Application of Actuarial Science to RAM

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Actuarial Science

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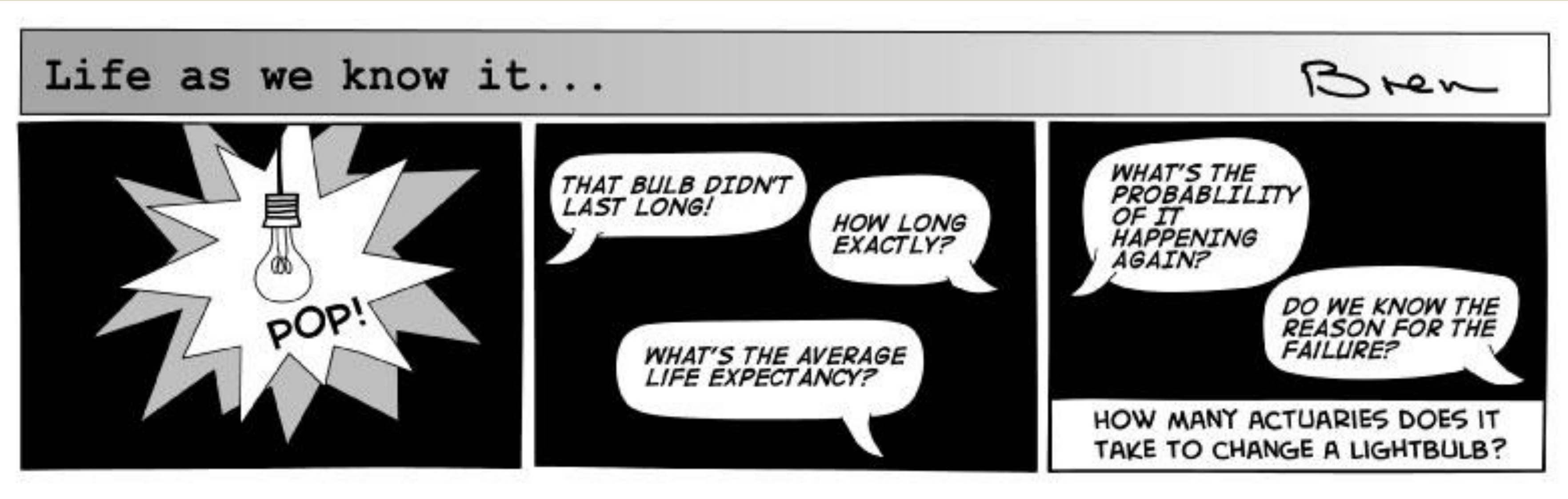
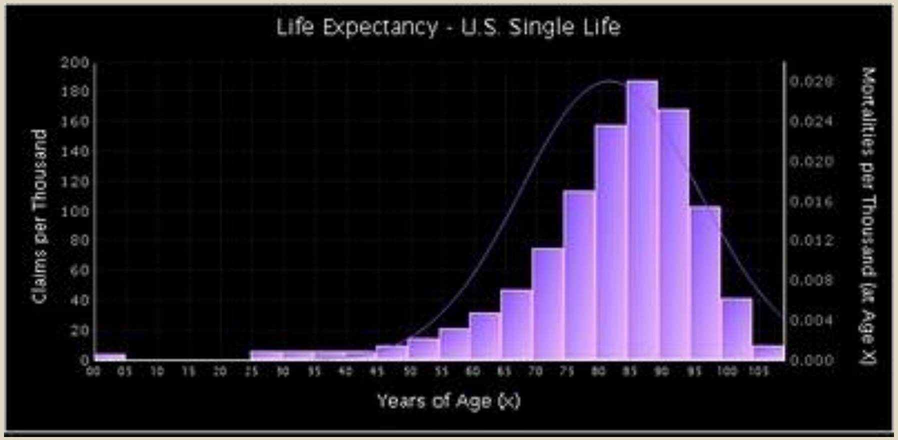
What is Actuarial Science?



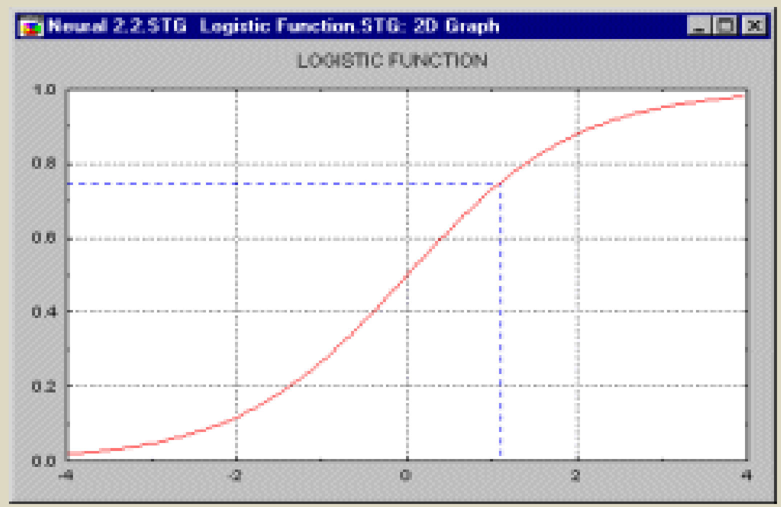
What is Actuarial Science?

- Actuarial Science – the discipline that applies mathematical and statistical methods to model and to assess risks, usually in insurance and finance.



What are Some Typical Actuarial Models?

- Occurrence Models – probability models that regress binomial occurrence versus transformed explanatory variables, i.e. Logit or Probit.
  - Logit, $p_1(x) = \dfrac{1}{1+e^{(-z'\theta)}} = \dfrac{1}{1+e^{(-\Sigma_{k=1}^{K} z_k \theta_k)}} = \text{logit}(z'\theta)$
  - Probit, $p_1(x) = \Phi(z'\theta)$

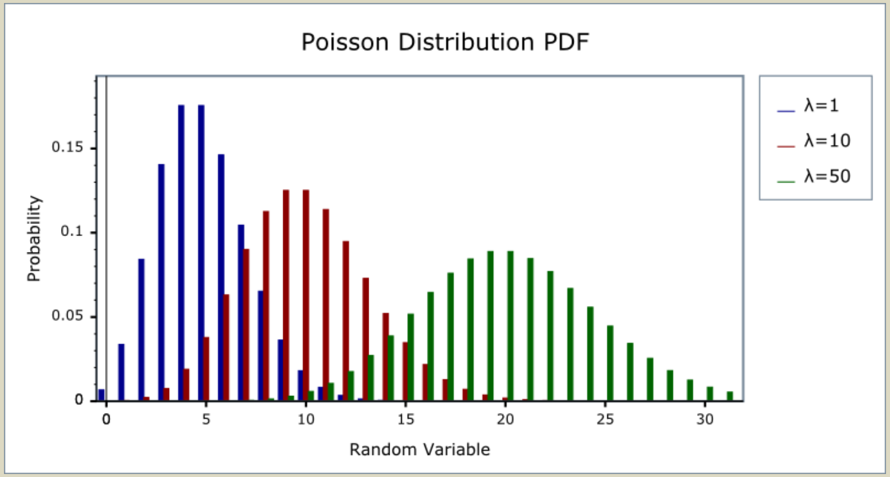


What are Some Typical Actuarial Models?

- Frequency Models – discrete count models that regress counts versus explanatory variables, i.e. Poisson.
  - Poisson Model for Counts

$$P[Y = y] = e^{(-\lambda)} \frac{\lambda^y}{y!}, \quad y = 0, 1, 2, \ldots$$

$$\lambda_i = e^{(z'_i \theta)}$$



What are Some Typical Actuarial Models?

- Severity/Duration Models – continuous models that regress dollar amounts or time versus explanatory variables, i.e. Pareto, Weibull, Exponential.
  - Exponential Model for Duration

$$\lambda(y) = \lambda, \qquad y \in \mathbb{R}^+$$

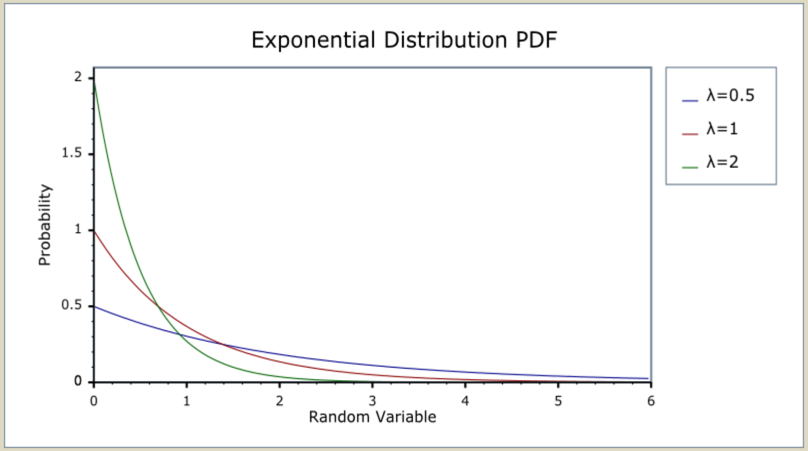
$$\lambda_i = \exp(Z_i'\theta),$$

$$S(y) = \exp(-\lambda y), \qquad f(y) = \lambda \exp(-\lambda y)$$

The mean and standard deviation of an exponential distribution are equal

$$E(Y) = \frac{1}{\lambda} \qquad V(Y) = \frac{1}{\lambda}$$

The expected residual lifetime is constant, equal to $\frac{1}{\lambda}$

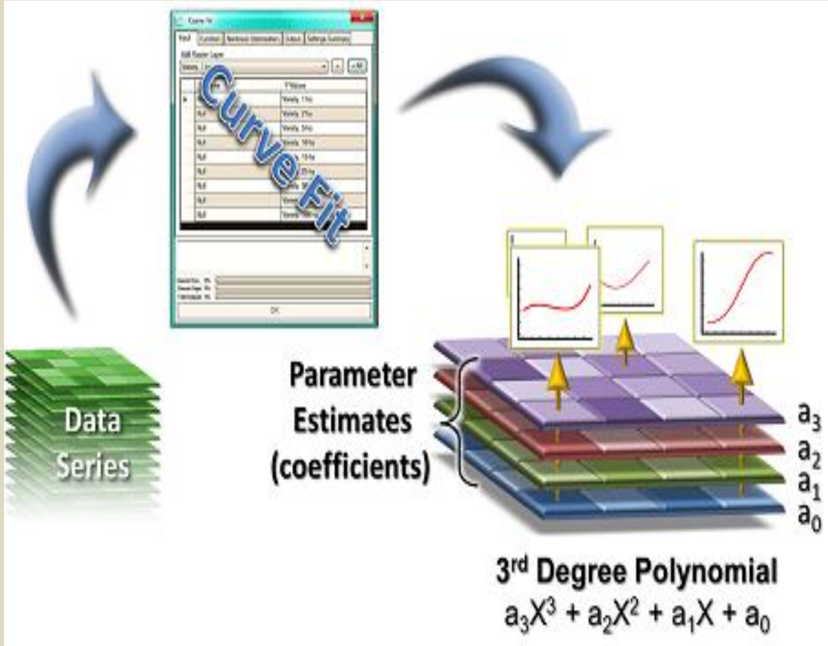


What are Some Typical Actuarial Tools?

