Simulating System Reliability using Monte Carlo Simulation

Presented by:

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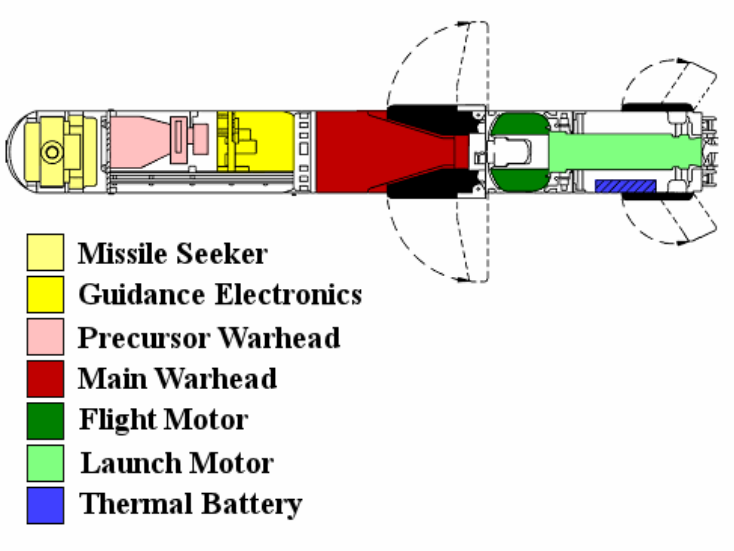
Table of Contents

System Reliability

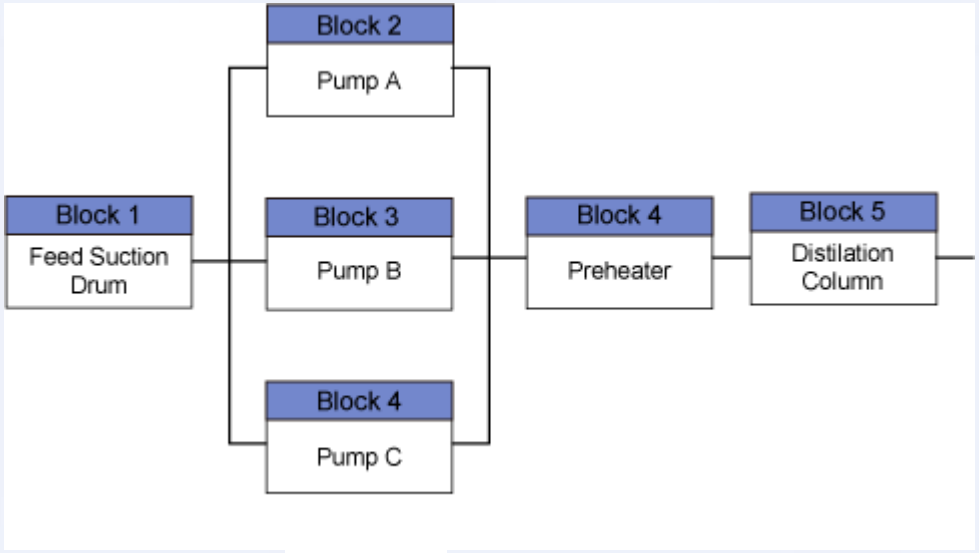


- Systems are comprised of multiple components specifically arranged for a design.
- If component reliabilities are known, or reasonably estimated, the reliability of the system can be estimated with certain confidence.
- This estimate is based on the component configuration, laws of probability, and key assumptions.

Reliability Block Diagrams

- The first step in estimating the system reliability is to create a Reliability Block Diagram (RBD)

- A RBD is a graphical representation of the components of a system and how they are related to each other in the system

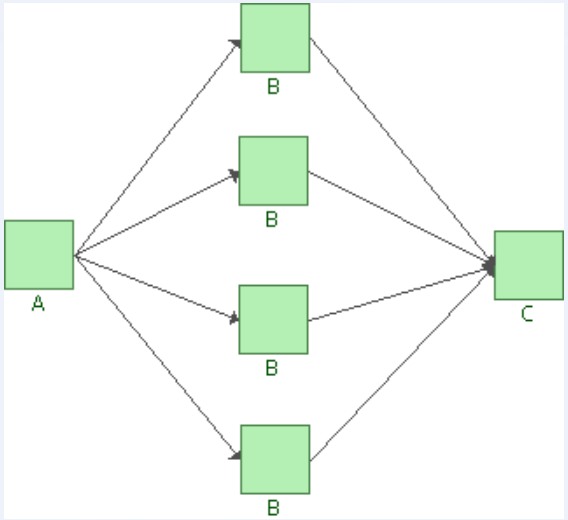


Reliability Block Diagrams

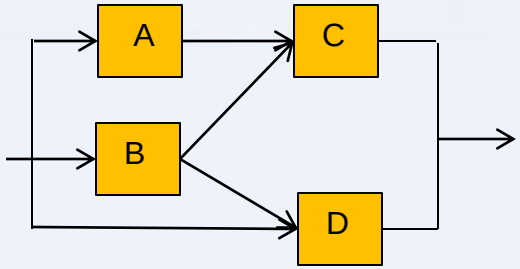
- Each complete RBD is unique to its design
- RBD's can be in several different arrangements

