

System Reliability

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Teaching:

Reliability Engineering

Electronics Manufacturing Systems

Quality Design and Control

Research:

Reliability of Electronic Components and Assemblies

Fatigue and Damage Accumulation in Solder Materials



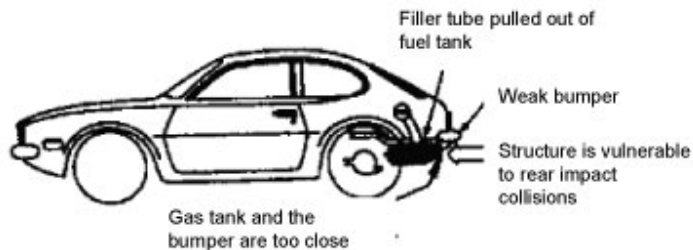


Agenda

- Introduction
- Failure Distributions
 - Constant Failure Rate (Exponential Distribution)
 - Time Dependent Failure Rate (Weibull Distribution)
- Reliability of Serial System
- Reliability of Parallel System
- Reliability of Combined System
- Reliability of Network System

Introduction

- Things Fail!
- 1978 - Ford Pinto: fuel tank fire in rear-end collisions
 - Deaths, lawsuits, and negative publicity (recall then discontinue production)



Introduction

- Things Fail!
- 2016 – Samsung Note 7
 - Battery catches fire
- Southwest Airlines (Louisville to Baltimore)
 - Burn the plane's carpet and caused some damage to its subfloor
- Reliability engineers attempt to study, characterize, measure, and analyze the failure in order to eliminate the likelihood of failures





Are Failures Random?

- Common approach taken in reliability is to treat failures as random or probabilistic events
- In theory, if we were able to comprehend the exact physics and chemistry of a failure process, failures could be predicted with certainty
- With incomplete knowledge of the physical/chemical processes which cause failures, failures will appear to occur at random over time
- This random process may exhibit a pattern which can be modeled by some probability distribution (i.e. Weibull)

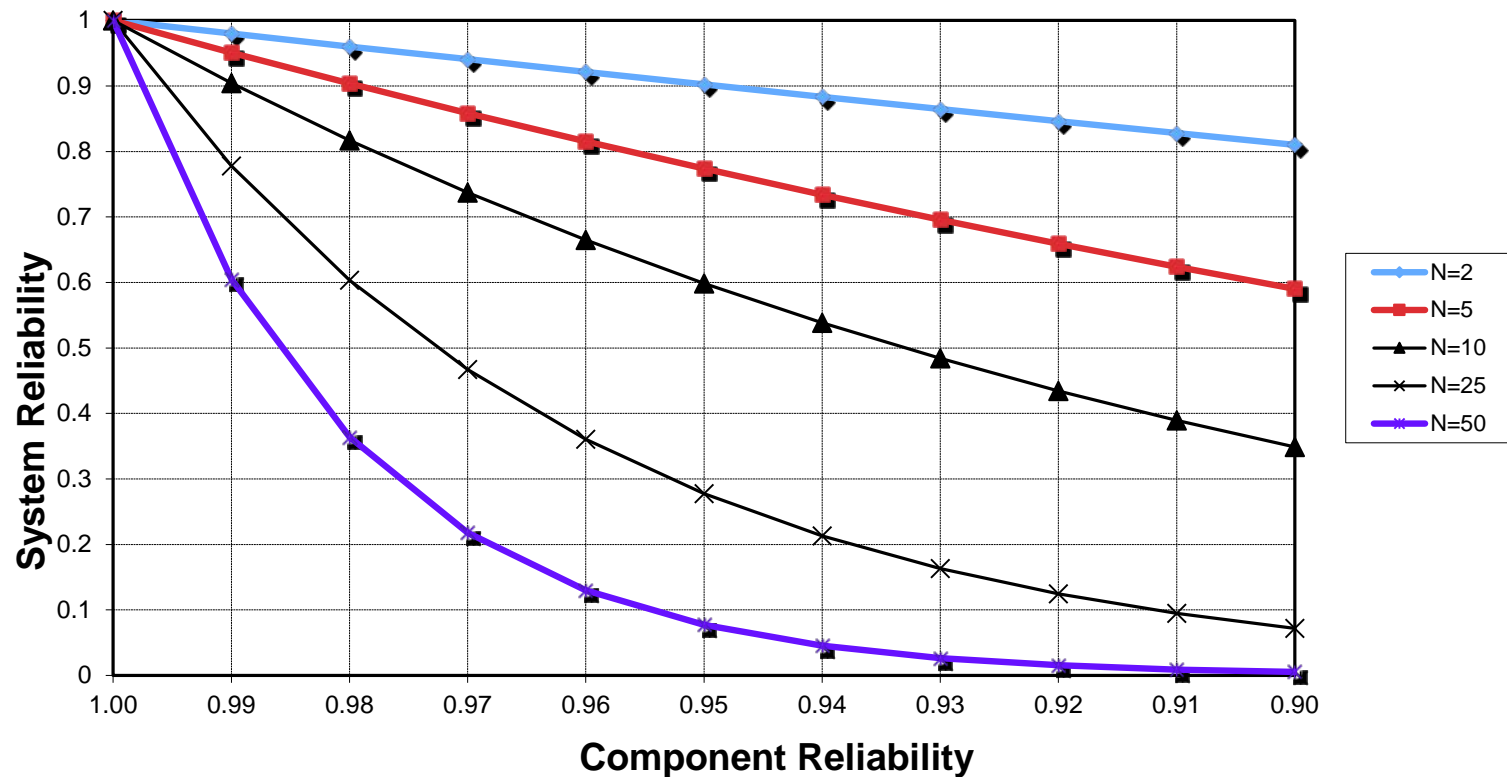


Reliability?

