq 1 \\ \lambda^2 p, & B < \mathrm{F} \leq A \\ \lambda^3 p, & 0 < \mathrm{F} \leq B \end{cases}, \qquad 0 \leq B \leq A \leq 1 \leq \lambda^1 \leq \lambda^2 \leq \lambda^3$$

$$\Lambda_{iq}^{1s} = \begin{cases} 1, & \sum_{k=1}^{l} \frac{r_{iq}^{ks}}{l \cdot R_{i,k}} > 0.66 \\ 0, & otherwise \end{cases}, \qquad \forall s, i, q$$

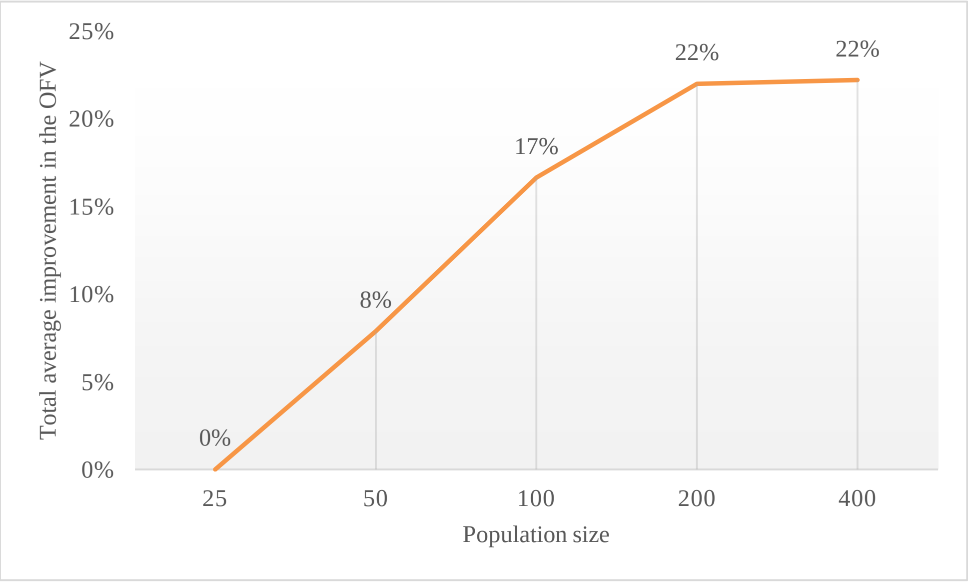
$$\Lambda_{iq}^{2s} = \begin{cases} 1, & 0.33 \leq \sum_{k=1}^{l} \frac{r_{iq}^{ks}}{l \cdot R_{i,k}} \leq 0.66 \\ 0, & otherwise \end{cases}, \qquad \forall s, i, q$$

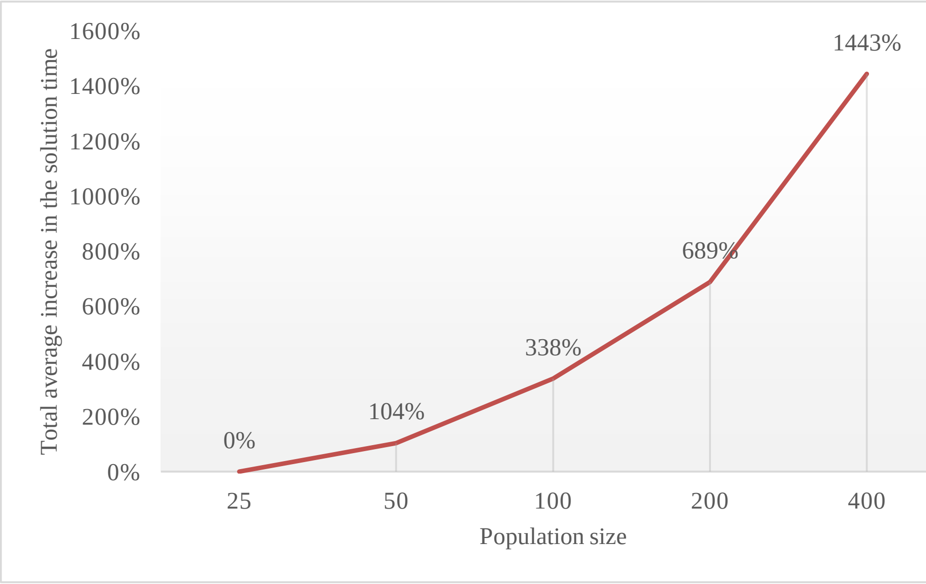
$$\Lambda_{iq}^{3s} = \begin{cases} 1, & \sum_{k=1}^{l} \frac{r_{iq}^{ks}}{l \cdot R_{i,k}} < 0.33 \\ 0, & otherwise \end{cases}, \qquad \forall s, i, q$$