- Series

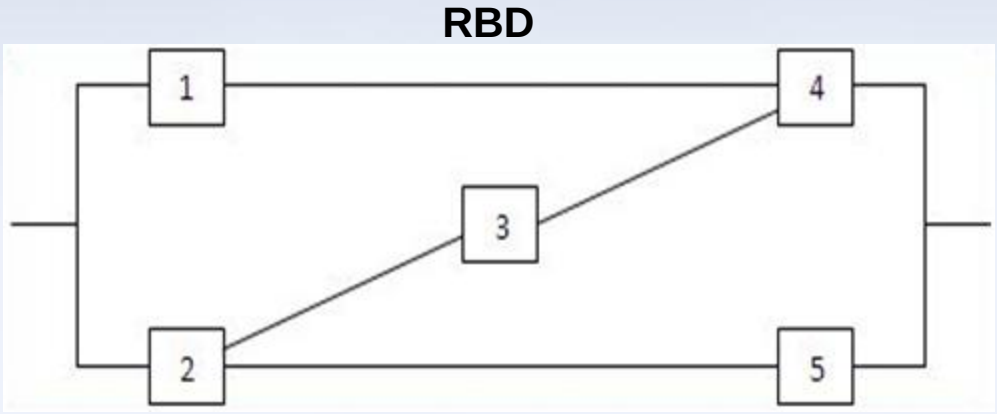
- Parallel



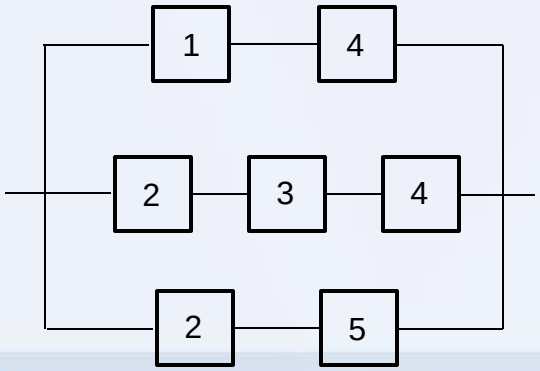
-Complex

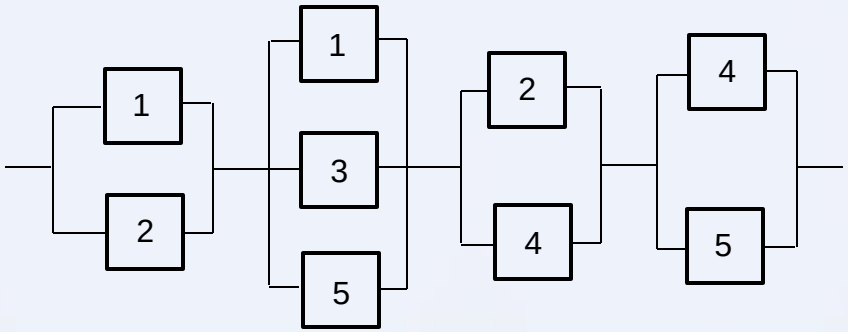


Simplifying RBD's



- Reduce the RBD into:
  - a parallel arrangement of series subsystems OR
  - a series arrangement of parallel subsystems





Structure Function

- Assumptions:
  - A component can only exist in 2 states – Functioning or Failed
  - Component failures are statistically independent

- Definition:
  - The state of component i, denoted by x_i, is:

$$x_i = \begin{cases} 0 & \text{if component } i \text{ has failed} \\ 1 & \text{if component } i \text{ is functioning} \end{cases}$$

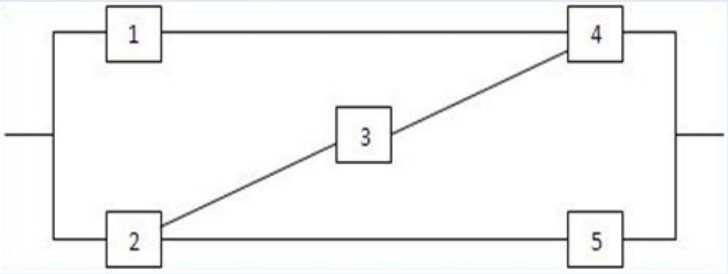
for $i = 1, 2, \ldots n$

- The structure function, $\varphi(\underline{x})$, relates the state of the system to the states of its individual components

$$\Phi(x_i) = \begin{cases} 0 & \text{if the system has failed when the state vector is } \underline{x} \\ 1 & \text{if the system is functioning when the state vector is } \underline{x} \end{cases}$$

Calculating the structure function

RBD



Minimal Path Set: $$\Phi(x_i) = 1 - \prod_{j=1}^{s} (1 - \prod_{i \in P_j} x_i)$$ Where s is the number of minimal path sets

$$\Phi(x_i) = 1 - (1 - x_1 x_4)(1 - x_2 x_3 x_4)(1 - x_2 x_5)$$

Minimal Cut Set: $$\Phi(x_i) = \prod_{j=1}^{k} \left[1 - \prod_{i \in C_j} (1 - x_i) \right]$$ Where k is the number of minimal cut sets

$$\Phi(x_i) = (1 - \overline{x_1 x_2})(1 - \overline{x_4 x_5})(1 - \overline{x_1 x_3 x_5})(1 - \overline{x_2 x_4})$$

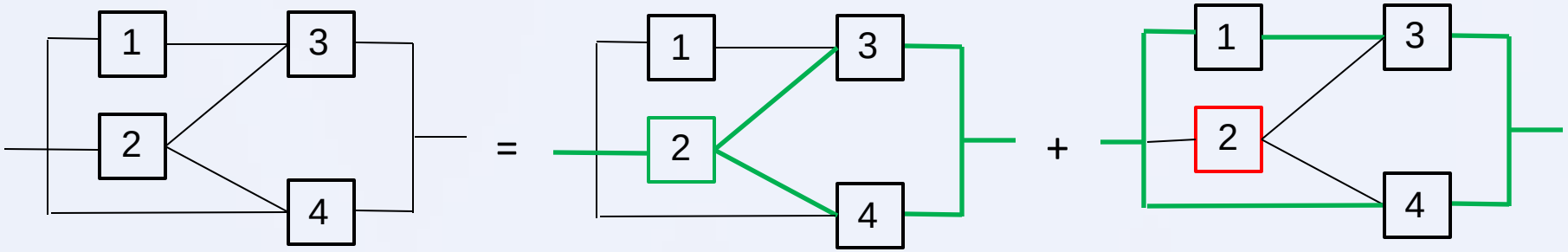
Calculating the System Reliability

- Using the structure function:

Cut Method ≤ Actual Reliability ≤ Path Method

$$\prod_{j=1}^{k}\left[1-\prod_{i\in C_j}(1-p_i)\right] \leq R(S) \leq 1-\prod_{j=1}^{s}\left(1-\prod_{i\in P_j} p_i\right)$$

- Calculating the actual reliability
  - Decomposition also known as the Keystone Method



Simulating System Reliability

- Software
  - Standard Microsoft Excel
  - VBA programming within Excel
  - Reliasoft
  - Raptor

Simulating System Reliability

- Benefits of using VBA code
  - Reuse
  - More precise calculations
  - Less likelihood of errors
  - Simple interface with user
  - Ability to run a set number of iterations
  - Ability to lock cells from the user
  - Faster simulations
  - Can run on any computer with Microsoft Office

Simulating System Reliability

- Information needed for simulation using VBA programming
  - Reliability Block Diagram
  - Failure rate of each component (λ)
  - Operating time of the system (t)

$$\text{Reliability} = e^{-\lambda t}$$

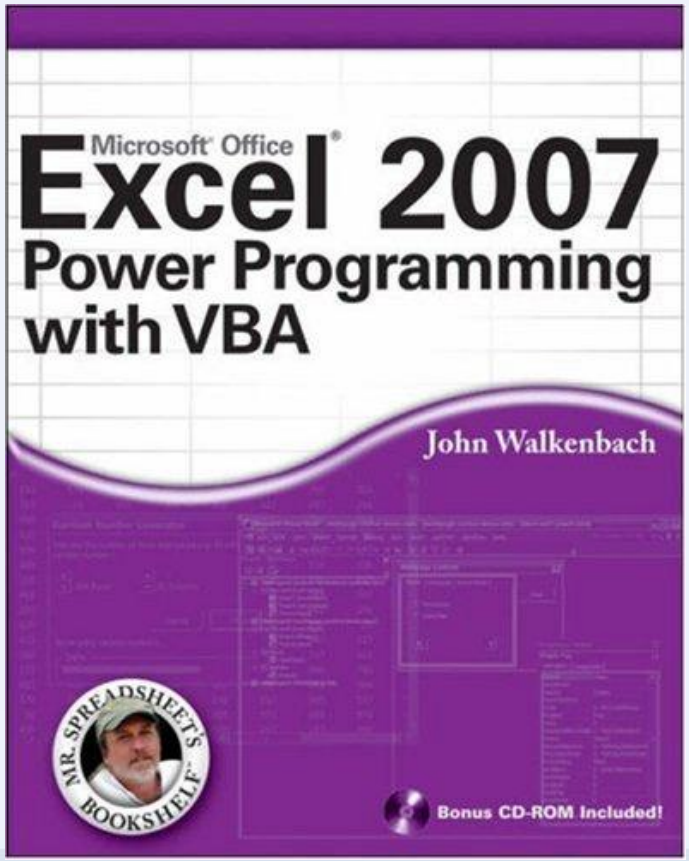
  - Number of iterations to run

Simulating System Reliability

Simulation demonstration in Excel

Helpful Reference

- VBA Programming reference



Conclusion

- Simulating reliability of a system is preferred to demonstrating reliability by testing
- Reliability Block Diagrams are specific to each system and must be considered in order to produce accurate reliability simulations
- Reliability estimates are based on the component configuration, laws of probability, and key assumptions
- Monte Carlo simulation in Excel is one way to predict a reasonable estimate of the system reliability
- Using VBA Code is an effective way to run simulations in Excel

Back-up Slides

Simulating System Reliability

Component	Lambda	Reliability	Time to Failure
1	0.0002	0.818730753	1937.570671
2	0.0002	0.818730753	2032.865688
3	0.0002	0.818730753	11023.61908
4	0.0002	0.818730753	566.6534217
5	0.0002	0.818730753	7615.211807

Component Working	Number of Iterations	Screen Updating?
1	10000	Off
0		
1		
1		
1		

Method	Structure Function	Check
keystone	1	0
paths	1	0
cuts	1	

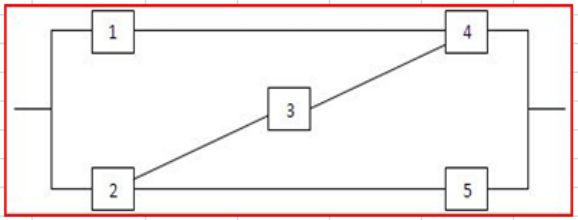
time: 1000 hrs

Paths	TTF of Path	TTF of System	Trials	MTTF	Successes	Failures	Simulated Reliability	Calculated Reliability
14	566.653422	2032.865688	10000	3873.899714	9158	842	0.9158	0.9093443
234	566.653422		Check					
25	2032.86569		0					
			0					