Period Life Table, 2007						
	Male			Female		
Exact age	Death probability [a]	Number of lives [b]	Life expectancy	Death probability [a]	Number of lives [b]	Life expectancy
0	0.007379	100,000	75.38	0.006096	100,000	80.43
1	0.000494	99,262	74.94	0.000434	99,390	79.92
2	0.000317	99,213	73.98	0.000256	99,347	78.95
3	0.000241	99,182	73.00	0.000192	99,322	77.97
4	0.000200	99,158	72.02	0.000148	99,303	76.99
5	0.000179	99,138	71.03	0.000136	99,288	76.00
6	0.000166	99,120	70.04	0.000128	99,275	75.01
7	0.000152	99,104	69.05	0.000122	99,262	74.02
8	0.000133	99,089	68.06	0.000115	99,250	73.03
9	0.000108	99,075	67.07	0.000106	99,238	72.04
10	0.000089	99,065	66.08	0.000100	99,228	71.04
11	0.000094	99,056	65.09	0.000102	99,218	70.05
12	0.000145	99,047	64.09	0.000120	99,208	69.06
13	0.000252	99,032	63.10	0.000157	99,196	68.07
14	0.000401	99,007	62.12	0.000209	99,180	67.08
15	0.000563	98,968	61.14	0.000267	99,160	66.09
16	0.000719	98,912	60.18	0.000323	99,133	65.11
17	0.000873	98,841	59.22	0.000369	99,101	64.13
18	0.001017	98,754	58.27	0.000401	99,064	63.15
19	0.001148	98,654	57.33	0.000422	99,025	62.18
20	0.001285	98,541	56.40	0.000441	98,983	61.20

Predictive Analytics for BIG DATA

Regression Based



Project Data

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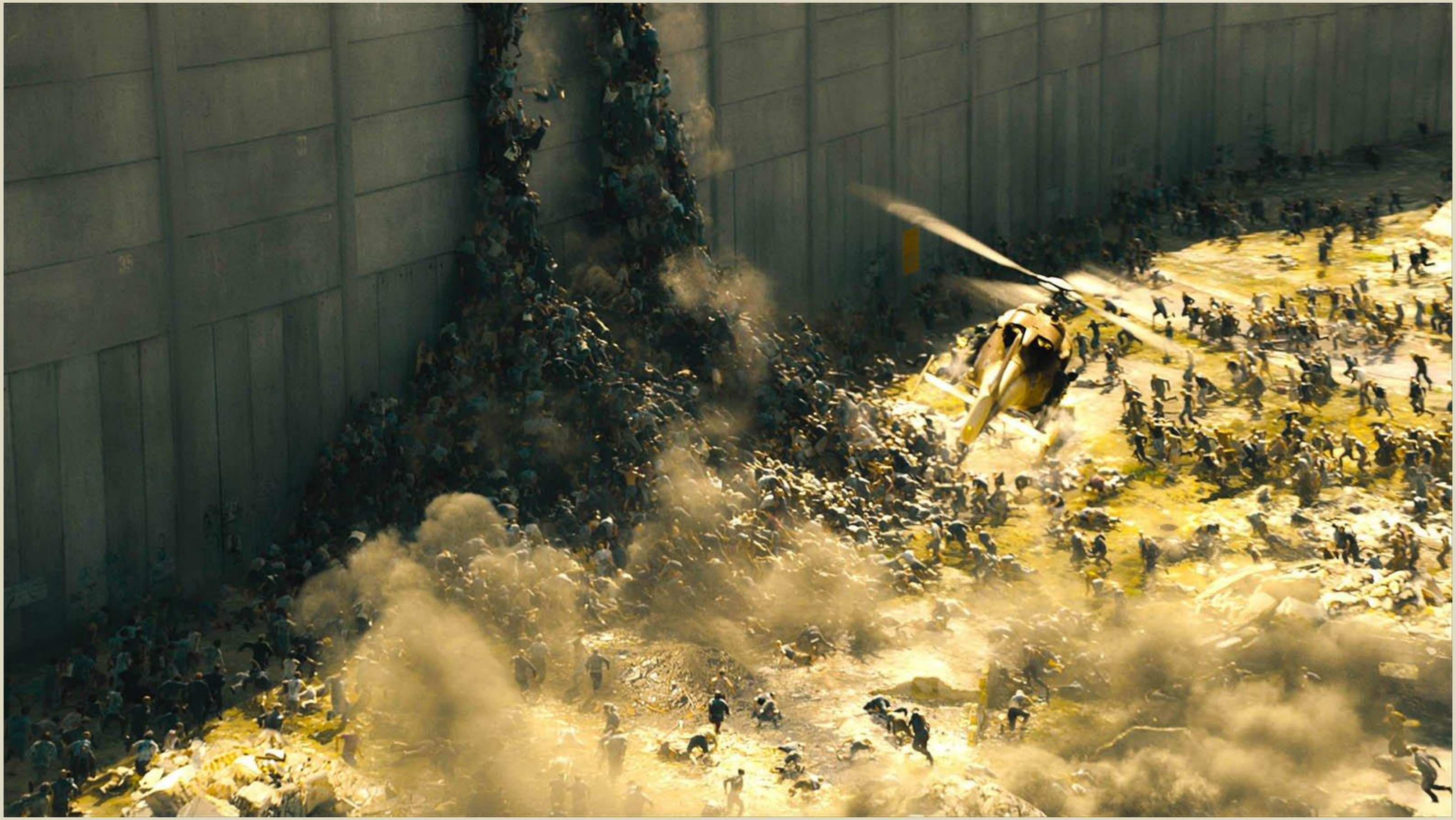
Can Actuarial Science be Applied to RAM?

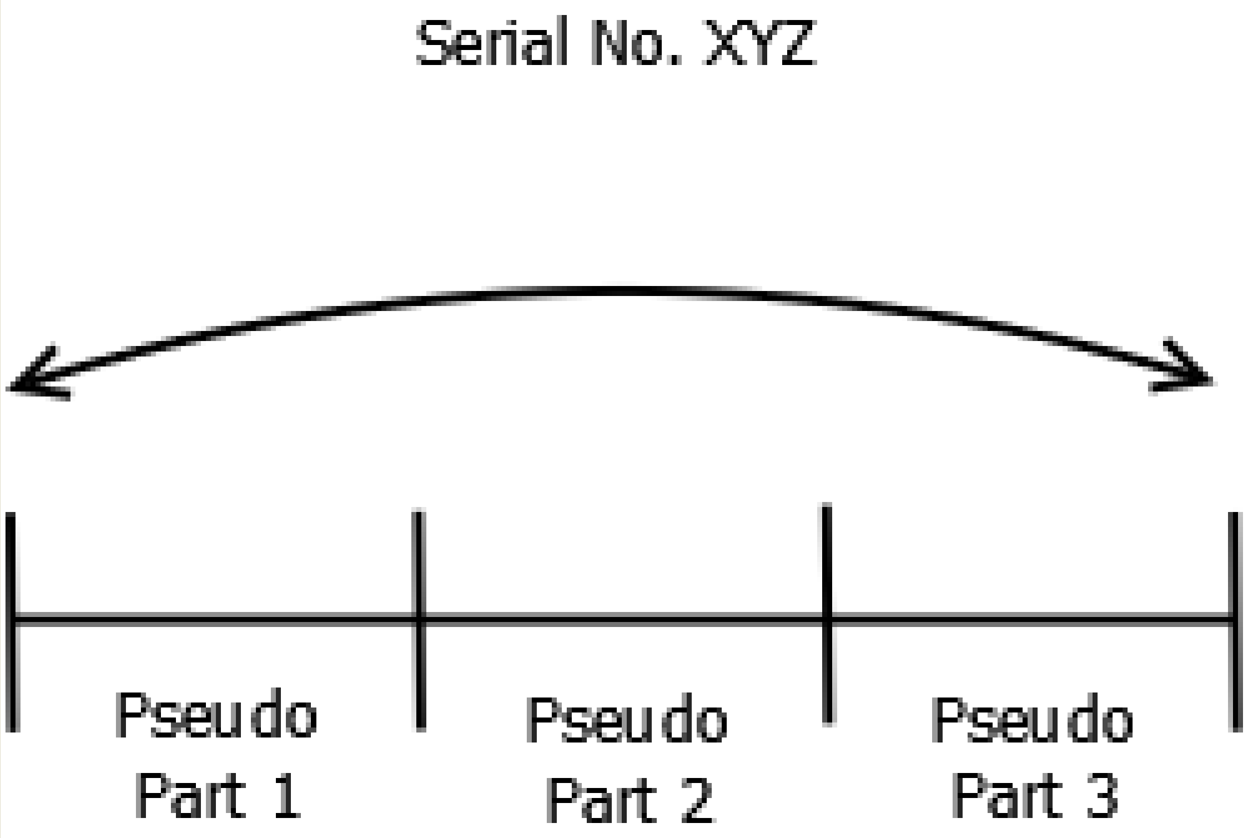
- In early 2011, AMRDEC asked if Actuarial Science can be applied to enhance RAM initiatives.
- On the surface, the answer seems to be obvious.
  - Manipulate BIG DATA.
  - Create life table for parts.
  - Provide predictive analytics based on external environment factors.
- But…

Challenges

- Part Life vs. Human Life (People Don't Come Back)



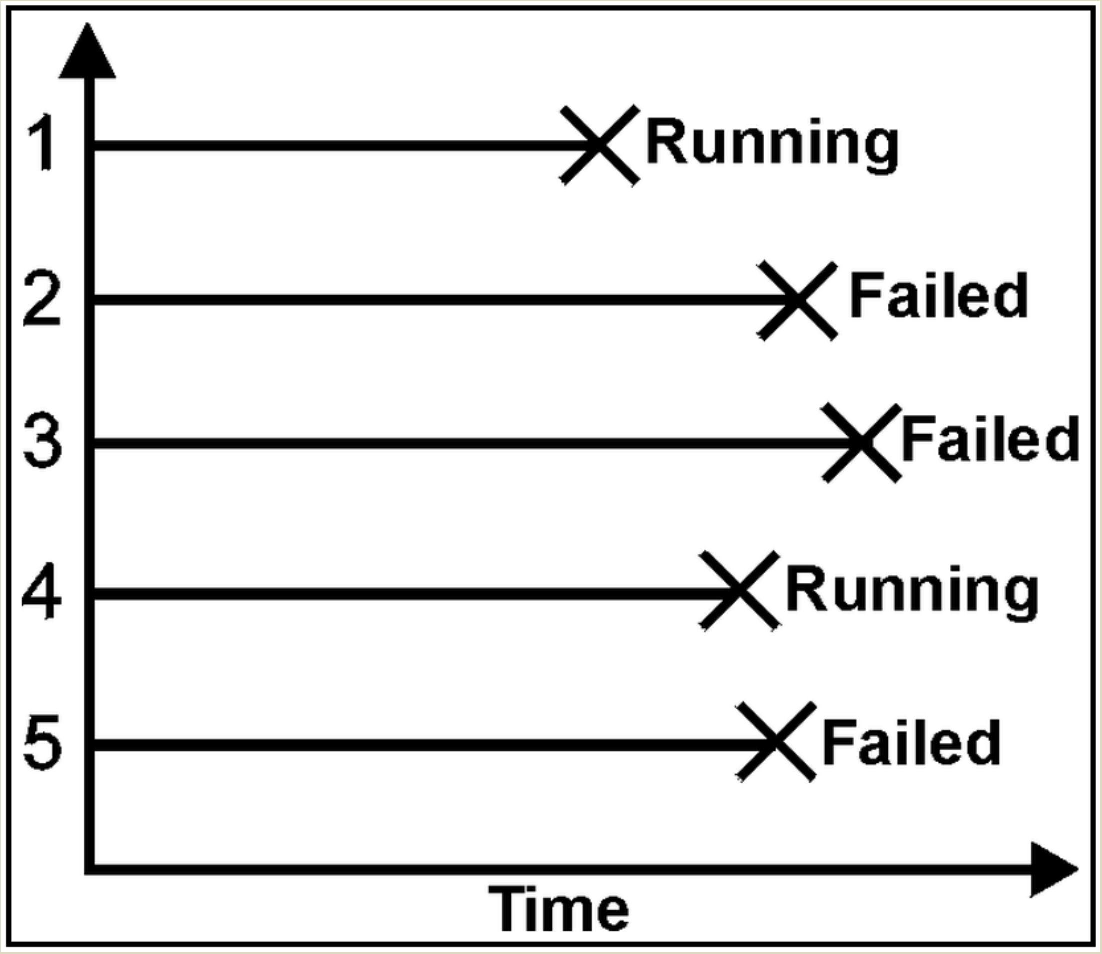
Pseudo Parts divided by failure



Data Sets Used

- 2410 Database
  - Copy 1,2,3
  - Flight Hours, ***TailNo***, Model, ***Dates***, UIC, Time Since New, Time Since Last Install, Overhaul, Time Since Overhaul, etc.
- 1352 Database
  - ***TailNo***, ***Dates***, Hours by TailNo
- CBM HUMS database
  - ***TailNo***, ***Dates***, many environmental factors
- Intersection: TailNo & Date

Illustration of Survival-Type Data



Necessity to Account for Censored Data

Year	Number in Study at Beginning of Year	Number Died During Year	Number Withdrawn (Censored)
[0-1)	200	3	16
[1-2)	181	5	14
[2-3)	162	6	13
[3-4)	143	7	11
[4-5)	125	7	12
[5-6)	106	5	9
[6-7)	92	6	8
[7-8)	78	8	5
[8-9)	65	7	6
[9-10)	52	7	7
[10-11)	38	6	7
[11-12)	25	9	6
[12-13)	10	7	3

Incorrect Method #1

Say we were asked to calculate the probability that a patient would survive 5 years. One incorrect way to calculate this probability is to throw out the data from the withdrawn patients (censored).

$$P(X > 5) = 1 - P(X < 5) = 1 - \frac{28\ \mathit{Deaths\ in\ 5\ Years}}{200 - 117(\mathit{censored})}$$ = **<u>66.3</u>**% Chance to Survive 5 Years

Incorrect Method #2

Another incorrect way is use 200 as the initial population, but to assume all the censoring falls at the end of the study.

$$P(X > 5) = 1 - P(X < 5) = 1 - \frac{28\ Deaths\ in\ 5\ Years}{200\ in\ Study}$$ = **<u>86.0</u>**% Chance to Survive 5 Years

Too optimistic or pessimistic?

Equation 1 leads to an overly pessimistic survival probability, while equation 2 leads to an overly optimistic survival probability.

The true survival probability is somewhere between these two incorrect estimates. This shows that statistical survival analysis techniques are necessary.