Complementary Slides: Population Size

Number of Jobs (n)	Population Size	Average Improvement in the OFV (Cost)	Average Increase in the Solution Time
4	25	0%	0%
	50	7%	77%
	100	15%	255%
	200	22%	546%
	400	27%	1215%
6	25	0%	0%
	50	6%	104%
	100	18%	399%
	200	22%	756%
	400	17%	1394%
8	25	0%	0%
	50	10%	131%
	100	17%	359%
	200	22%	764%
	400	23%	1720%

Complementary Slides: Population Size





Performance Evaluation, n=4

Test Problem	CPLEX (SMIP)		Simulation-Optimization (SO)			Gap	
	OFV	Time (sec.)	OFV	Time (sec.)	Iterations	OFV	Time
1	6210.44	96	6263.10	414	45	0.85%	332.04%
2	6534.53	111	6938.77	443	47	6.19%	298.53%
3	5556.52	121	5577.18	504	55	0.37%	315.17%
4	6598.37	98	7166.55	450	48	8.61%	357.97%
5	5710.22	113	5710.27	455	49	0.00%	302.32%
6	5211.41	89	5360.73	550	60	2.87%	521.18%
7	5604.63	136	5604.68	332	36	0.00%	144.24%
…							
27	6122.64	103	6664.98	411	44	8.86%	300.46%
28	5452.83	124	5452.88	586	64	0.00%	374.55%
29	7393.03	145	7734.09	441	48	4.61%	203.53%
30	6329.74	117	6329.80	601	67	0.00%	412.40%
Average	**6148.92**	**120**	**6465.91**	**485**	**52**	**5.23%**	**318.67%**
Minimum	**5079.85**	**85**	**5079.90**	**332**	**36**	**0.00%**	**138.78%**
Maximum	**7393.03**	**180**	**7932.77**	**751**	**84**	**23.70%**	**559.98%**

Performance Evaluation, n>4

N of Jobs (n)	CPLEX Avg. Time	SO Avg. Time	Avg. N of Iter.	Gap in the Solution Time			Gap in the OFV		
				Avg.	Min.	Max.	Avg.	Min.	Max.
4	120	485	52	318.67%	138.78%	559.98%	5.23%	0.00%	23.70%
5	2383	777	68	-63.78%	-79.81%	-21.89%	9.43%	0.00%	32.46%
6	3002	1091	79	-63.66%	-78.43%	-53.80%	0.99%	-15.18%	21.19%
7	3001	1430	88	-52.34%	-73.58%	-44.63%	-10.57%	-34.84%	17.08%
8	3001	1828	98	-39.09%	-52.89%	-35.11%	-23.38%	-43.39%	-6.48%
9	3000	2081	98	-30.63%	-44.06%	-25.03%	-26.64%	-42.29%	-8.22%

Performance Evaluation, n=10

- 5/30 problems unsolved by Gurobi under limited time
- SO always had a better solution
  - Min. -45.94%
  - Max. -6.18%
  - Avg. -26.34%

Complementary Slides: Sensitivity to the input data

Setting	$DDTF$	MIF	No. of Problems Solved	Gap in the OFV		
				Avg.	Min.	Max.
1	3	50	30	10.42%	0.00%	26.63%
2	4	50	30	9.41%	0.00%	32.29%
3	5	50	30	11.82%	0.29%	30.77%
4	4	40	30	15.39%	0.49%	32.40%
5	4	50	30	11.79%	0.80%	53.47%
6	4	60	30	11.92%	0.00%	37.32%