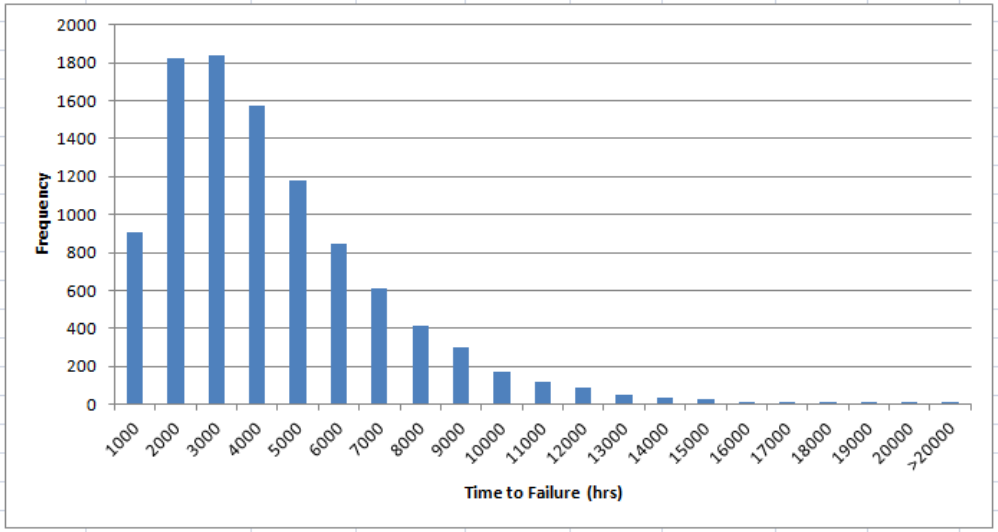
Cuts	TTF of Cut	TTF of System
12	2032.86569	2032.865688
45	7615.21181	
24	2032.86569	
135	11023.6191	

Histogram BIN	Frequency	Cumulative Frequency
1000	0	905
2000	0	1823
3000	1	1836
4000	0	1572
5000	0	1182
6000	0	849
7000	0	607
8000	0	417
9000	0	296
10000	0	167
11000	0	119
12000	0	86
13000	0	52
14000	0	33

System



Reset Data and Run New Simulation



Sheet1 Sheet2 Sheet3

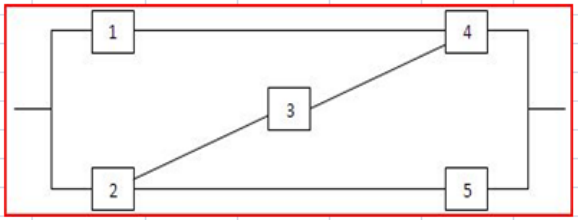
Simulating System Reliability

Component	Lambda	Reliability	Time to Failure
1	0.0002	0.818730753	1937.570671
2	0.0002	0.818730753	2032.865688
3	0.0002	0.818730753	11023.61908
4	0.0002	0.818730753	566.6534217
5	0.0002	0.818730753	7615.211807

Component Working	Number of Iterations	Screen Updating?
1	10000	Off
0		
1		
1		
1		

Method	Structure Function	Check
keystone	1	0
paths	1	0
cuts	1	

System



time 1000 hrs

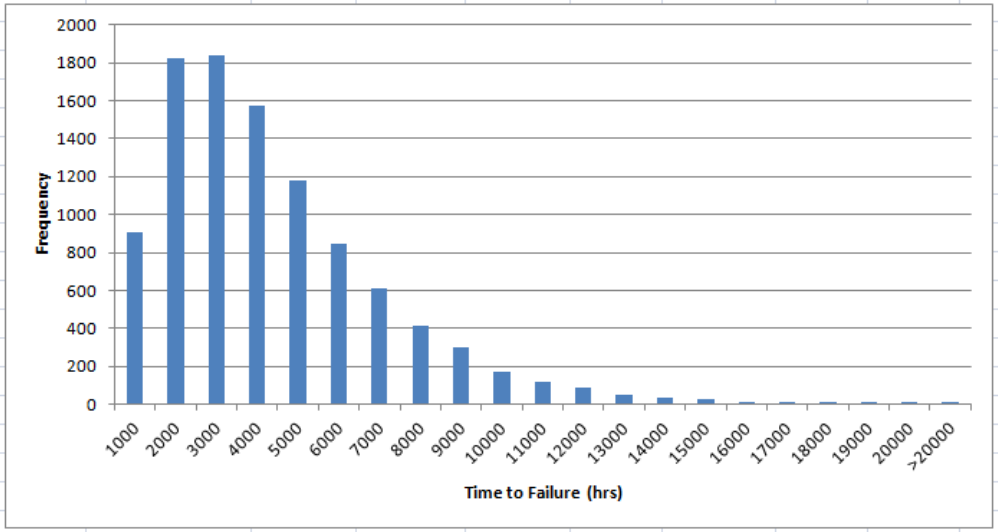
Paths	TTF of Path	TTF of System	Trials	MTTF	Successes	Failures	Simulated Reliability	Calculated Reliability
14	566.653422	2032.865688	10000	3873.899714	9158	842	0.9158	0.9093443
234	566.653422		Check					
25	2032.86569		0					
			0					

Cuts	TTF of Cut	TTF of System
12	2032.86569	2032.865688
45	7615.21181	
24	2032.86569	
135	11023.6191	

Reset Data and Run New Simulation

Start button

Histogram BIN	Frequency	Cumulative Frequency
1000	0	905
2000	0	1823
3000	1	1836
4000	0	1572
5000	0	1182
6000	0	849
7000	0	607
8000	0	417
9000	0	296
10000	0	167
11000	0	119
12000	0	86
13000	0	52
14000	0	33