- Reliability is the probability that a component or system will perform a required function for a given period of time when used under stated operating conditions
- $R(t)$ = It is the probability of non-failure
- More focus on reliability
 - System complexity
 - Cost of failures
 - Public awareness of product quality and reliability
 - New regulations concerning product liability

Complexity and Reliability

For serial System





The Reliability Function, $R(t)$

- Reliability is defined as the probability that a system (component) will function over some time period t
- Let T = a random variable, the time to failure of a component
- $R(t)$ is the probability that the time to failure is greater than or equal to t

$$R(t) = Pr\{T \geq t\}$$

where $R(t) \geq 0$, $R(0) = 1$, and

$$\lim_{t \rightarrow \infty} R(t) = 0$$

Often called the **SURVIVAL FUNCTION**



The Failure Function, $F(t)$

- $F(t)$ is the probability that a failure occurs before time t

$$F(t) = 1 - R(t) = \Pr \{T < t\}$$

where $F(0) = 0$ and $\lim_{t \rightarrow \infty} F(t) = 1$

- It is the cumulative distribution function (CDF) of the failure distribution

Reliability

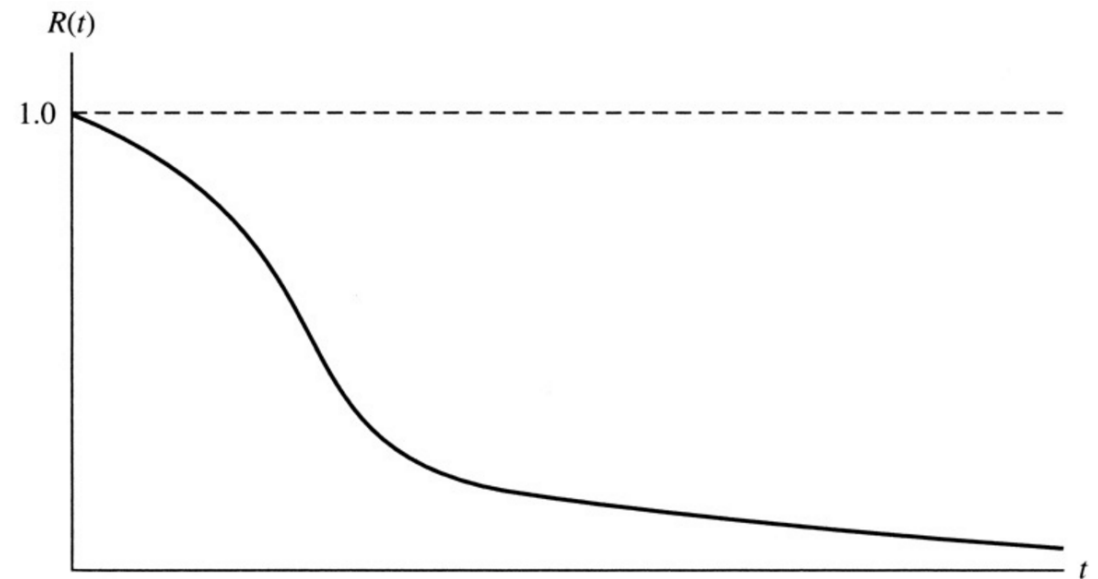
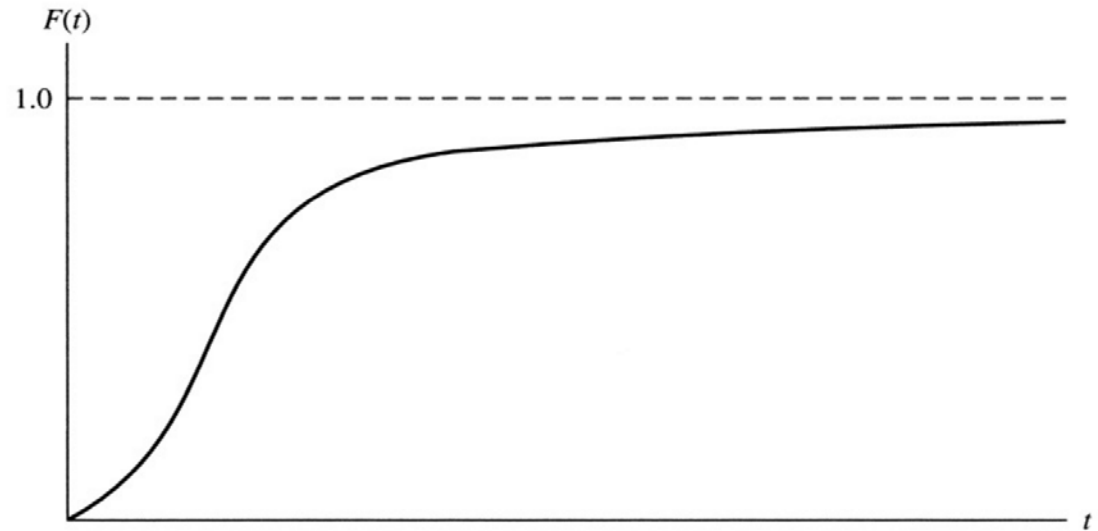
Failure function