Complementary Slides: Sensitivity to the input data

Setting	DDTF	MIF	The solution time of CPLEX				The solution time of the SO algorithm			
			Avg. (sec.)	Min. (sec.)	Max. (sec.)	Variance	Avg. (sec.)	Min. (sec.)	Max. (sec.)	Variance
1	4	40	74	6	259	4359	576	317	1013	26188
2	4	50	198	43	419	8264	551	323	759	11790
3	4	60	204	96	426	5991	510	229	670	7920
4	3	40	301	42	1179	54610	566	362	832	12494
5	4	40	207	31	396	6723	526	267	728	12320
6	5	40	200	48	464	9320	525	353	722	8576
Standard Deviation			**66**	**27**	**300**	**17839**	**24**	**47**	**112**	**6072**

Complementary Slides: Maintenance Schedule

Machine	10	50	100	250	500	1000
Excavators	✓	✓	✓	✓	✓	✓
Wheel Loaders	✓	✓	✓	✓	✓	✓
(Haul) Trucks	✓	✓			✓	✓

Task No.	Maintenance Activity Interval (hours)		
	250	500	1000
1	Air Conditioner - Test	**Axle Oil (Front) - Change**	**Axle Oil (Front) - Change**
2	Axle Bearings (Front) - Lubricate	**Axle Oil (Rear) - Change**	**Axle Oil (Rear) - Change**
3	Axle Oil Level (Front) - Check	**Final Drive Oil - Change**	Battery Hold-Down - Tighten
4	Axle Oil Level (Rear) - Check	**Transmission Oil - Change**	Drum Brakes - Inspect
5	Braking System - Test	Drive Shaft Support Bearing Lubricant - Check	**Final Drive Oil - Change**
6	Condenser (Refrigerant) - Clean	Fuel System Priming Pump - Operate	Overhead Guard - Inspect
7	Cooling System Hoses - Inspect	Fuel System Secondary Filter - Replace	**Transmission Oil - Change**
8	Engine Oil and Filter - Change	Fuel Tank Cap and Strainer - Clean	
9	Final Drive Oil Level - Check	Fuel System Primary Filter/Water Separator-	
10	Swing Bearing - Lubricate	Element - Replace	
11	Transmission Oil Level - Check		
12	V-Belts - Inspect/Adjust/Replace		

Complementary Slides: Case Study

Task	Steps
Transmission Oil Level - Check	**1. Remove filler plug (1).** 2. Check the lubricant level. The lubricant level should be at the bottom of the opening for filler plug (1). 3. If necessary, fill the gearbox with lubricant to the bottom of the opening for filler plug (1). **4. Clean filler plug (1).** **5. Inspect the O-ring seal. If damage or wear is noticed on the O-ring seal, replace the seal.** **6. Install filler plug (1).**
Transmission Oil - Change	1. Remove the dirt that is around filler plug (1) and around drain plug (2). 2. Remove drain plug (2). Drain the lubricant into a suitable container. 3. Clean drain plug (2). 4. Inspect the O-ring seal. If damage or wear is noticed on the O-ring seal, replace the seal. 5. Install drain plug (2). **6. Remove filler plug (1).** 7. Fill the gearbox with lubricant to the bottom of the filler plug opening. **8. Clean filler plug (1).** **9. Inspect the O-ring seal. If damage or wear is noticed on the O-ring seal, replace the seal.** **10. Install filler plug (1).**

Complementary Slides: SMIP

$$\text{minimize } E[Z_s] = \sum_{s=1}^{S} \Pr(s) \left[\sum_{j=1}^{n} \sum_{q=1}^{n} \Pi_{jq}^{s} + \sum_{i=1}^{m} \sum_{q=1}^{n} \sum_{r=1}^{o=2^{l}-1} \phi_{iqr} (SP'_{ir} + e'^{s}_{ir} WF) \right], \tag{3}$$