Sheet1 Sheet2 Sheet3

Simulating System Reliability

Component	Lambda	Reliability	Time to Failure	Component Working	Number of Iterations	Screen Updating?		Method	Structure Function	Check
1	0.0002	0.818730753	150.4361041	1	10000	Off		keystone	1	0
2	0.0002	0.818730753	5196.808947	1				paths	1	0
3	0.0002	0.818730753	118.44062	1				cuts	1	
4	0.0002	0.818730753	6233.726133	0						
5	0.0002	0.818730753	6873.422759	1						

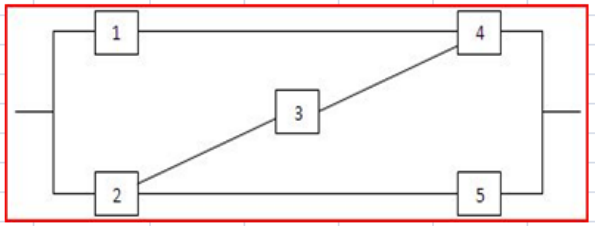
time	1000	hrs

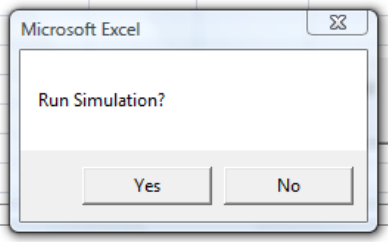
Paths	TTF of Path	TTF of System	Trials	MTTF	Successes	Failures	Simulated Reliability	Calculated Reliability
14	150.436104	5196.808947	10000	3916.587074	9098	902	0.9098	0.9093443
234	118.44062		Check					
25	5196.80895		0					
			0					

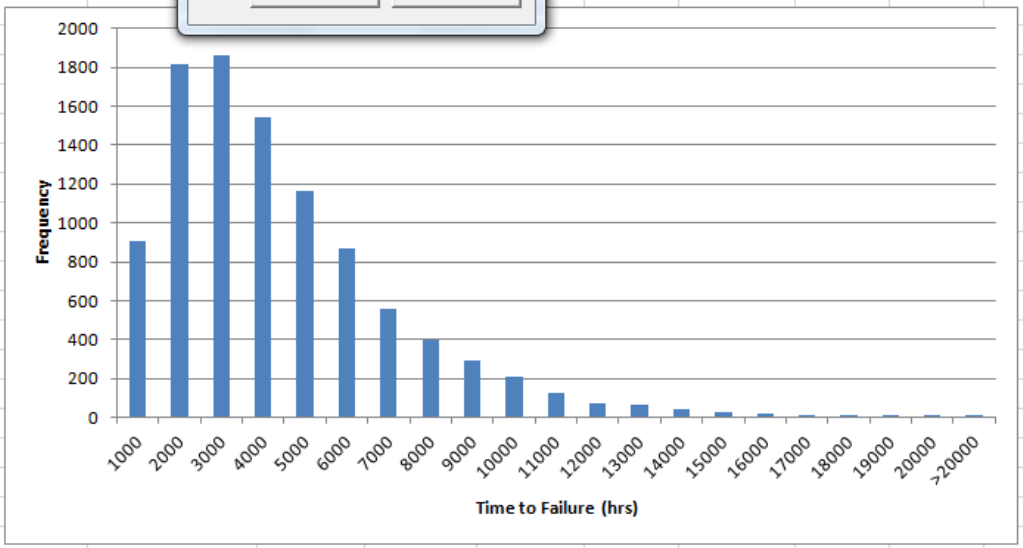
Cuts	TTF of Cut	TTF of System
12	5196.80895	5196.808947
45	6873.42276	
24	6233.72613	
135	6873.42276	

Histogram BIN	Frequency	Cumulative Frequency
1000	0	907
2000	0	1820
3000	0	1862
4000	0	1545
5000	0	1161
6000	1	868
7000	0	556
8000	0	400
9000	0	289
10000	0	205
11000	0	126
12000	0	72
13000	0	66
14000	0	44

System







Simulating System Reliability

Component	Lambda	Reliability	Time to Failure
1	0.0001	0.860707976	23719.10347
2	0.0002	0.740818221	6799.008462
3	0.0003	0.637628152	169.1308462
4	0.0004	0.548811636	2225.449691
5	0.0005	0.472366553	51.1641836

time	1500	hrs

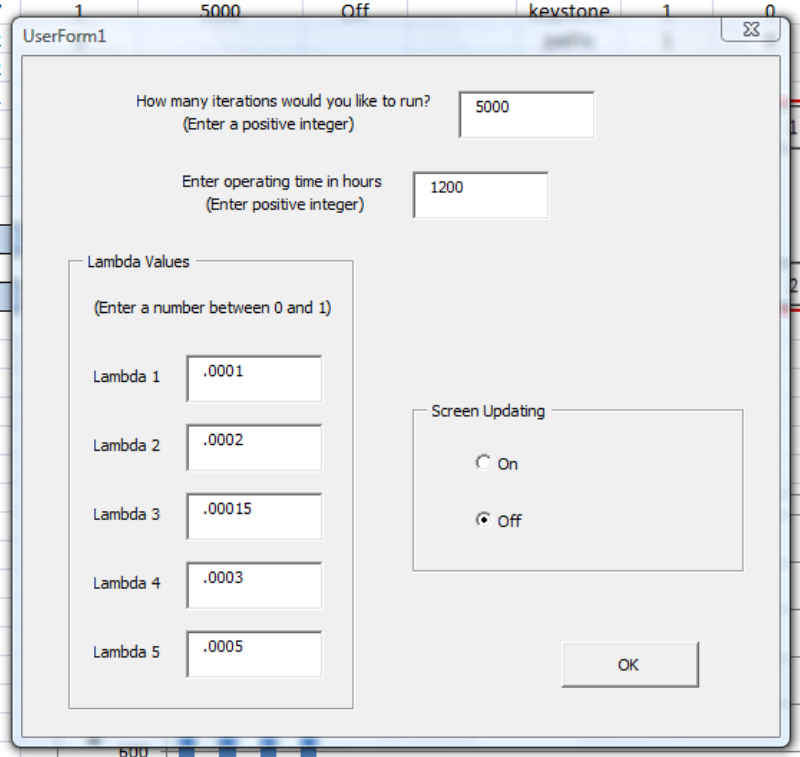
Paths	TTF of Path	TTF of System	Trials
14	2225.44969	2225.449691	5000
234	169.130846		Check
25	51.1641836		0
			0