Kaplan-Meier Estimator

The Kaplan–Meier estimator is the nonparametric maximum likelihood estimate of the survival function, *S*(*t*). The Kaplan Meier is different from the empirical distribution in that it can take into account censored data.

t_i	n_i	d_i	c_i
0	15	0	0
2	15	2	1
3	12	1	2
5	9	1	1
10	7	2	0

Kaplan-Meier Estimator : $$\hat{S}(t) = \prod_{t_i < t} \frac{n_i - d_i}{n_i}$$

Kaplan-Meier Estimator

Year	Number in Study at Beginning of Year	Number Died During Year	Number Withdrawn (Censored)	KM Survival Function
[0-1)	200	3	16	98.5%
[1-2)	181	5	14	95.8%
[2-3)	162	6	13	92.2%
[3-4)	143	7	11	87.7%
[4-5)	125	7	12	82.8%
[5-6)	106	5	9	78.9%
[6-7)	92	6	8	73.8%
[7-8)	78	8	5	66.2%
[8-9)	65	7	6	59.1%
[9-10)	52	7	7	51.1%
[10-11)	38	6	7	43.0%
[11-12)	25	9	6	27.5%
[12-13)	10	7	3	8.3%

What is censored?

A censored observation occurs when the failure condition is not met. For helicopter parts, are we talking about supply or reliability?

We determine censoring by chargeable vs. non-chargeable failure codes. Which failure codes are chargeable for supply, and for reliability?

Supply Failure Event

A failure from Supply's perspective is one that takes the part out of commission and requires repair. The non-chargeable failure codes with respect to supply failure are limited to those that represent no actual failure.

Non-Chargeable Examples:
FC 799 – Serviceable, no defect
FC 804 – Removed for scheduled maintenance

Reliability Failure Event

A failure from Reliability's perspective is one that due to inherent properties of the part, rather than environmental, combat, or misuse. The non-chargeable failure codes with respect to reliability failure include far more failure codes.

Non-Chargeable Examples:
FC 731: Battle Damage
FC 917: Bird Strike

R.E.A.R.M.

Repair Event Analysis and Recording Machine

REARM Engine

- Uses the R statistical programming tool
- Input Excel 2410 and 1352 database
- Automatically correct (some) errors in data
- Output a list of clean sequences with accompanying data

Parametric Distributions

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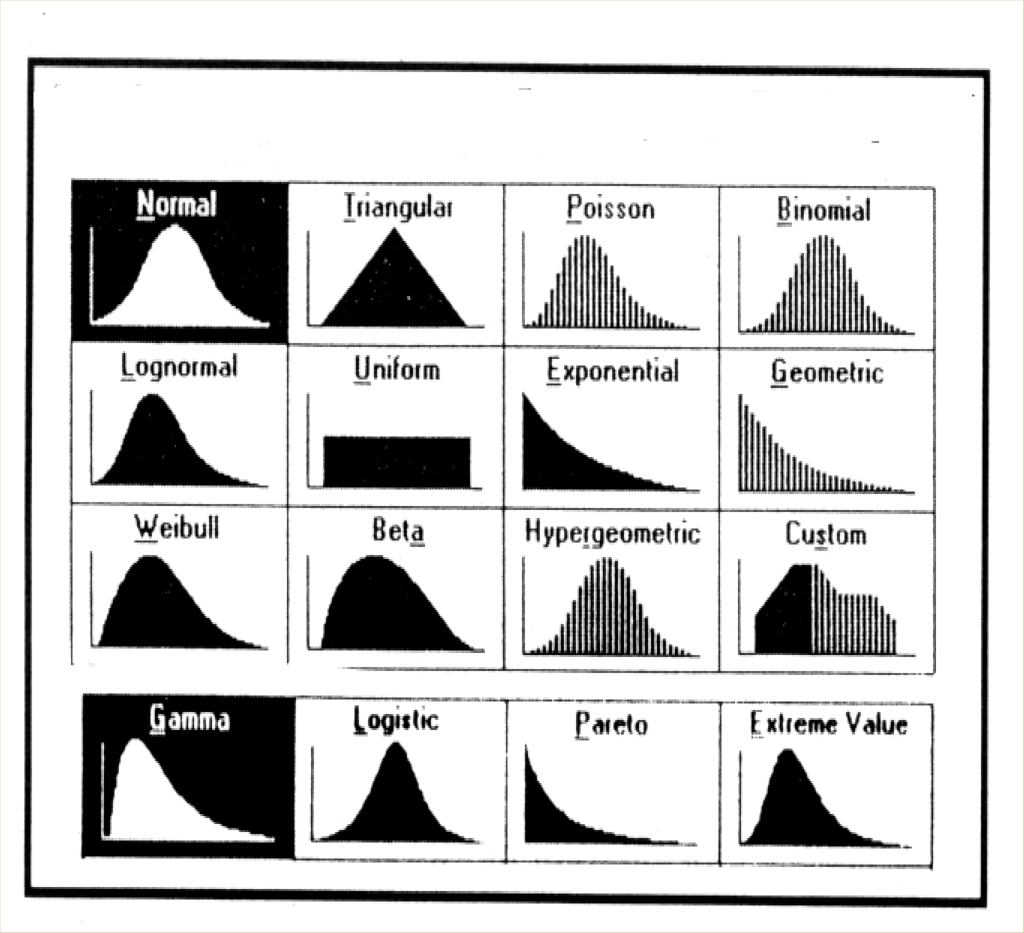
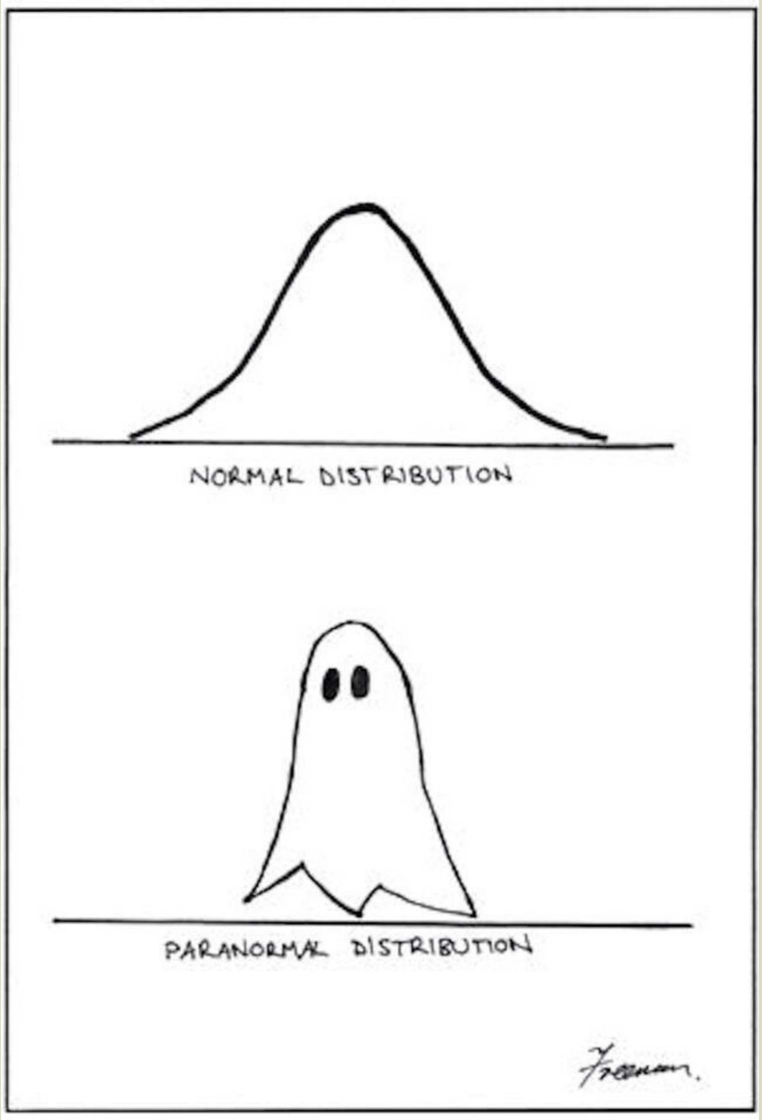
Possible Distributions

- Normal
- Lognormal
- Exponential
- Beta
- Weibull
- Gamma
- Negative Binomial
- Cauchy