$$F(t) = \int_0^t f(t') dt'$$

$f(t)$ is Probability
Density Function

Reliability function

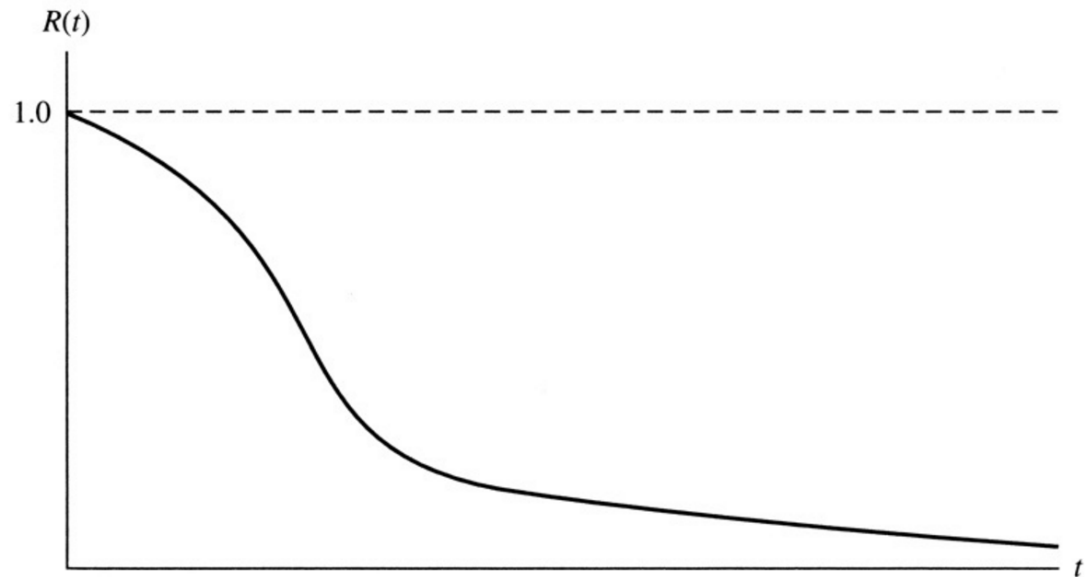
$$R(t) = \int_t^{\infty} f(t') dt'$$



Mean Time to Failure

- It is the average time of survival

$$MTTF = \int_0^{\infty} R(t) dt$$



(a)



Failure Rate Function, $\lambda(t)$

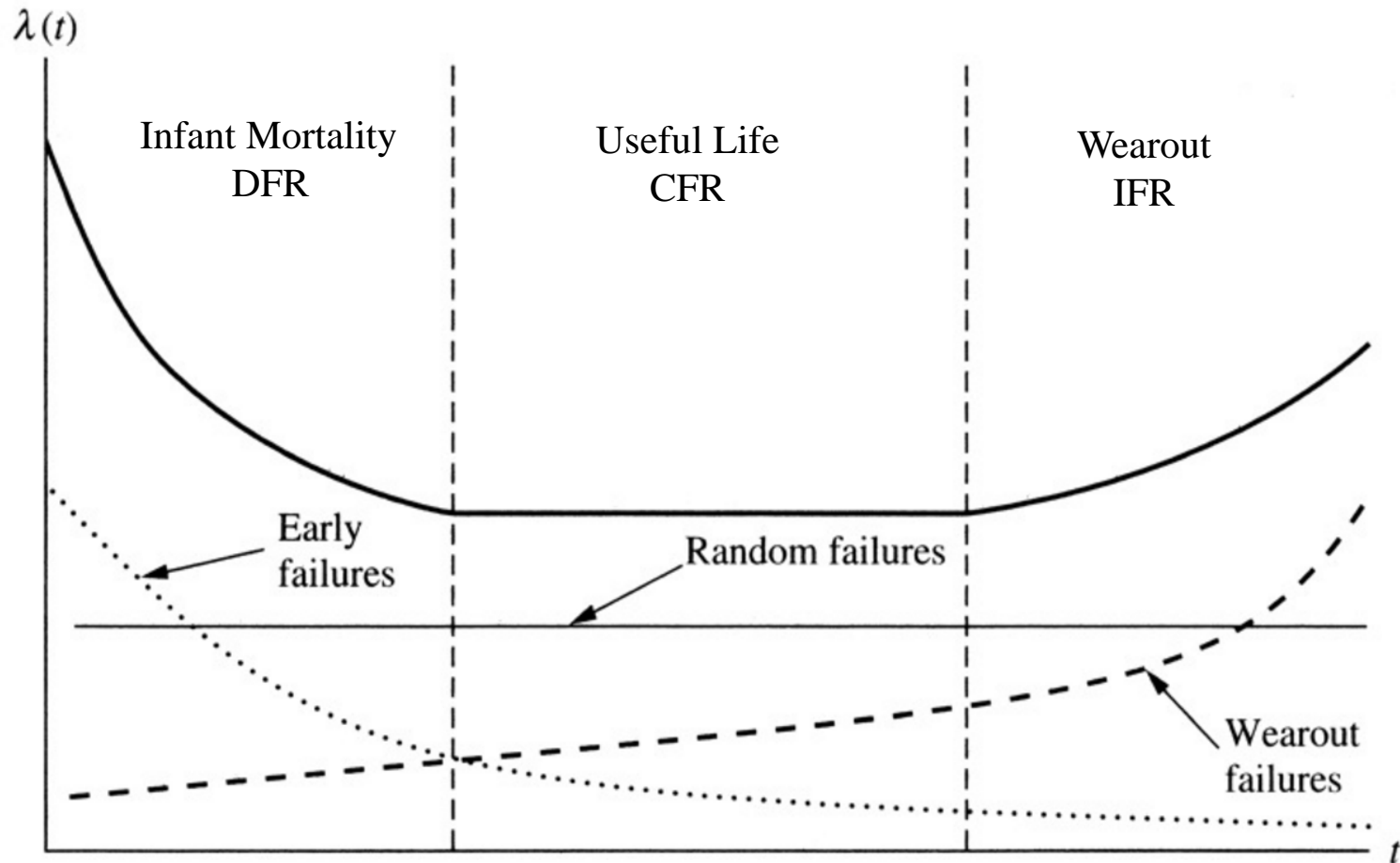
- Failure rate is expressed as a function of time
- Mathematically, failure rate equals probability density function divided by reliability function:

$$\lambda(t) = \frac{f(t)}{R(t)}$$

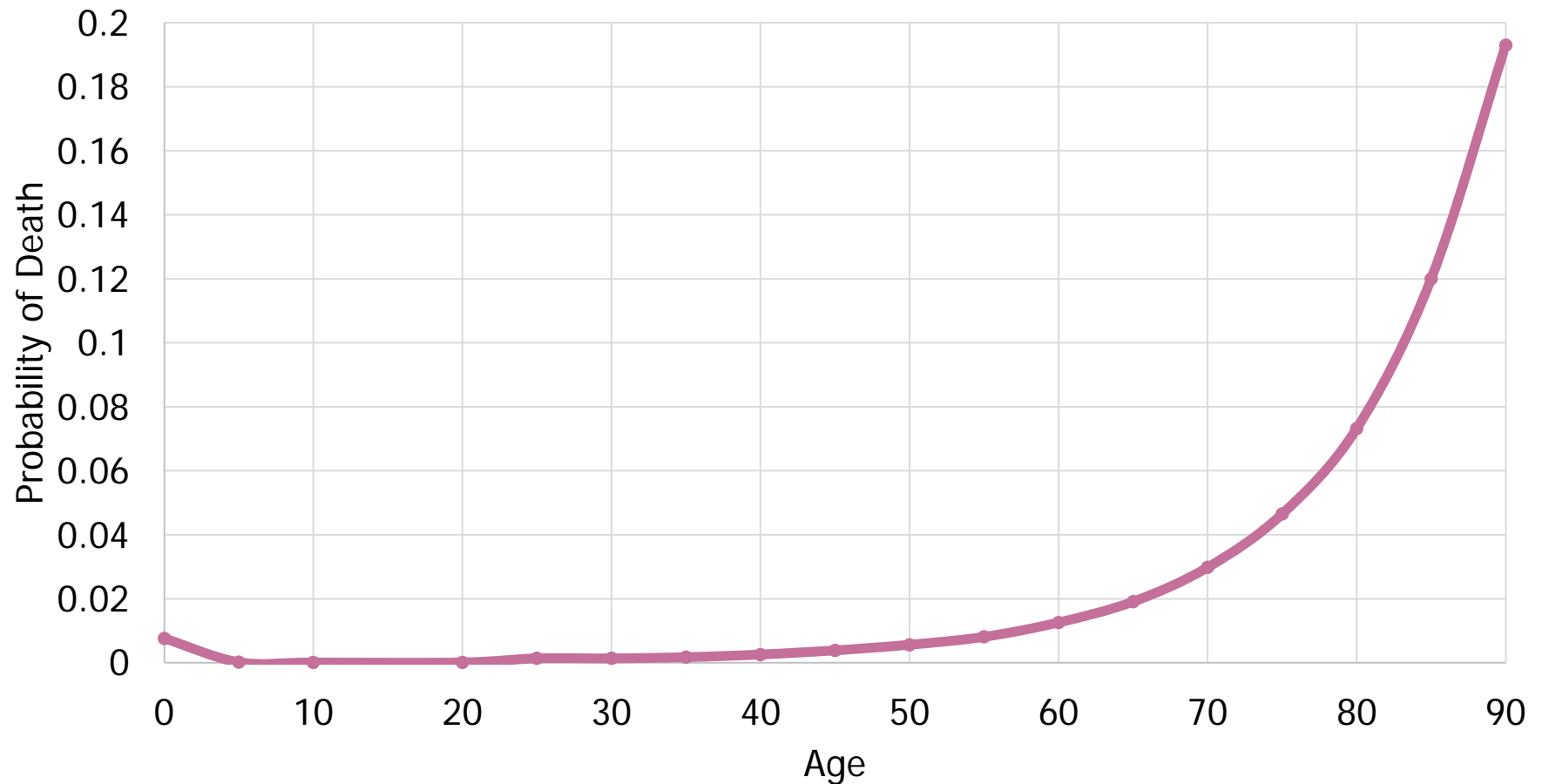
$$R(t) = \int_t^{\infty} f(t') dt' = \exp \left[- \int_0^t \lambda(t') dt' \right]$$

- Failure rates can be characterized as:
 - Increasing Failure Rate (IFR) when $\lambda(t)$ increasing
 - Decreasing Failure Rate (DFR) when $\lambda(t)$ decreasing
 - Constant Failure Rate (CFR) when $\lambda(t)$ increasing constant

Bathtub Curve



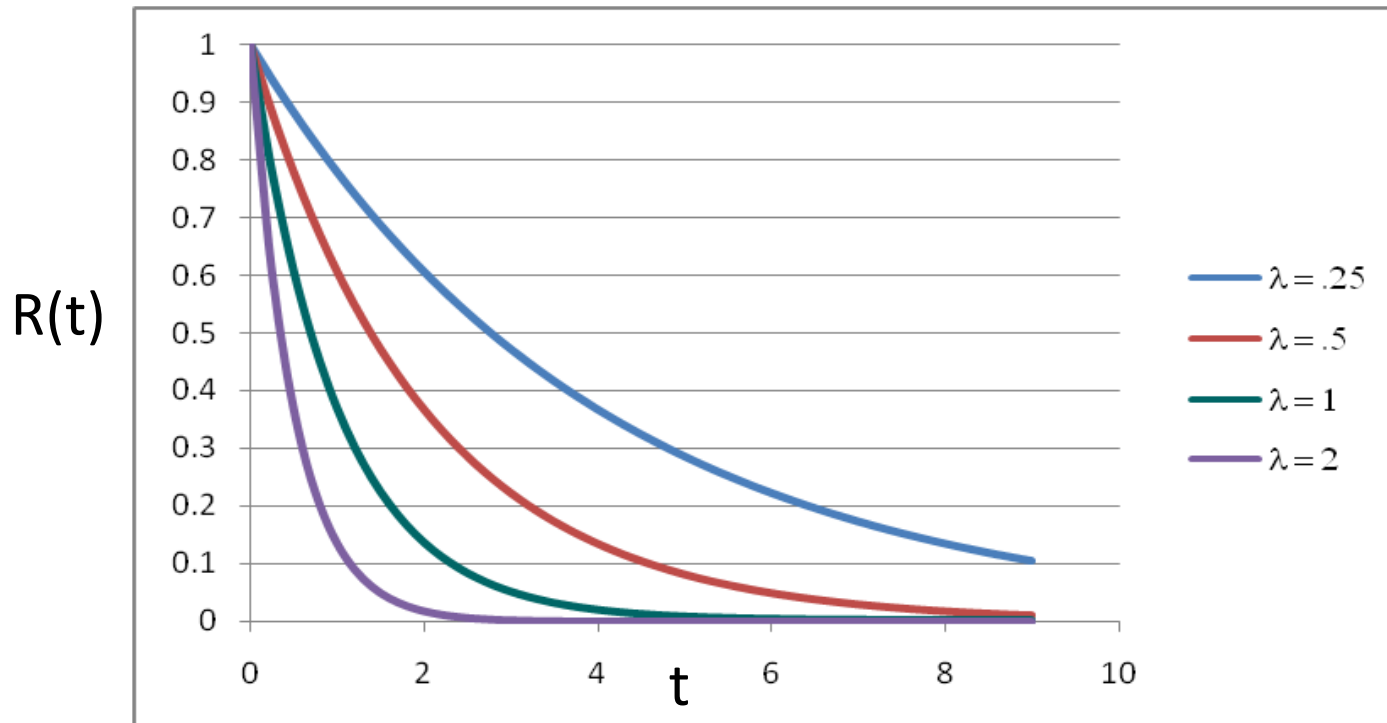
Human Mortality Curve



Exponential Distribution

- A failure distribution that has a constant failure rate is called an exponential probability distribution