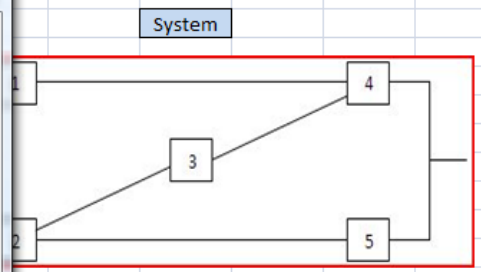
Cuts	TTF of Cut	TTF of System
12	23719.1035	2225.449691
45	2225.44969	
24	6799.00846	
135	23719.1035	

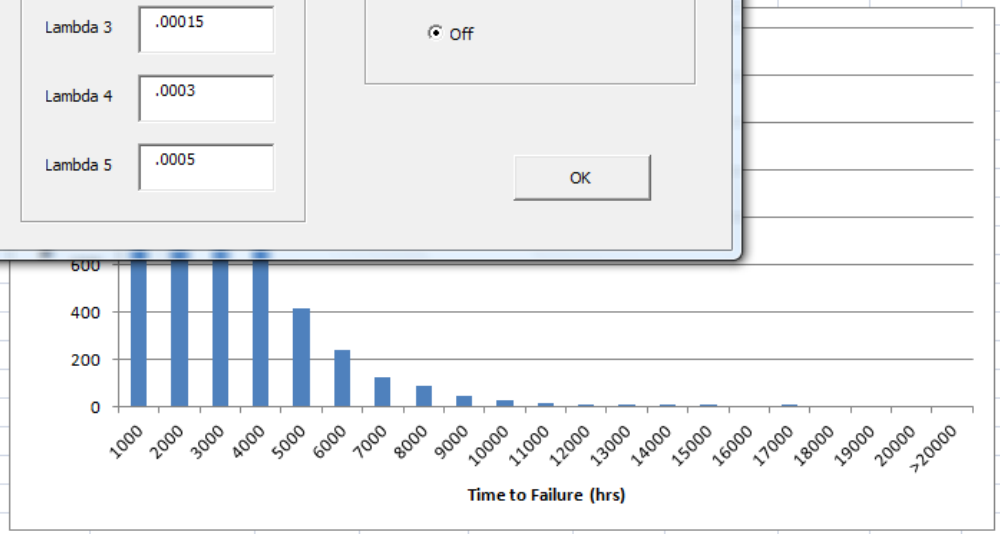
Histogram BIN	Frequency	Cumulative Frequency
1000	0	925
2000	0	1351
3000	1	1021
4000	0	729
5000	0	415
6000	0	241
7000	0	125
8000	0	90
9000	0	45
10000	0	26
11000	0	15
12000	0	7
13000	0	5
14000	0	3

Component Working	Number of Iterations	Screen Updating?
1	5000	Off

Method	Structure Function	Check
keystone	1	0







Sheet1 Sheet2 Sheet3

Simulating System Reliability

Component	Lambda	Reliability	Time to Failure
1	0.0001	0.886920437	2548.013698
2	0.0002	0.786627861	11484.34793
3	0.00015	0.835270211	715.3815777
4	0.0003	0.697676326	4160.641988
5	0.0005	0.548811636	1615.281084

Component Working	Number of Iterations	Screen Updating?
1	5000	Off

Method	Structure Function	Check
keystone	1	0
paths	1	0
cuts	1	

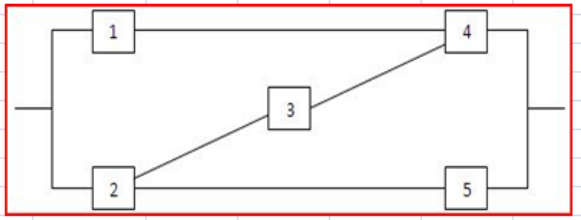
time 1200 hrs

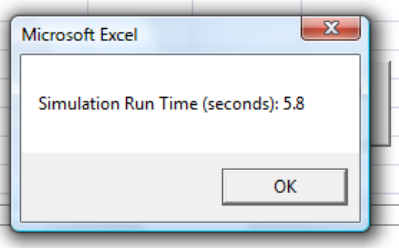
Paths	TTF of Path	TTF of System	Trials	MTTF	Successes	Failures	Simulated Reliability	Calculated Reliability
14	2548.0137	2548.013698	5000	3171.140537	3984	1016	0.7968	0.8067466
234	715.381578		Check					
25	1615.28108		0					
			0					