Subject to:

$$\sum_{q=1}^{n} x_{jq} = 1, \qquad j = 1, \ldots, n \tag{4}$$

$$\sum_{j=1}^{n} x_{jq} = 1, \qquad q = 1, \ldots, n \tag{5}$$

$$ST_{11}^{s} = 0, \qquad s = 1, \ldots, S \tag{6}$$

$$ST_{i1}^{s} = \sum_{i'=1}^{i-1} \rho_{i'1}^{s}, \qquad i = 2, \ldots, m, s = 1, \ldots, S \tag{7}$$

$$ST_{1q}^{s} = FT_{1(q-1)}^{s} + \sum_{r=1}^{o} \phi_{1qr} e'^{s}_{1r}, \qquad q = 2, \ldots, n, s = 1, \ldots, S \tag{8}$$

$$ST_{iq}^{s} \geq FT_{i(q-1)}^{s} + \sum_{r=1}^{o} \phi_{iqr} e'^{s}_{1r}, \qquad i = 2, \ldots, m, q = 2, \ldots, n, s = 1, \ldots, S \tag{9}$$

$$ST_{iq}^{s} \geq FT_{(i-1)q}^{s}, \qquad i = 2, \ldots, m, q = 2, \ldots, n, s = 1, \ldots, S \tag{10}$$

$$FT_{iq}^{s} = ST_{iq}^{s} + \rho_{iq}^{s}, \qquad i = 1, \ldots, m, q = 1, \ldots, n, s = 1, \ldots, S \tag{11}$$

$$r_{i1}^{ks} = R_{i,k}, \qquad i = 1, \ldots, m\ , s = 1, \ldots, S, k = 1, \ldots, l \tag{12}$$

$$r_{iq}^{ks} \geq \sum_{j=1}^{n} \gamma_{ijq}^{s}, \qquad s = 1, \ldots, S, i = 1, \ldots, m, q = 1, \ldots, n, k = 1, \ldots, l \tag{13}$$

Complementary Slides: SMIP

$$r_{iq}^{ks} \geq r_{i(q-1)}^{ks} - \sum_{j=1}^{n} \gamma_{ij(q-1)}^{s} - y_{iqk}K, \qquad s = 1,\dots,S, i = 1,\dots,m, q = 2,\dots,n, k = 1,\dots,l \quad (14)$$

$$r_{iq}^{ks} \leq r_{i(q-1)}^{ks} - \sum_{j=1}^{n} \gamma_{ij(q-1)}^{s} + y_{iqk}K, \qquad s = 1,\dots,S, i = 1,\dots,m, q = 2,\dots,n, k = 1,\dots,l \quad (15)$$

$$r_{iq}^{ks} \geq R_{i,k} - K(1 - y_{iqk}), \qquad s = 1,\dots,S, i = 1,\dots,m, q = 2,\dots,n, k = 1,\dots,l \quad (16)$$

$$r_{iq}^{ks} \leq R_{i,k} + K(1 - y_{iqk}), \qquad s = 1,\dots,S, i = 1,\dots,m, q = 2,\dots,n, k = 1,\dots,l \quad (17)$$

$$t_q^s \geq FT_{mq}^s - \sum_{j=1}^{n} x_{jq} d_j, \qquad s = 1,\dots,S, q = 1,\dots,n \quad (18)$$

$$\Pi_{jq}^s - \pi_j t_q^s \geq -K(1 - x_{jq}), \qquad s = 1,\dots,S, j = 1,\dots,n, q = 1,\dots,n \quad (19)$$

$$\Pi_{jq}^s - \pi_j t_q^s \leq K(1 - x_{jq}), \qquad s = 1,\dots,S, j = 1,\dots,n, q = 1,\dots,n \quad (20)$$

$$\Pi_{jq}^s \geq -K x_{jq}, \qquad s = 1,\dots,S, j = 1,\dots,n, q = 1,\dots,n \quad (21)$$

$$\Pi_{jq}^s \leq K x_{jq}, \qquad s = 1,\dots,S, j = 1,\dots,n, q = 1,\dots,n \quad (22)$$

$$\rho_{iq}^s = \sum_{j=1}^{n} \sum_{h=1}^{H} u_{ijq}^{hs} \lambda^h p_{ij}, \qquad s = 1,\dots,S, i = 1,\dots,m, q = 1,\dots,n \quad (23)$$