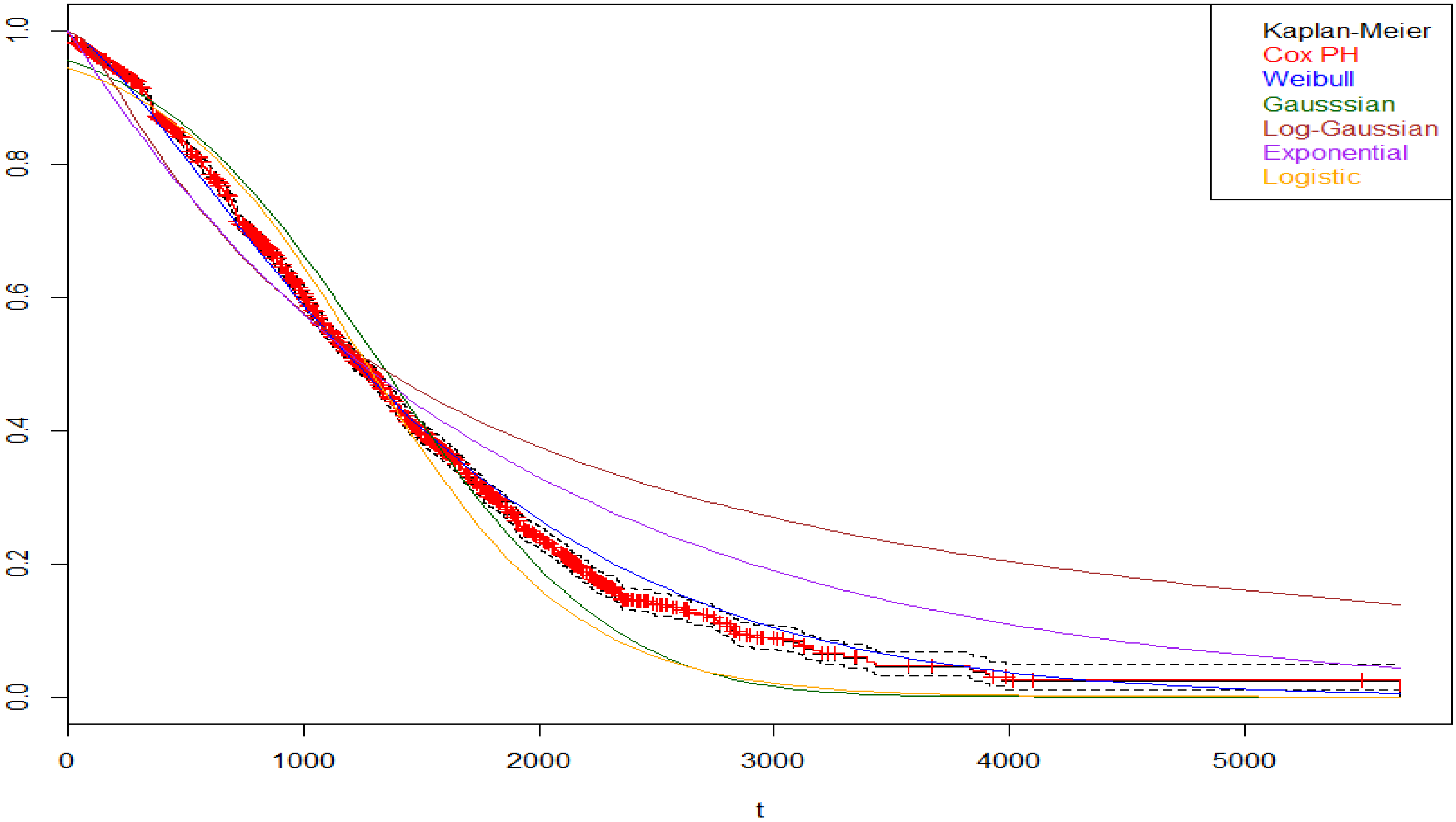


Possible Distributions



Survival Analysis Tools – KM Curves

Comparison of Parametric Survival Functions Zero-Truncated



Survival Regression

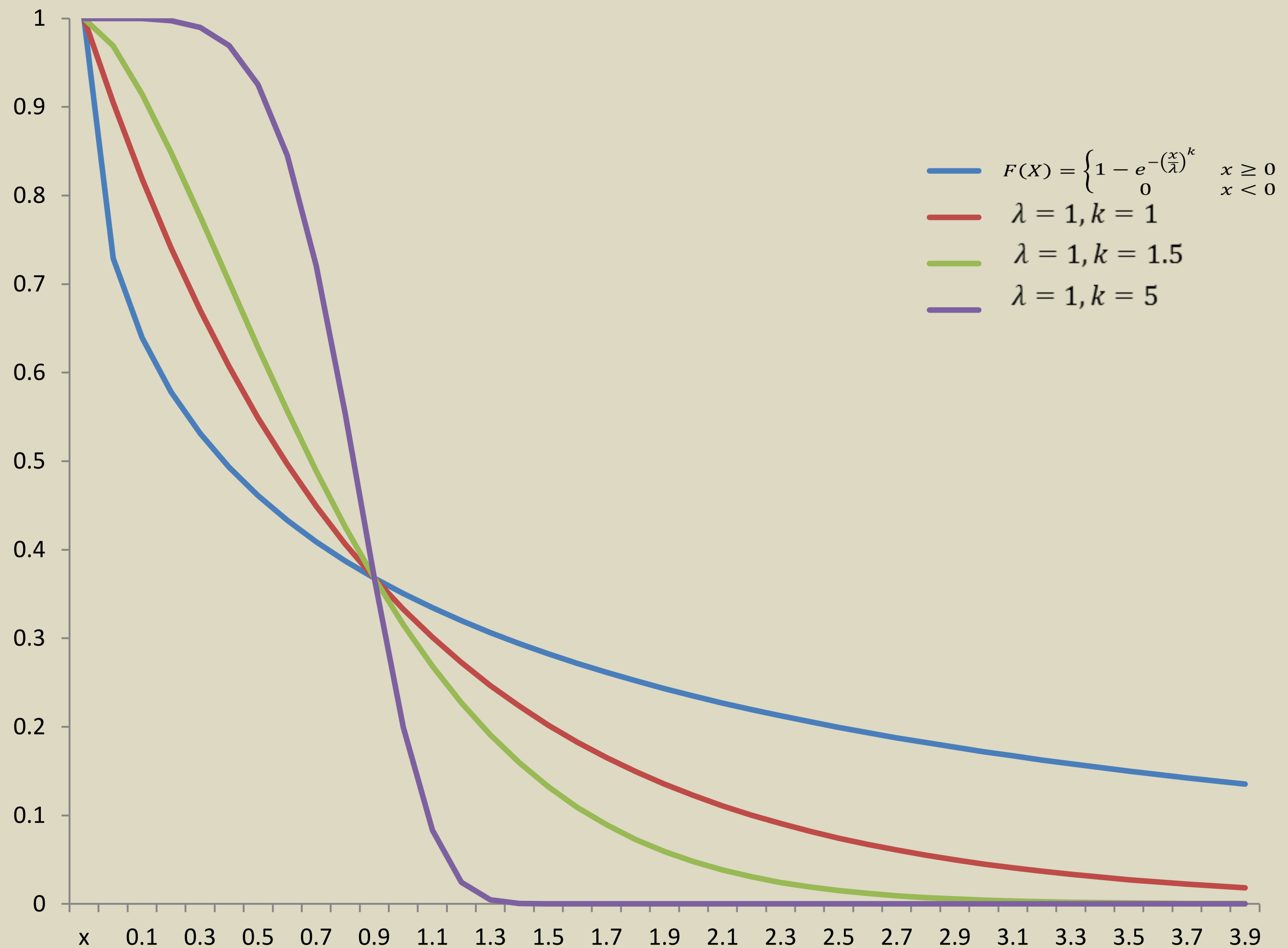
Survival Regression
Fits a parametric survival distribution to the model (Gaussian, Weibull, Logistic, etc.)

<u>Weibull CDF</u>

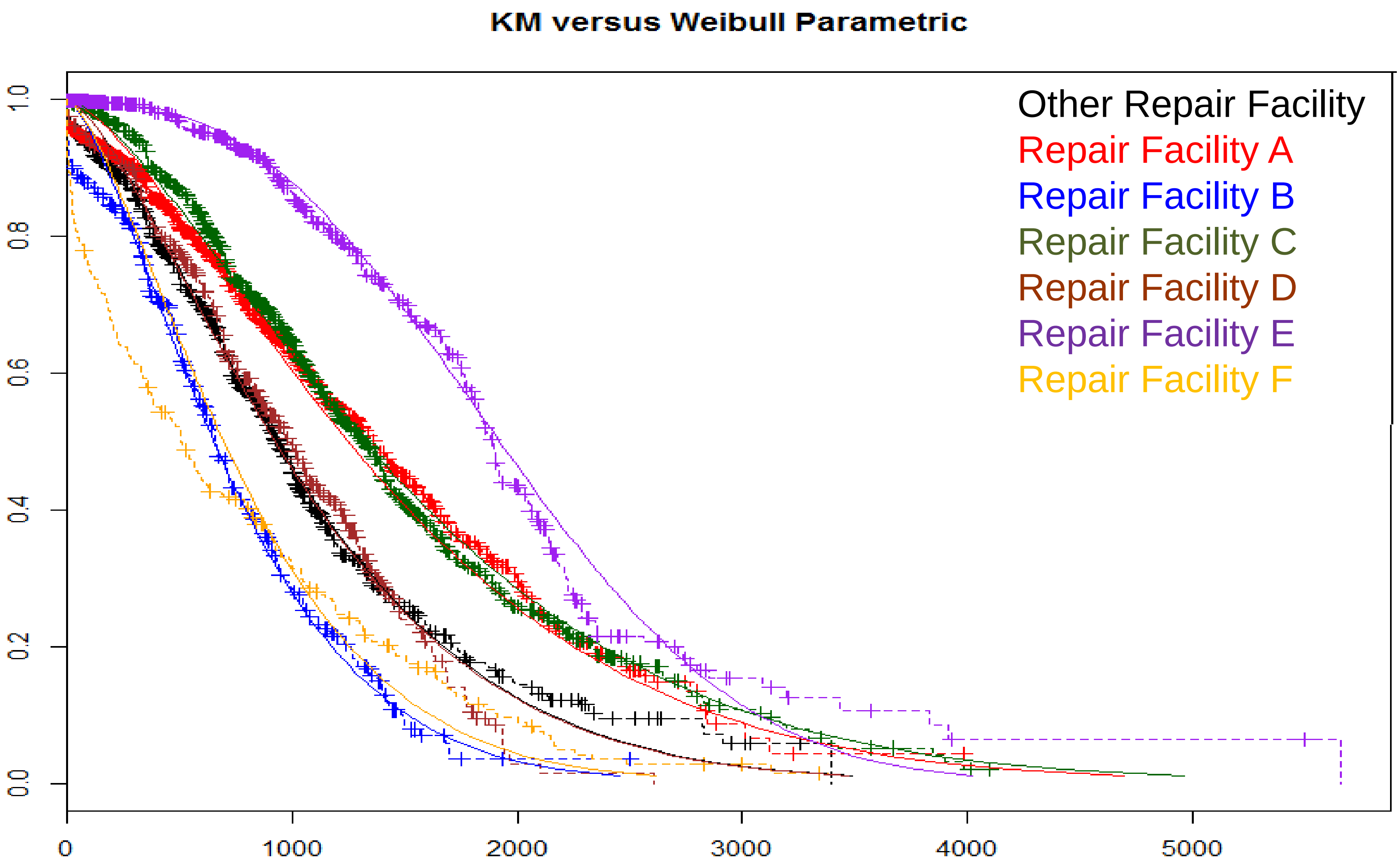
$$F(X) = \begin{cases} 1 - e^{-\left(\frac{x}{\lambda}\right)^k} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

This regression can fit a lambda (shape) to a survival distribution, then adjust k (scale) to the effects of different covariates.

Weibull Parameters- Adjusting Scale



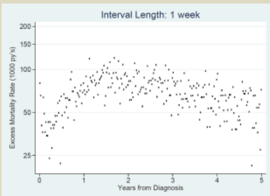
Survival Analysis Tools – KM Curves



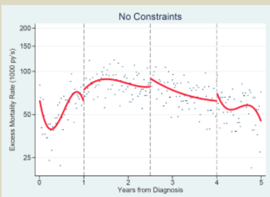
Flexible Survival Regression

- Parametric Models have advantages for
  - Prediction.
  - Extrapolation.
  - Quantification (e.g., absolute and relative differences in risk).
  - Modelling time-dependent effects.
  - Understanding.
  - Complex models in large datasets (time-dependent effects /multiple time-scales)
  - All cause, cause-specific or relative survival.
- The estimates obtained from flexible parametric survival models are incredibly similar to those obtained from a Cox model.
- An important feature of flexible parametric models is the ability
- to model time-dependent effects, i.e., there are non-proportional
- hazards
  - Time-dependent effects are modeled using splines, but will
  - generally require fewer knots than the baseline.
  - This is because we are now modeling deviation from the baseline hazard rate.
  - Also possible to split time to estimate hazard ratio in different intervals.

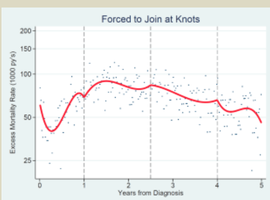
Constructing the Flexible Parametric Model