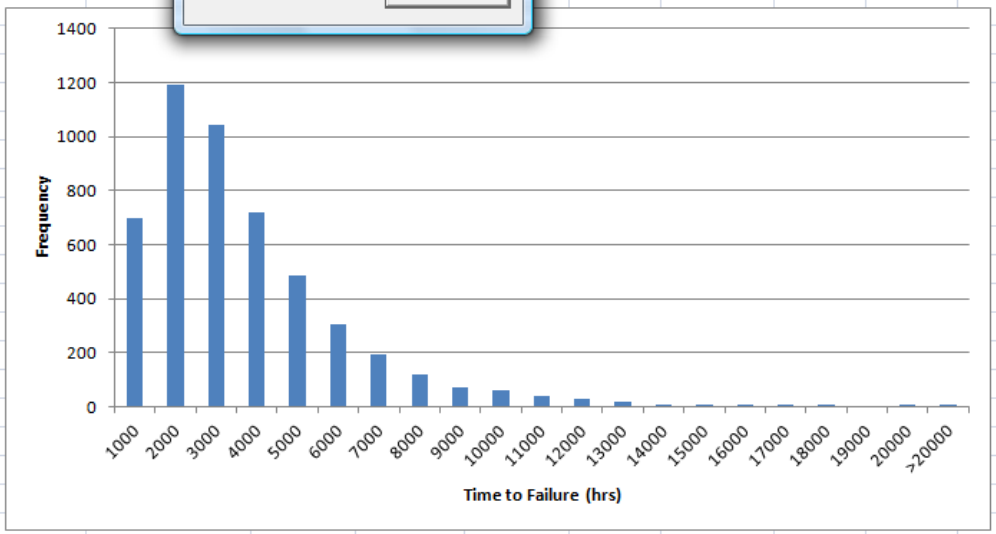
Cuts	TTF of Cut	TTF of System
12	11484.3479	2548.013698
45	4160.64199	
24	11484.3479	
135	2548.0137	

Histogram BIN	Frequency	Cumulative Frequency
1000	0	697
2000	0	1190
3000	1	1042
4000	0	721
5000	0	487
6000	0	304
7000	0	194
8000	0	120
9000	0	72
10000	0	62
11000	0	38
12000	0	27
13000	0	17
14000	0	9

System







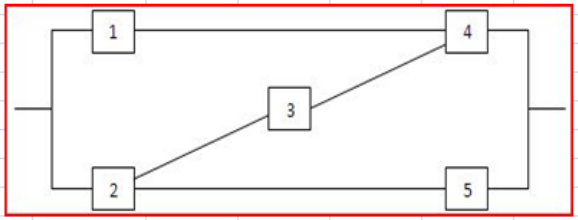
Sheet1 Sheet2 Sheet3

Simulating System Reliability

Component	Lambda	Reliability	Time to Failure	Component Working	Number of Iterations	Screen Updating?
1	0.0001	0.886920437	2548.013698	1	5000	Off
2	0.0002	0.786627861	11484.34793	1		
3	0.00015	0.835270211	715.3815777	1		
4	0.0003	0.697676326	4160.641988	1		
5	0.0005	0.548811636	1615.281084	0		

Method	Structure Function	Check
keystone	1	0
paths	1	0
cuts	1	

System



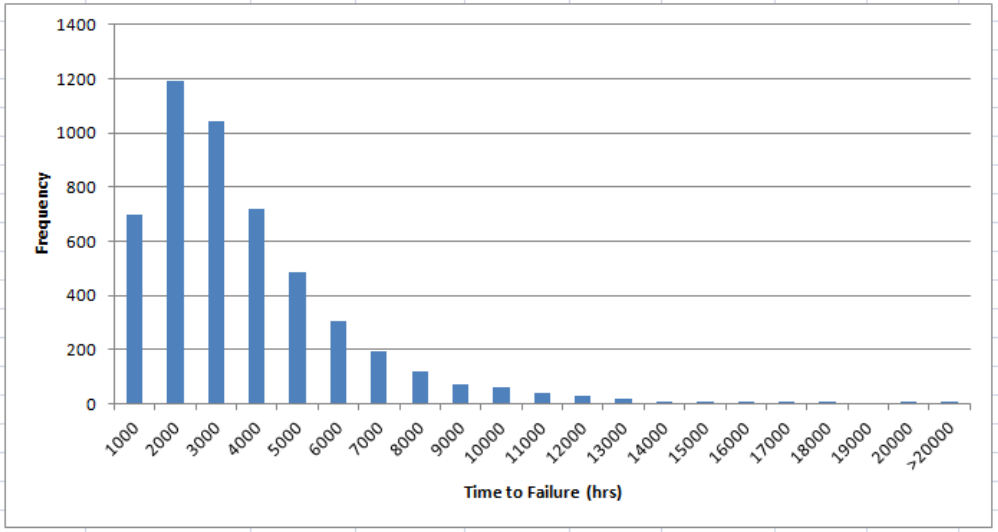
time 1200 hrs

Paths	TTF of Path	TTF of System	Trials	MTTF	Successes	Failures	Simulated Reliability	Calculated Reliability
14	2548.0137	2548.013698	5000	3171.140537	3984	1016	0.7968	0.8067466
234	715.381578		Check					
25	1615.28108		0					
			0					