$$u_{ijq}^{hs} \leq \Lambda_{iq}^{hs}, \qquad s = 1,\dots,S, i = 1,\dots,m, j = 1,\dots,n, q = 1,\dots,n, h = 1,\dots,H \quad (24)$$

$$u_{ijq}^{hs} \leq x_{jq}, \qquad s = 1,\dots,S, i = 1,\dots,m, j = 1,\dots,n, q = 1,\dots,n, h = 1,\dots,H \quad (25)$$

$$u_{ijq}^{hs} \geq x_{jq} + \Lambda_{iq}^{hs} - 1, \qquad s = 1,\dots,S, i = 1,\dots,m, j = 1,\dots,n, q = 1,\dots,n, h = 1,\dots,H \quad (26)$$

Complementary Slides: SMIP

$$\sum_{h=1}^{H} \Lambda_{iq}^{hs} = 1, \qquad s = 1,\dots,S, i = 1,\dots,m, q = 1,\dots,n \tag{27}$$

$$\sum_{k=1}^{l} \frac{r_{iq}^{ks}}{l \cdot R_{i,k}} > \frac{H-1}{l} \Lambda_{iq}^{1s} - K \sum_{h=2}^{H} \Lambda_{iq}^{hs}, \qquad s = 1,\dots,S, i = 1,\dots,m, q = 1,\dots,n \tag{28}$$

$$\sum_{k=1}^{l} \frac{r_{iq}^{ks}}{l \cdot R_{i,k}} \le \frac{h}{l} \Lambda_{iq}^{hs} + K \left(\sum_{h'=1}^{H} \Lambda_{iq}^{h's} - \Lambda_{iq}^{hs} \right), \qquad h = 2,\dots,H-1, s = 1,\dots,S, i = 1,\dots,m, q = 1,\dots,n \tag{29}$$

$$\sum_{k=1}^{l} \frac{r_{iq}^{ks}}{l \cdot R_{i,k}} \ge \frac{h-1}{l} \Lambda_{iq}^{hs} - K \left(\sum_{h'=1}^{H} \Lambda_{iq}^{h's} - \Lambda_{iq}^{hs} \right), \qquad h = 2,\dots,H-1, s = 1,\dots,S, i = 1,\dots,m, q = 1,\dots,n \tag{30}$$

$$\sum_{k=1}^{l} \frac{r_{iq}^{ks}}{l \cdot R_{i,k}} < \frac{1}{l} \Lambda_{iq}^{Hs} + K \sum_{h=1}^{H-1} \Lambda_{iq}^{hs}, \qquad s = 1,\dots,S, i = 1,\dots,m, q = 1,\dots,n \tag{31}$$

$$\gamma_{ijq}^{s} \le x_{jq} K, \qquad s = 1,\dots,S, i = 1,\dots,m, j = 1,\dots,n, q = 1,\dots,n \tag{32}$$

$$\gamma_{ijq}^{s} \le \rho_{iq}^{s}, \qquad s = 1,\dots,S, i = 1,\dots,m, j = 1,\dots,n, q = 1,\dots,n \tag{33}$$

$$\gamma_{ijq}^{s} \ge \rho_{iq}^{s} + (x_{jq} - 1)K, \qquad s = 1,\dots,S, i = 1,\dots,m, j = 1,\dots,n, q = 1,\dots,n \tag{34}$$

$$\sum\nolimits_{r=1}^{o} b_r \phi_{iqr} = \sum_{k=1}^{l} a_k y_{iqk}\,, \qquad i = 1,\dots,m, q = 1,\dots,n \tag{35}$$

$$x_{jq}, y_{iqk}, \Lambda_{iq}^{h}, u_{ijq}^{hs}, \phi_{iqr} \in \{0,1\}, \qquad s = 1,\dots,S, j = 1,\dots,n, q = 1,\dots,n, i = 1,\dots,m, h = 1,\dots,H \tag{36}$$

$$r_{iq}^{ks}, t_q^s, \Pi_{jq}^s, ST_{iq}^s, FT_{iq}^s, \rho_{iq}^s, \gamma_{ijq}^s \ge 0, \qquad s = 1,\dots,S, j = 1,\dots,n, q = 1,\dots,n, i = 1,\dots,m, h = 1,\dots,H \tag{37}$$