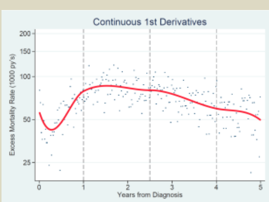
- Piecewise Hazard Function



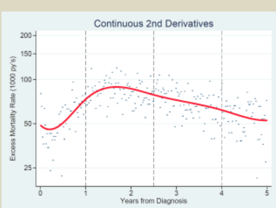
- No Continuity Corrections



- Function Forced to Join at Knots

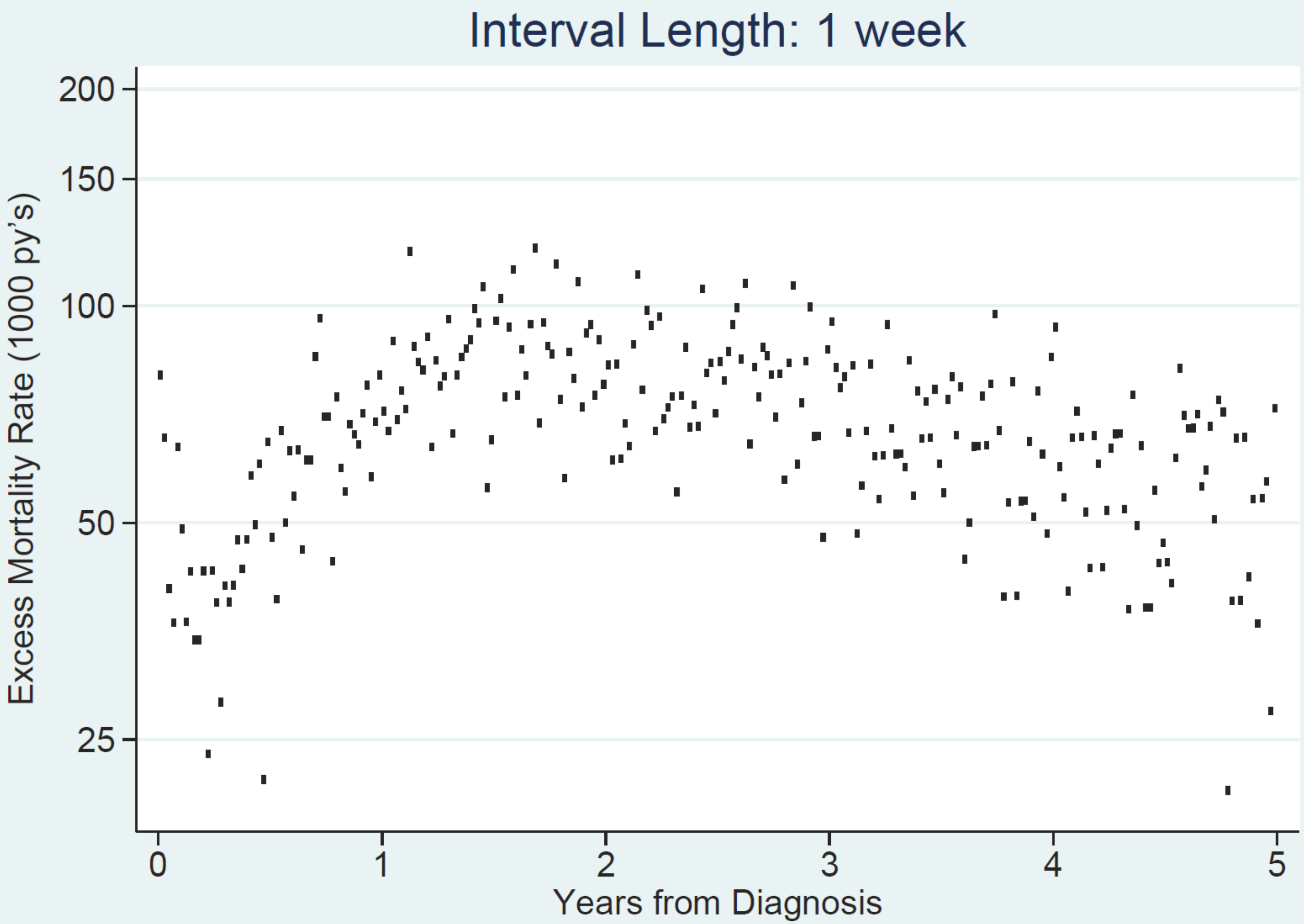


- Continuous at First Derivative

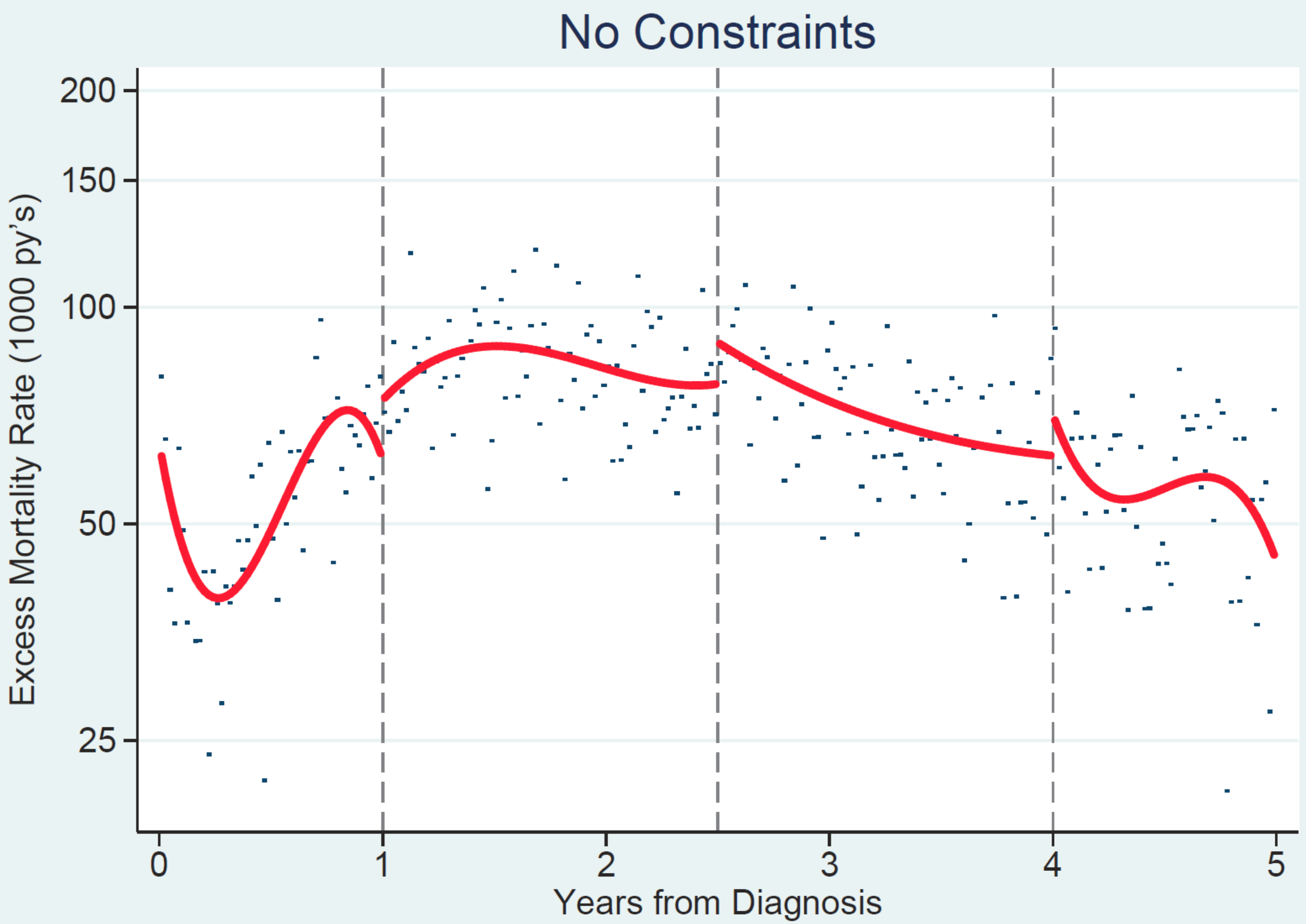


- Continuous at Second Derivative

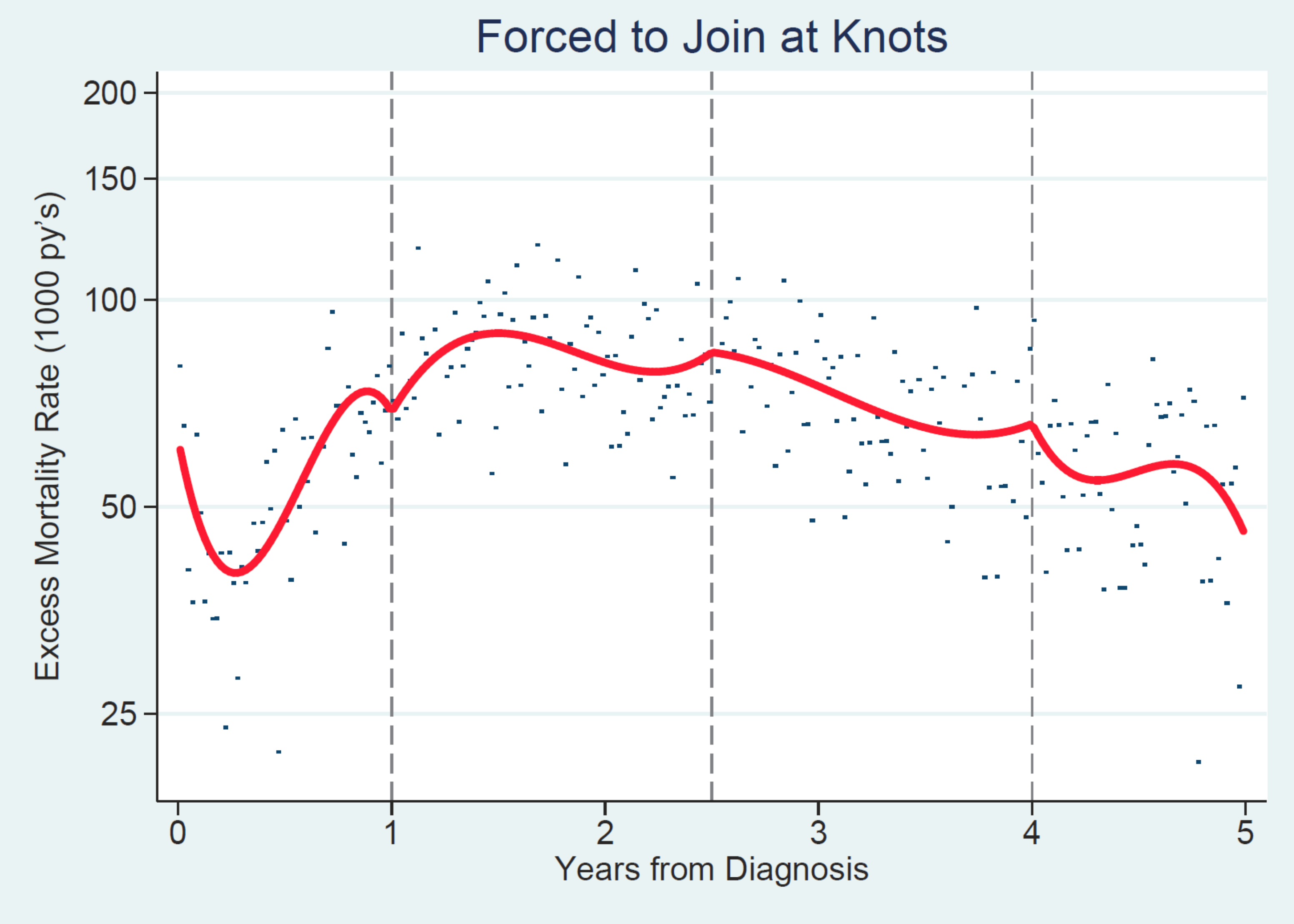
Piecewise Hazard Function



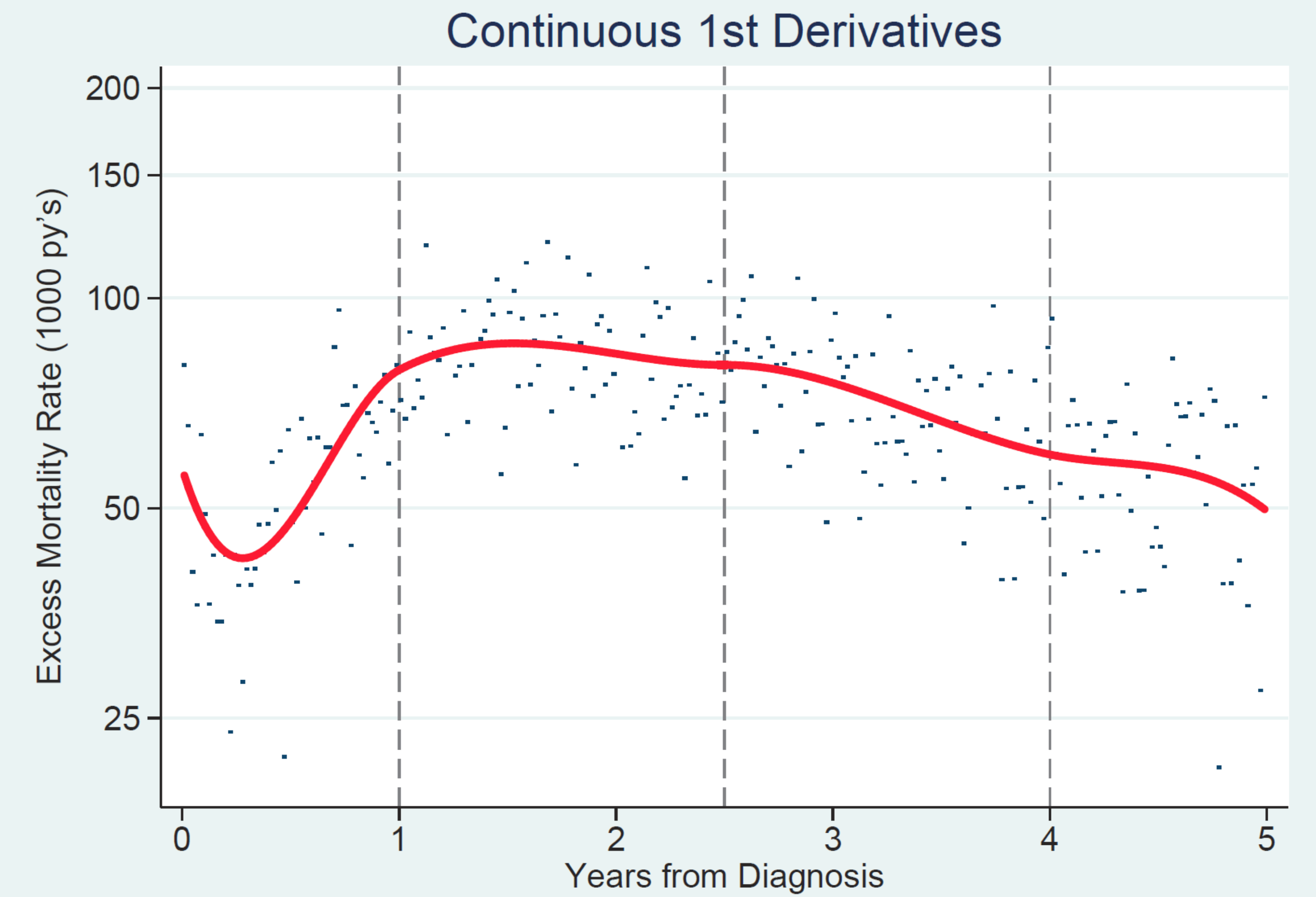
No Continuity Corrections



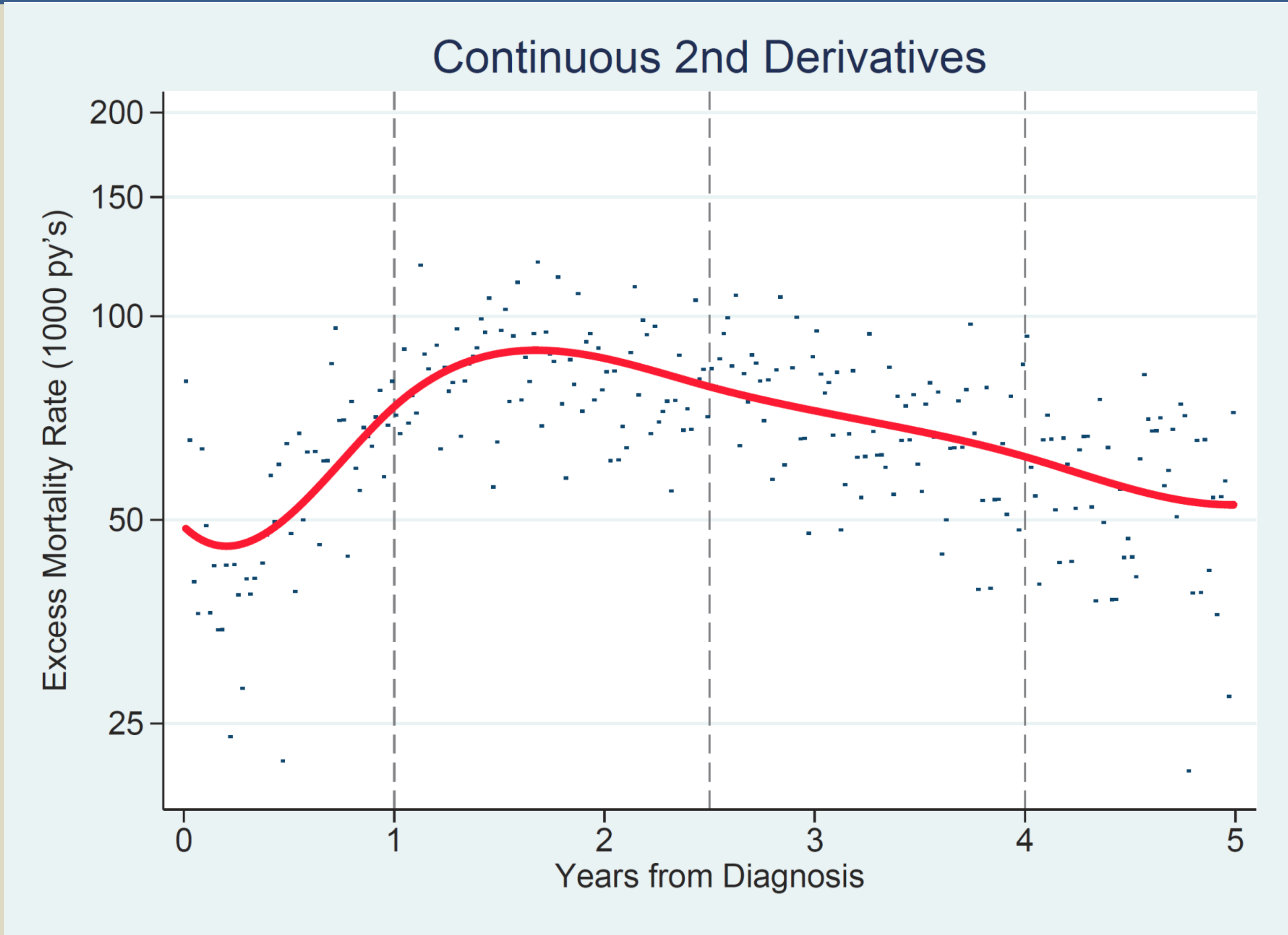
Function Forced to Join at Knots



Continuous at First Derivative



Continuous at Second Derivative



Risk Classification

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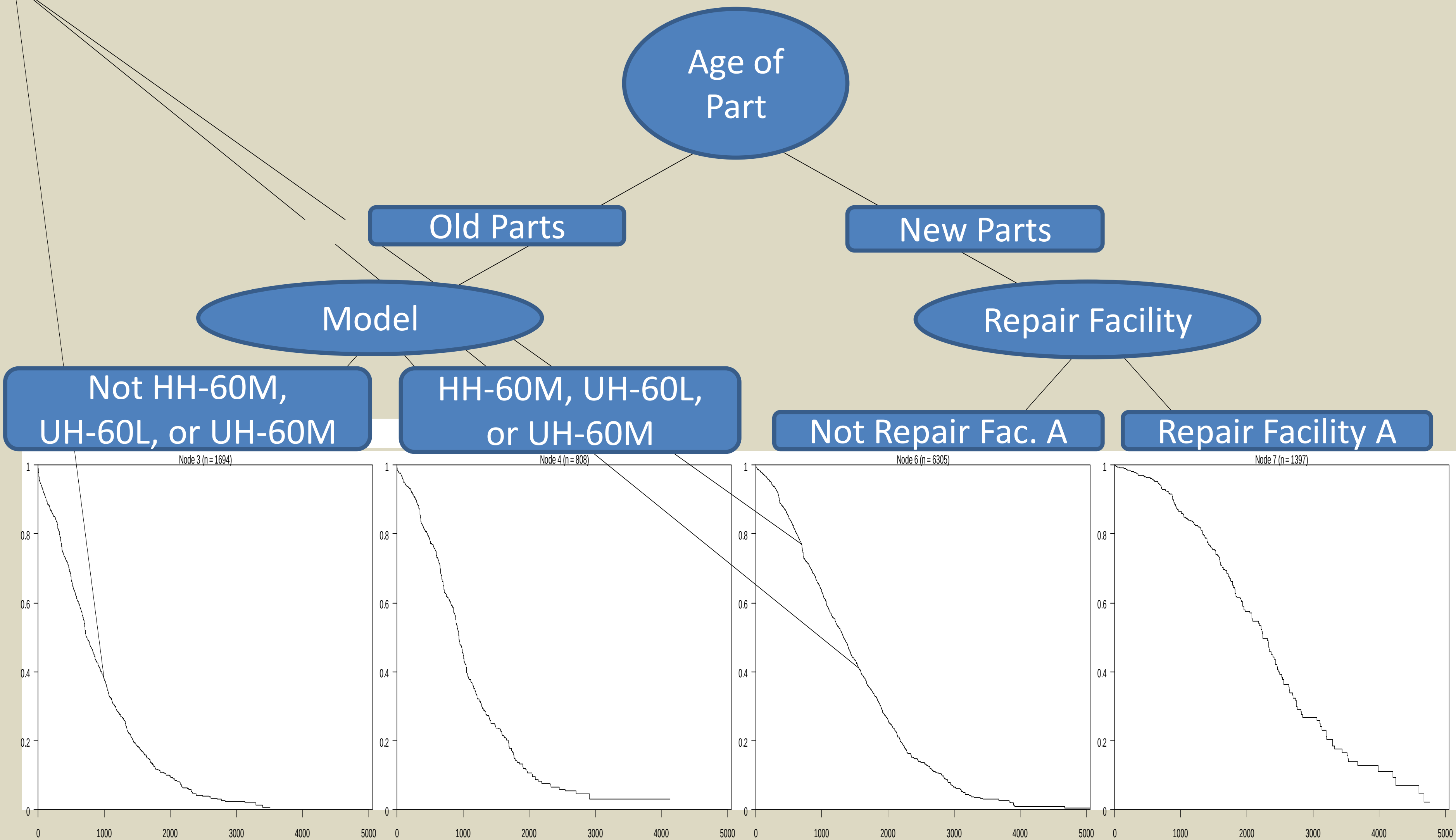
Risk Classification Chart – Life Insurance

Category	Preferred	Standard	Substandard
Smoking	Non-Smokers	Non-Smokers	Smokers
Body Mass Index	18.5-24.9	25 to 29.9	Less than 18.5 Greater than 30
Driving Record	No Tickets	No Major Tickets (DWI)	Many Tickets or a Major Ticket

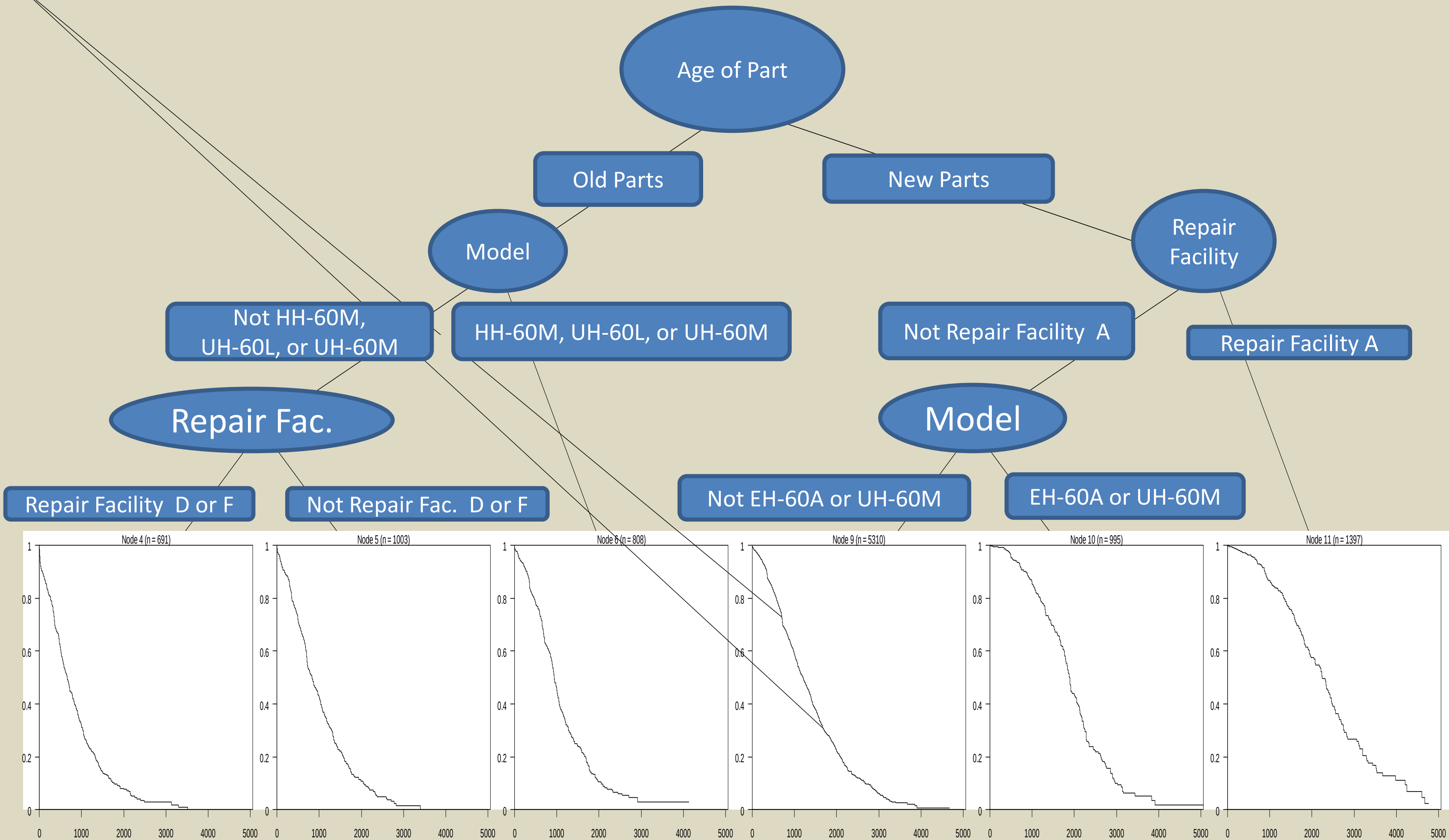
Conditional Inference

- Algorithm
  - Variable Selection Step 1:
    - Permutation based significance test in order to select the variable,
    - Choose the covariate with lowest p-value below than a pre-specified significance value, i.e. 0.05
      - Choosing the p-value is a unique parameter which determines the size of the tree
      - P-values are used to make comparisons between variables that are categorical and numerical
  - Splitting Procedure Step 2:
    - Explore all possible splits
    - Goodness of a split is evaluated again by a permutation –based test
  - Recursively repeat steps 1 and 2 until no more splits can be determined

Conditional Inference Tree



Conditional Inference Tree



Cox Proportional Hazards Model

Alternatively, we can differentiate levels of risk factors using the Cox Proportional Hazards model

The Cox Proportional Hazards model is as follows:

$$h_i(t) = h_0(t) * e^{(\beta_1 X_1 + B_2 X_2 + \ldots)}$$

Where: $h_i(t)$ is the hazard rate of a component i at time t,
$h_0(t)$ is the baseline hazard rate at time t,