Reset Data and Run New Simulation

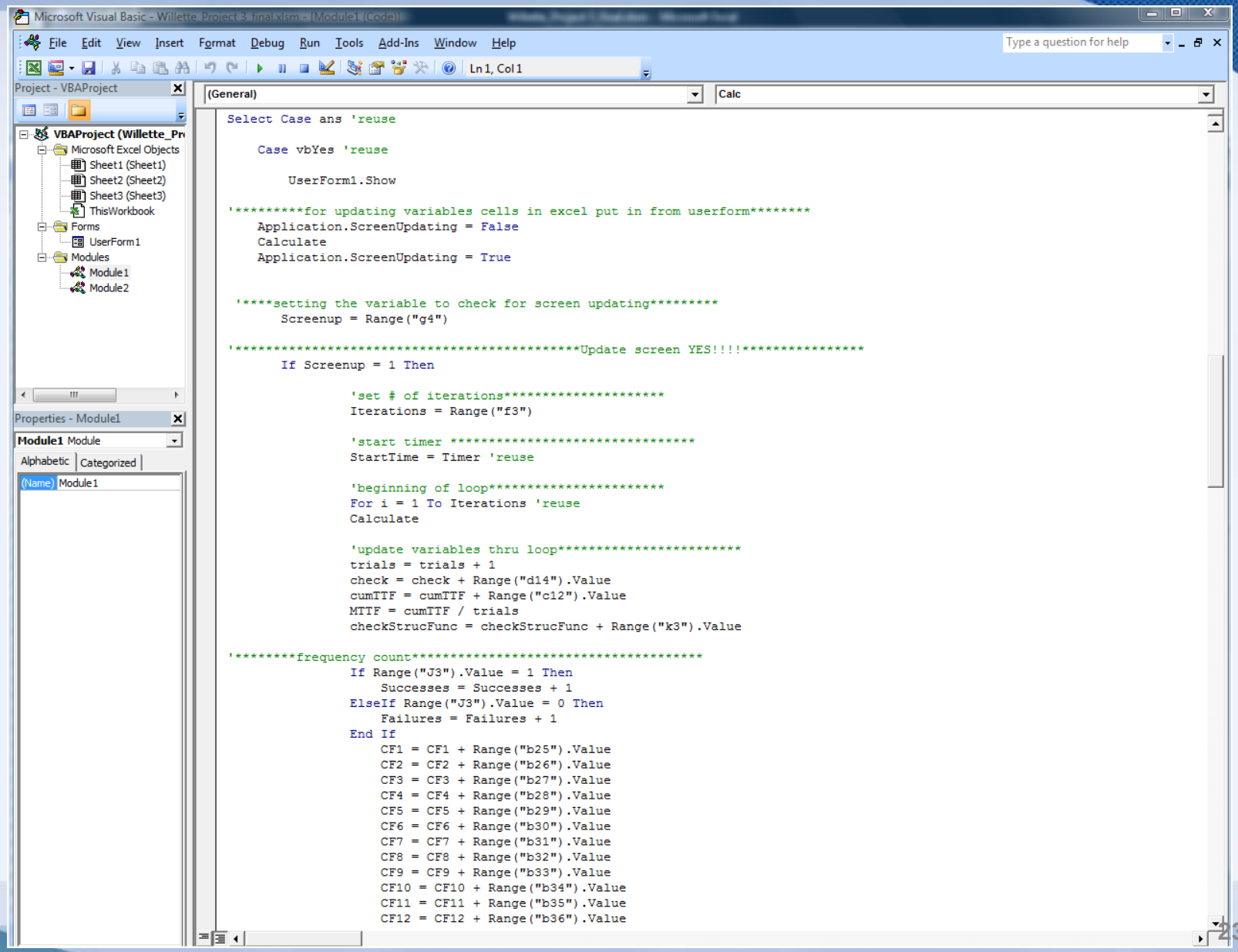
Cuts	TTF of Cut	TTF of System
12	11484.3479	2548.013698
45	4160.64199	
24	11484.3479	
135	2548.0137	

Histogram BIN	Frequency	Cumulative Frequency
1000	0	697
2000	0	1190
3000	1	1042
4000	0	721
5000	0	487
6000	0	304
7000	0	194
8000	0	120
9000	0	72
10000	0	62
11000	0	38
12000	0	27
13000	0	17
14000	0	9



Sheet1 Sheet2 Sheet3

VBA Code


Microsoft Visual Basic - Willette_Project 3_final.xlsm - [Module1 (Code)]
File Edit View Insert Format Debug Run Tools Add-Ins Window Help
Type a question for help
Ln 1, Col 1
Project - VBAProject
VBAProject (Willette_Pr
Microsoft Excel Objects
Sheet1 (Sheet1)
Sheet2 (Sheet2)
Sheet3 (Sheet3)
ThisWorkbook
Forms
UserForm1
Modules
Module1
Module2
Properties - Module1
Module1 Module
Alphabetic Categorized
(Name) Module1
(General)
Calc
Select Case ans 'reuse
Case vbYes 'reuse
UserForm1.Show
'*********for updating variables cells in excel put in from userform********
Application.ScreenUpdating = False
Calculate
Application.ScreenUpdating = True
'****setting the variable to check for screen updating*********
Screenup = Range("g4")
'***Update screen YES!!!!*****************
If Screenup = 1 Then
'set # of iterations********************
Iterations = Range("f3")
'start timer *******************************
StartTime = Timer 'reuse
'beginning of loop**********************
For i = 1 To Iterations 'reuse
Calculate
'update variables thru loop************************
trials = trials + 1
check = check + Range("d14").Value
cumTTF = cumTTF + Range("c12").Value
MTTF = cumTTF / trials
checkStrucFunc = checkStrucFunc + Range("k3").Value
'********frequency count*************************************
If Range("J3").Value = 1 Then
Successes = Successes + 1
ElseIf Range("J3").Value = 0 Then
Failures = Failures + 1
End If
CF1 = CF1 + Range("b25").Value
CF2 = CF2 + Range("b26").Value
CF3 = CF3 + Range("b27").Value
CF4 = CF4 + Range("b28").Value
CF5 = CF5 + Range("b29").Value
CF6 = CF6 + Range("b30").Value
CF7 = CF7 + Range("b31").Value
CF8 = CF8 + Range("b32").Value
CF9 = CF9 + Range("b33").Value
CF10 = CF10 + Range("b34").Value
CF11 = CF11 + Range("b35").Value
CF12 = CF12 + Range("b36").Value