$$\lambda(t) = \lambda \longrightarrow R(t) = \exp(-\lambda t)$$





Weibull Distribution

- The most useful probability distributions in reliability is the Weibull
- Used to model increasing, decreasing, or constant failure rates
- The Weibull failure rate function:

$$\lambda(t) = at^b$$

- $\lambda(t)$ is increasing for $b > 0$, decreasing for $b < 0$
constant for $b = 0$



Weibull Distribution

- For mathematical convenience it is better to express $\lambda(t)$ in the following manner:

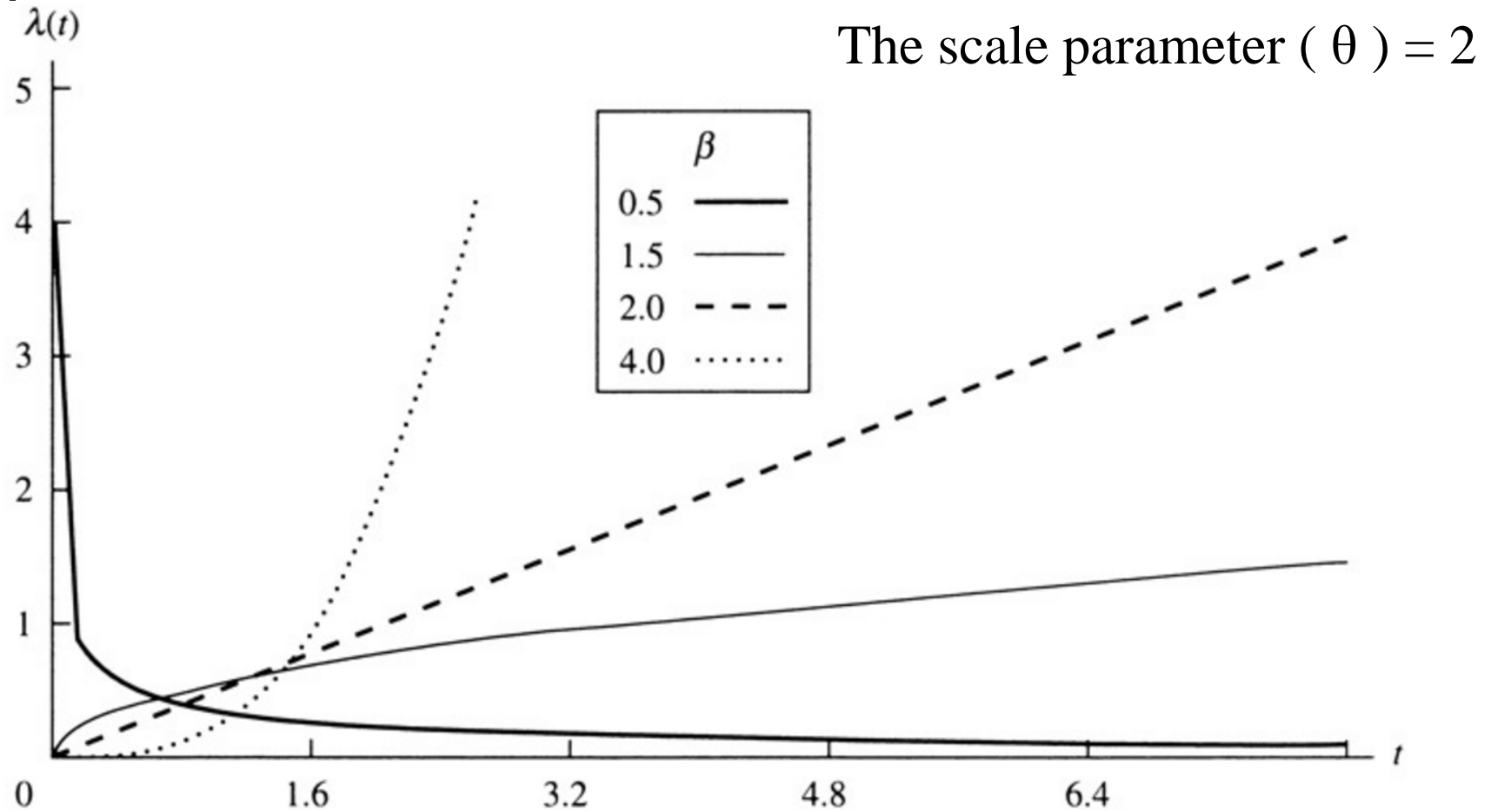
$$\lambda(t) = at^b \quad \longrightarrow \quad \lambda(t) = \frac{\beta}{\theta} \left(\frac{t}{\theta} \right)^{\beta-1}$$

β is the shape parameter

θ is the scale parameter (characteristic life)

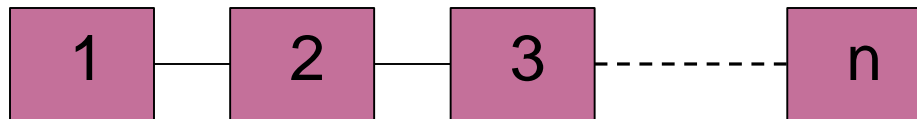
$$R(t) = e^{-\left(\frac{t}{\theta} \right)^\beta}$$

Weibull Distribution



Reliability of Serial System

- Reliability Block Diagram



- How do we calculate the Reliability of this system?
- Go back to the basic probability:

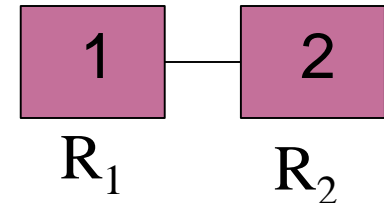
E_1 = the event, component 1 does not fail

E_2 = the event, component 2 does not fail

$P\{E_1\} = R_1$ and $P\{E_2\} = R_2$ where

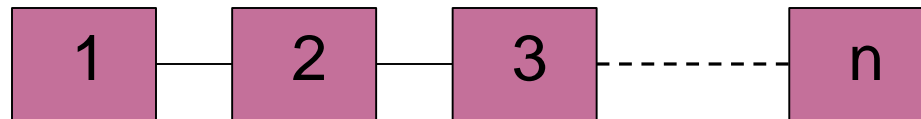
Therefore assuming independence:

$$R_s = P\{E_1 \cap E_2\} = P\{E_1\} P\{E_2\} = R_1 R_2$$



Reliability of Serial System

- Reliability Block Diagram



- Generalizing to n mutually independent components in series:

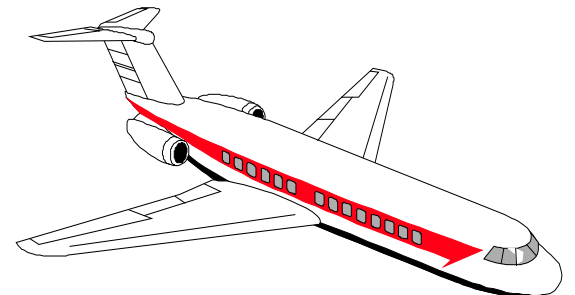
$$R_s(t) = R_1 R_2 \dots R_n$$

- For Serial System:

$$R_s(t) \leq \min \{R_1, R_2, \dots, R_n\}$$

Exercise

- The failure distribution of the main landing gear of a commercial airliner is Weibull with a shape parameter of 1.6 and a characteristic life of 10,000 landings.
- The nose gear also has a Weibull distribution with a shape parameter of 0.90 and a characteristic life of 15,000 landings.
- What is the reliability of the landing gear system if the system is to be overhauled after 1,000 landings?



Exercise

- For Weibull: $R(t) = \exp - \left(\frac{t}{\theta}\right)^\beta$
- What is the system reliability after 1000 landing?

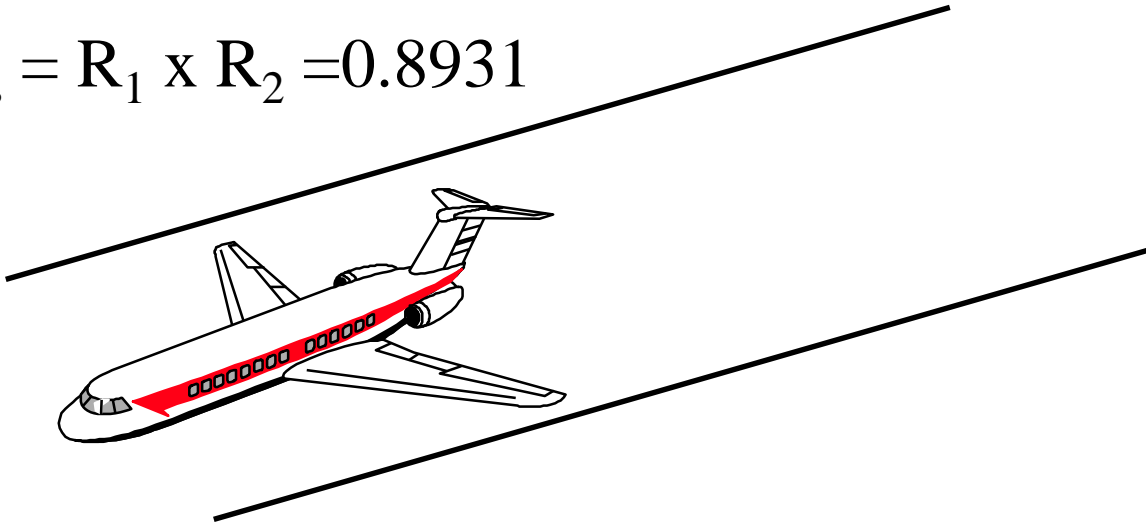
$$\begin{array}{l} R_1 \\ \theta = 10,00 \\ \beta = 1.6 \end{array}$$

$$\begin{array}{l} R_2 \\ \theta = 15,00 \\ \beta = 0.9 \end{array}$$

$$R_1 = \exp - \left(\frac{1,000}{10,000}\right)^{1.6} = 0.975$$

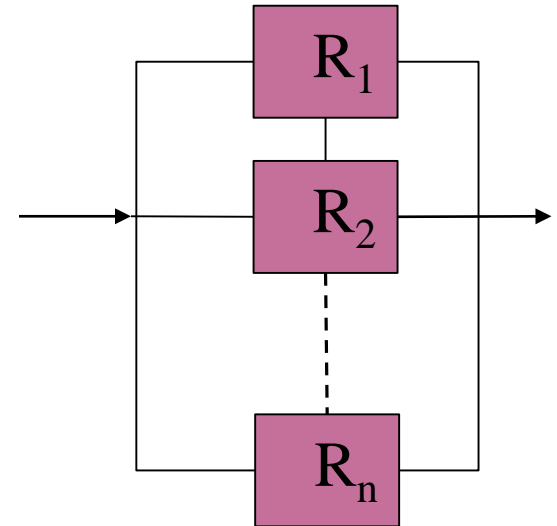
$$R_2 = \exp - \left(\frac{1,000}{15,000}\right)^{0.9} = 0.916$$