β_1 is the coefficient of the first covariate of interest,
X_1 is the first covariate of interest,
β_2 is the coefficient of second covariate of interest, and
X_2 is the second covariate of interest.

Difference in Hazard Rate by Repair Level

Cox Proportional Hazards by Previous Repair Level

n=12053 number of events=4113

Baseline = UH-60A

	Beta	exp(Beta)	P-Value	
MH-60K	0.19473	1.21498	0.00381	**
UH-60L	-0.27767	0.75754	0.112	

Significance codes: 0 ‘***’, 0.001 ‘**’, 0.01 ‘*’, 0.05 ‘.’

Constructing the Parametric Model

- Before the data of the old parts are fitted to the Weibull distribution, they are first split into 4 different classes (New, Preferred, Standard, and Substandard).
- The new parts with Times since new (TSN) values of zero are segmented from the other old parts and classified as new. For the old parts, they are classified as ether “preferred”, “standard”, or “substandard” depending on four risk factors.
- These four risk factors are:
  - Helicopter Model
  - Previous Repair Facility
  - Times since new (TSN)
  - Previous Chargeable Failure Code
- RLH then uses the Cox Proportional Hazard Regression to measure the average hazard rate for each levels of the four risk factors and then rank them. The results are displayed on the next slide.

Constructing the Parametric Model

Risk Factors	Rank	Elements
Previous Repair Facility (UIC)	Above, Above Average (AAA)	UIC-A
	Above Average (AA)	UIC-B, UIC-C, UIC-D
	Average (A)	All Other UICs
	Below Average (BA)	UIC-F
Helicopter Model	Above, Average (AA)	HH-60G, EH-60L, UH-60M
	Average (A)	UH-60L and all other helicopter models
	Below Average (BA)	MH-60K
Previous Failure Code	Average (A)	Previous Failure Code is not 2 (Air Leak) or 520 (Pitted)
	Below Average (BA)	Previous Failure Code is 2 or 520
Time Since New (TSN)	Above Average (AA)	Less than 1500 flight hours
	Average (A)	Between 1500 and 3500 flight hours (inclusive)
	Below Average (BA)	Over 3500 flight hours

Constructing the Parametric Model

- There are 72 possible different combinations of these risk factors and each one is labeled as category 1,2, …., 72.

Category	Previous UIC	Time Since New (TSN)	Previous Failure Code	Model
8	AA	A	A	AA
12	A	AA	A	AA
32 (Baseline)	A	A	A	A

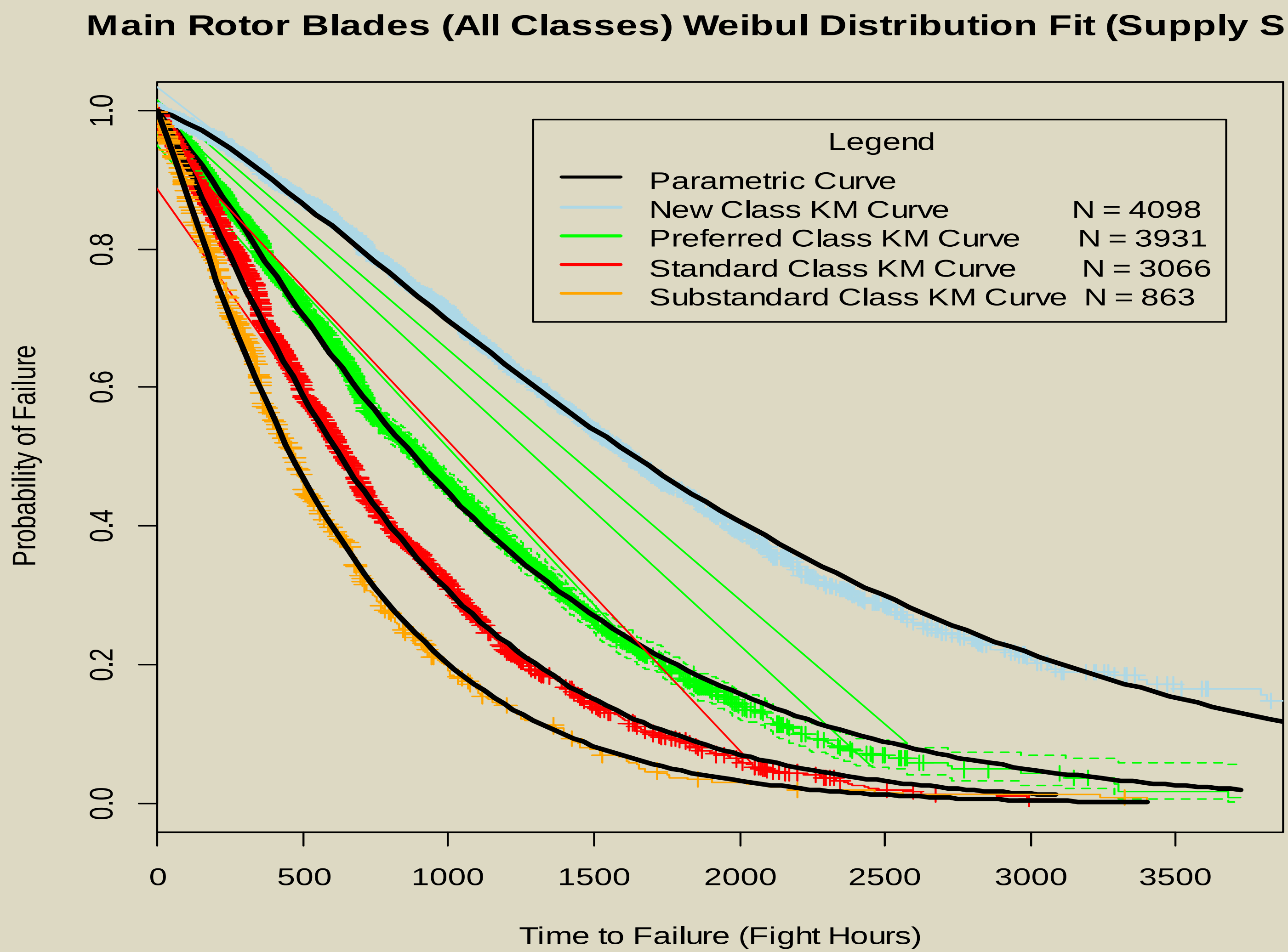
- The magnitude of each risk factor's effect on survival rate may differ. The idea of creating these categories is to evaluate the overall effect of all four risk factors. These 72 categories are entered into a Cox PH regression equation with baseline set as category 32 and the results are shown in the next slide.

Constructing the Parametric Model

Class	Categories Included	Number of Observations	Class
New	All Parts with TSN = 0	4241	New
Preferred	7, 19, 20, 25, 26, 27 ,29, 31	1821	Preferred
Standard	All Other Categories	3200	Standard
Substandard	33, 35, 36, 38, 44, 50	896	Substandard

- The categories with a hazard rate that is lower than the baseline on average are put into the “Preferred” class, the categories with a hazard rate that is higher than the baseline on average are put into the “Substandard” class, and all other categories are put in the, “Standard” class. The new class contains all the new parts, regardless of their risk factors. RLH then constructed the parametric models by fitted the Weibull distribution to each of the four classes.

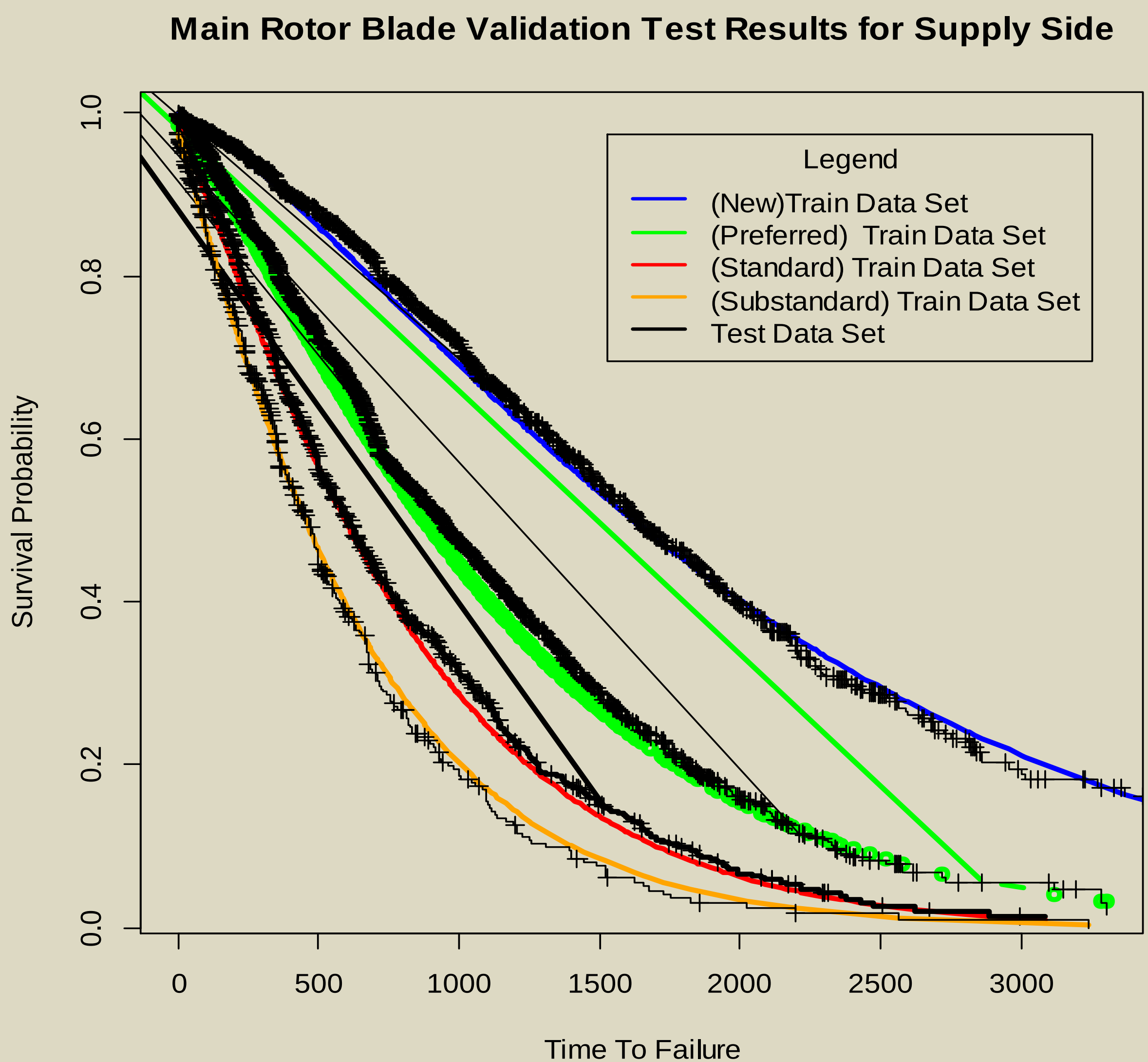
Parametric Model: Goodness of Fit



Parametric Model: Validation Testing

- The objective is to test if the four parametric models serves as an accurate representation of the real data.
- The "Train" data set, which 65% of the data randomly chosen, is fitted to a Weibull distribution to create a parametric model. The rest of the data, the "Test" data set, is used to plot a KM curve.
- The parametric curve is then compared with the KM curve.
- RLH then conducts the Kolmogorov-Smirnov (KS) Test between the KM and the parametric curve.
- The KS test can compare a sample data set with a reference distribution and determine the how likely that sample data is drawn from the reference distribution. In this case, the sample data is the "Test" data set and the reference distribution is the parametric model.

Parametric Model: Validation Testing Results



Parametric Model: Validation Testing Results

- KS test results:

Class	Test Statistic (D)	P-Value
New	0.0367	0.8355
Preferred	0.0262	0.9113
Standard	0.0356	0.8688
Substandard	0.0593	0.7655

- The null hypothesis is that the sample data is drawn from the reference distribution. The results shows that the p-values for all four models are greater than the alpha, 0.05. Therefore, the null hypothesis is not rejected and the test indicates that the "Test" data set is likely to have been drawn from the parametric model.

Monte Carlo Method
And
Inventory Simulation

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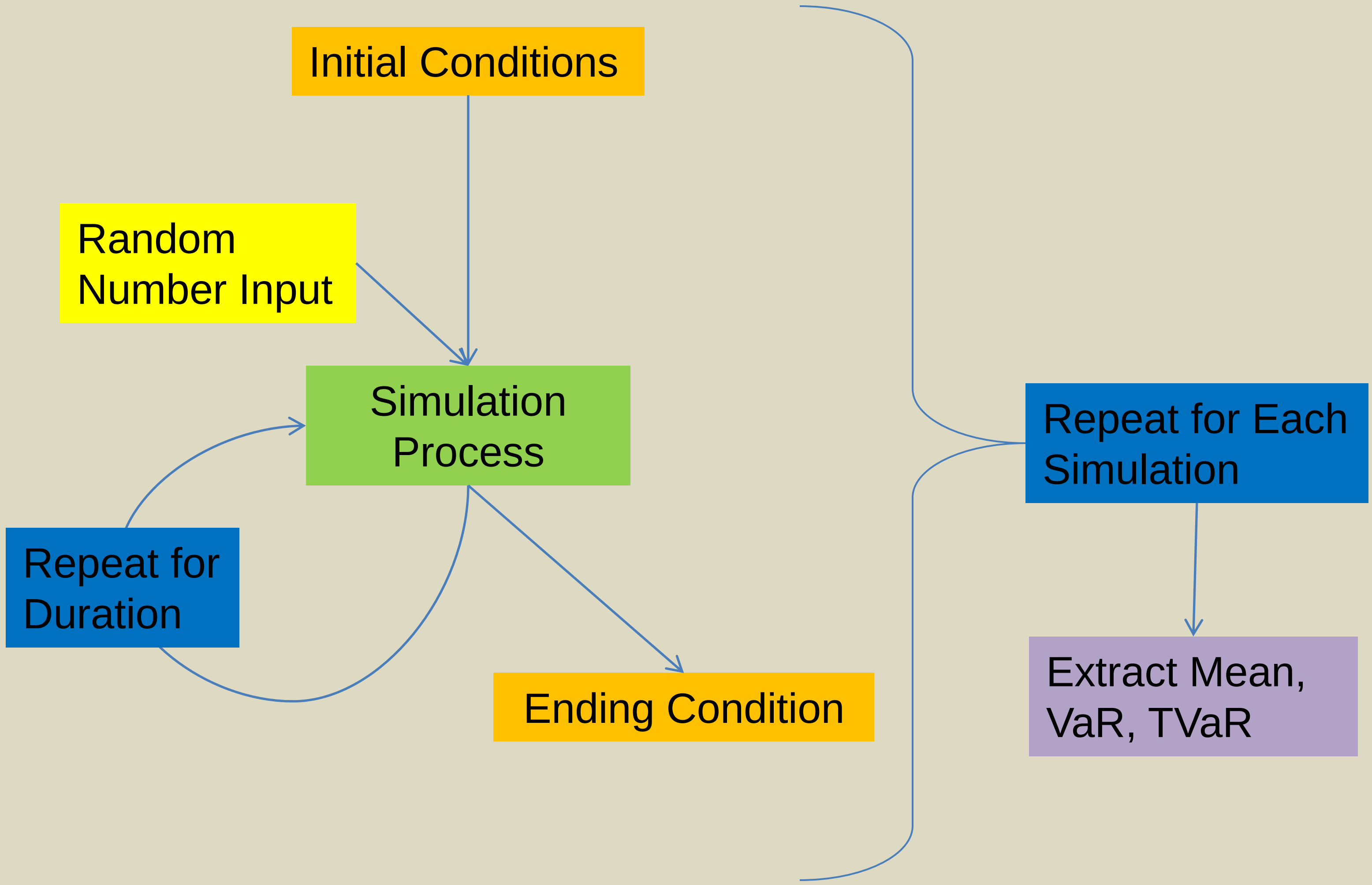
Monte Carlo Simulation Basics