$$R_s = R_1 \times R_2 = 0.8931$$



Reliability of Parallel System

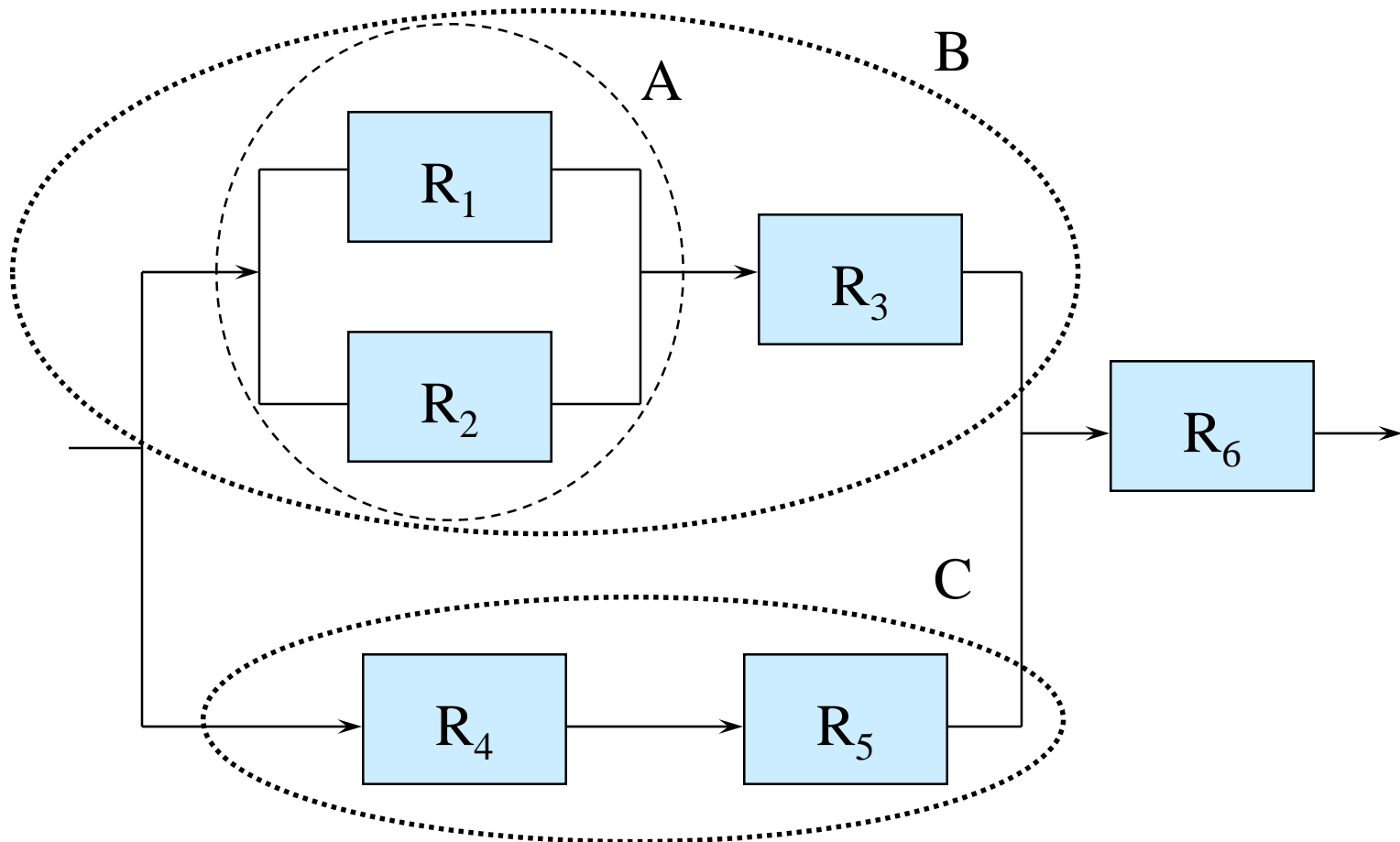
- Reliability Block Diagram
- Reliability of parallel system is the probability that at least one component does NOT fail!



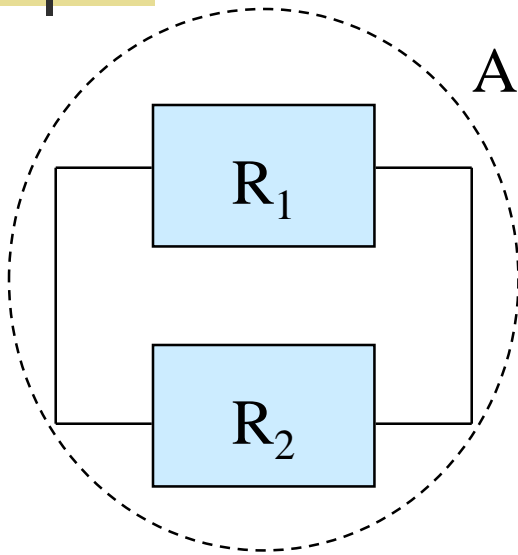
$$R_s(t) = 1 - [(1 - R_1)(1 - R_2) \dots (1 - R_n)]$$