Monte Carlo Simulation Basics

http://genedan.com/tag/brownian-motion/

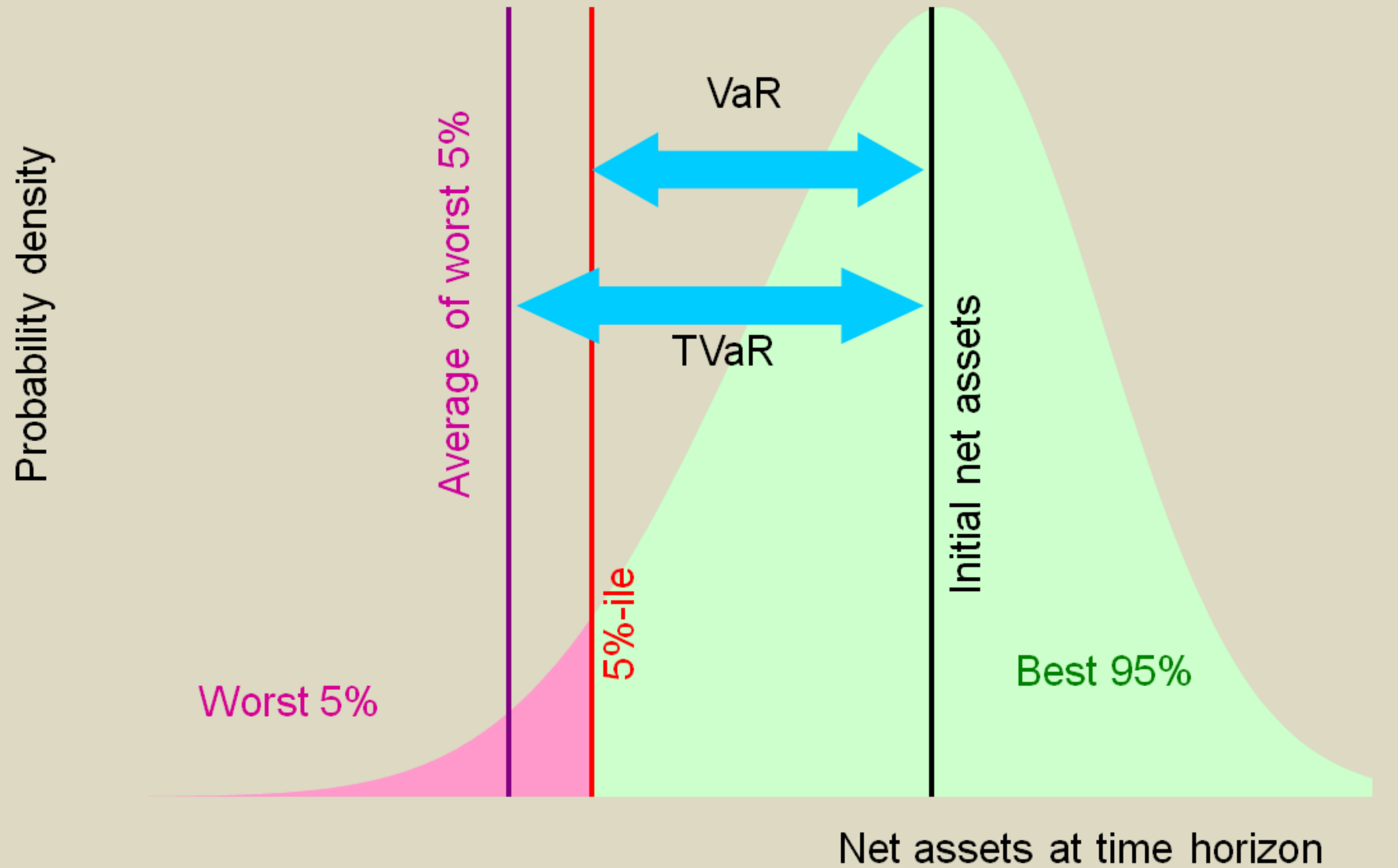
Heads - 50%
Tails - 50%

50 Flips per Simulation

Simulation Time

1	10
20	250

Simulation Basics: Value at Risk and Conditional Value at Risk



In our simulation, assets might be considered to be spare inventory.

http://www.nematrian.com/R.aspx?p=TailValueAtRisk

Supply Simulation Initial Conditions and Assumptions

- 200 Helicopters or ‘Slots’ of random frame composition
- 220 Parts with randomized covariates (TSLI, UIC)
- Method to classify parts into risk categories
- Hazard rate parameters for risk classes estimated from 2410
- Average flight hours per month for each airframe based on 1352 Dataset

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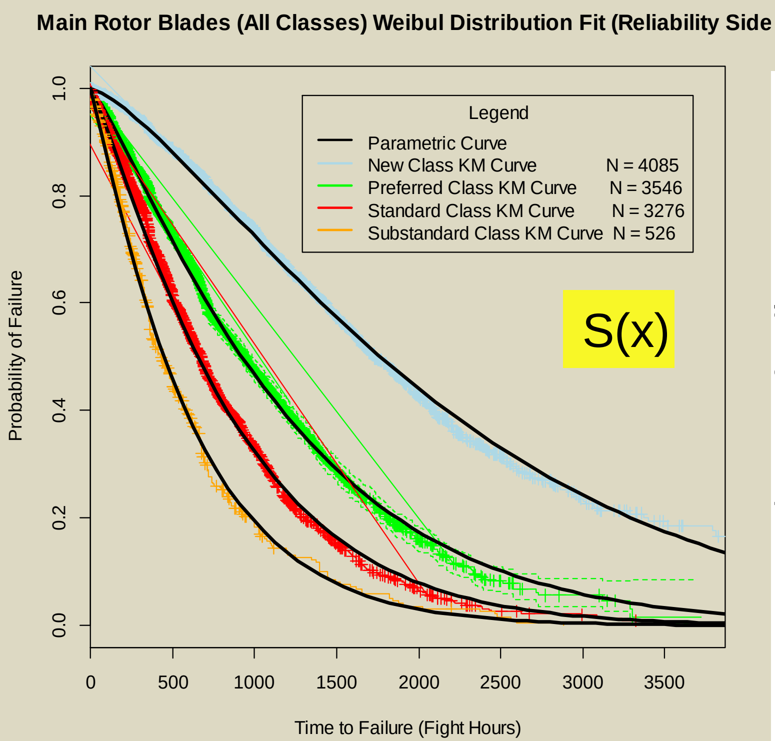
- Monthly simulation, deaths/installs happen at end of month
- On failure parts have 20% chance of “True Death”
- Some failures are minor and repairs happen within a month

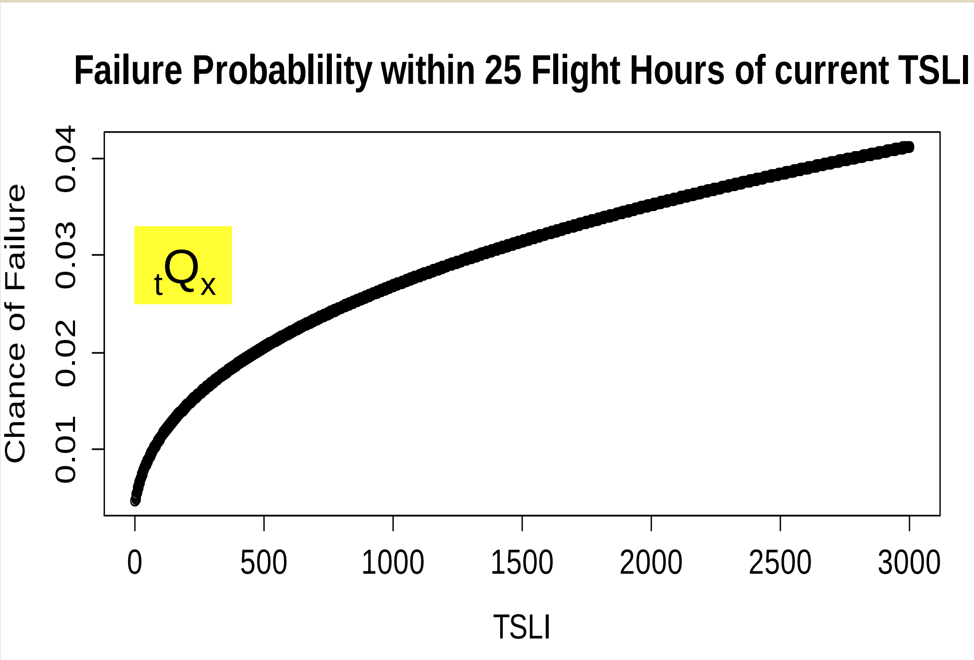
Supply Simulation: Monthly Survival

- Since simulation is done in 1 month intervals, survivorship is only calculated assuming part survives the average flight hours per month of the airframe

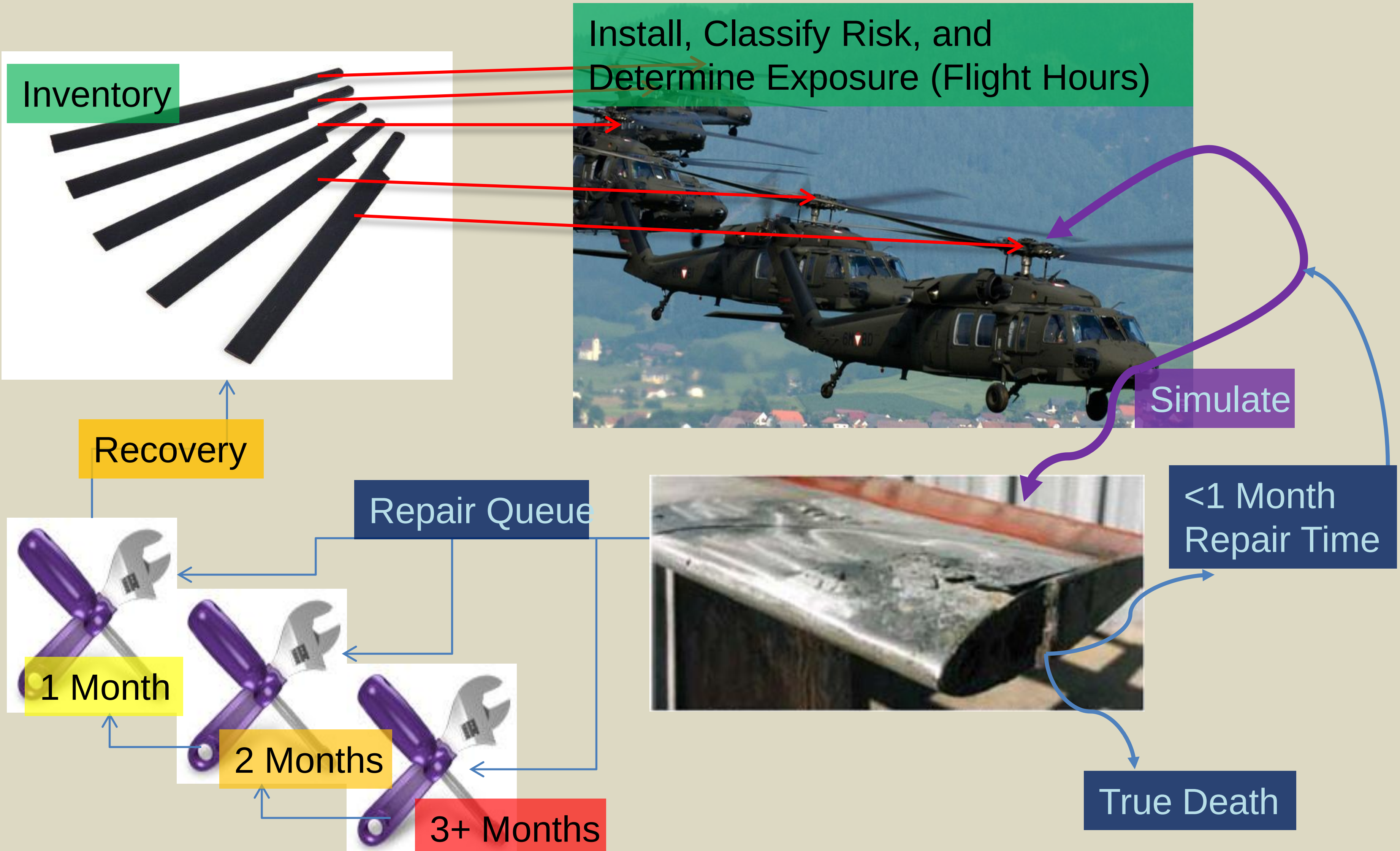
$${}_tP_x = P(x + t \mid x + t > X) = S(x + t) / S(X)$$

$${}_tQ_x = 1 - {}_tP_x$$





Supply Simulation Process



Supply Simulation: Sample Run and Results

____Monte Carlo Sample Statistics_____________________

• Percent of Simulations where inventory ran out:	5.9%
• Average number of Failures:	36.2
• Average Instant Repair rate on Failure:	21.5%
• Average Deaths:	7.3
• Average Remaining Inventory:	5.2

____Tail Statistics _____________________________________

• 95% Percentile of Failures:	46
• Conditional Expected Value of Failures over 95th percentile:	48.2
• 5th Percentile of Remaining Inventory:	-1
• Conditional Expected Value of Remaining Inventory over 95th percentile:	6.3

Supply Simulation: Improvements

- Matching assumptions and process to general practices
- Reason for Failure (Specific Fail Code)
  – Associated distribution for repair times
- Nonchargeable removals
  – Allows parts to change helicopters without failing first
- Back testing Methods

Questions

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Thank you!

Contact

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