- For Parallel System:
 $R_s(t) \geq \max \{R_1, R_2, \dots, R_n\}$

Combined Series - Parallel Systems

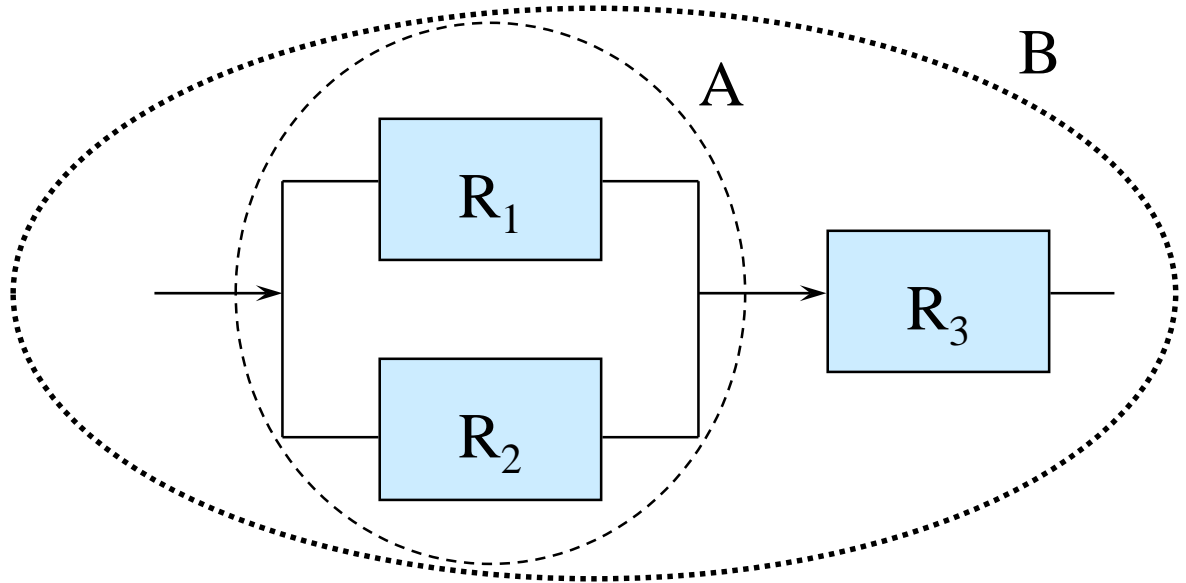


Combined Series - Parallel Systems

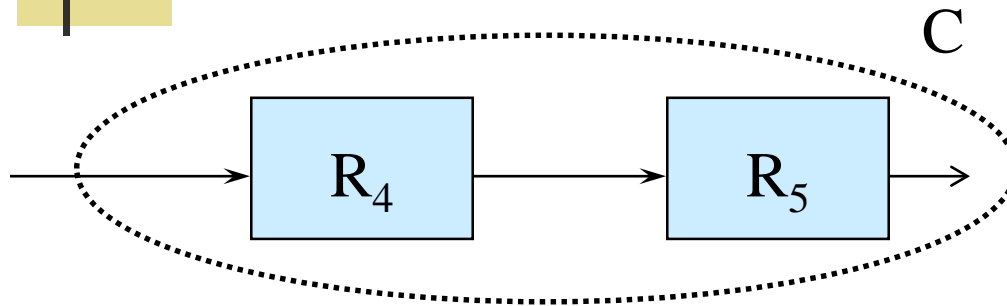


$$R_A = [1 - (1 - R_1) (1 - R_2)]$$

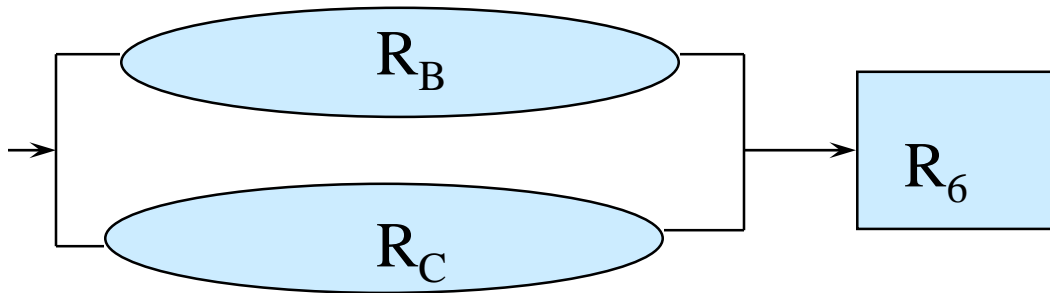
$$R_B = R_A R_3$$



Combined Series - Parallel Systems

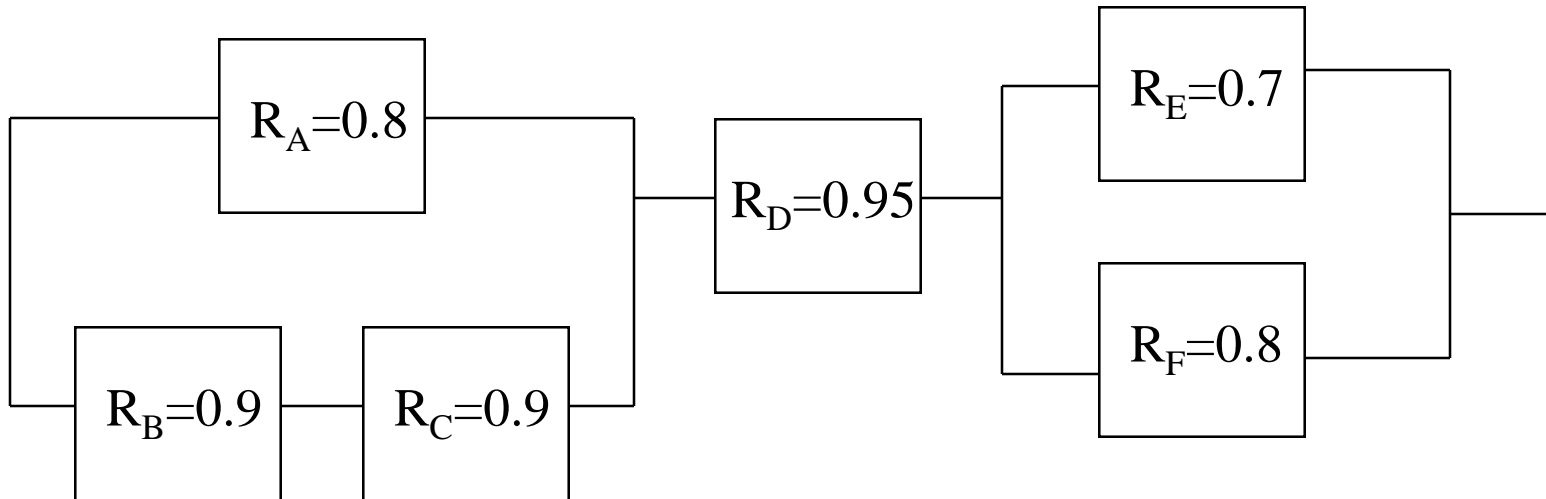


$$R_C = R_4 R_5$$



$$R_s = [1 - (1 - R_B) (1 - R_c)] R_6$$

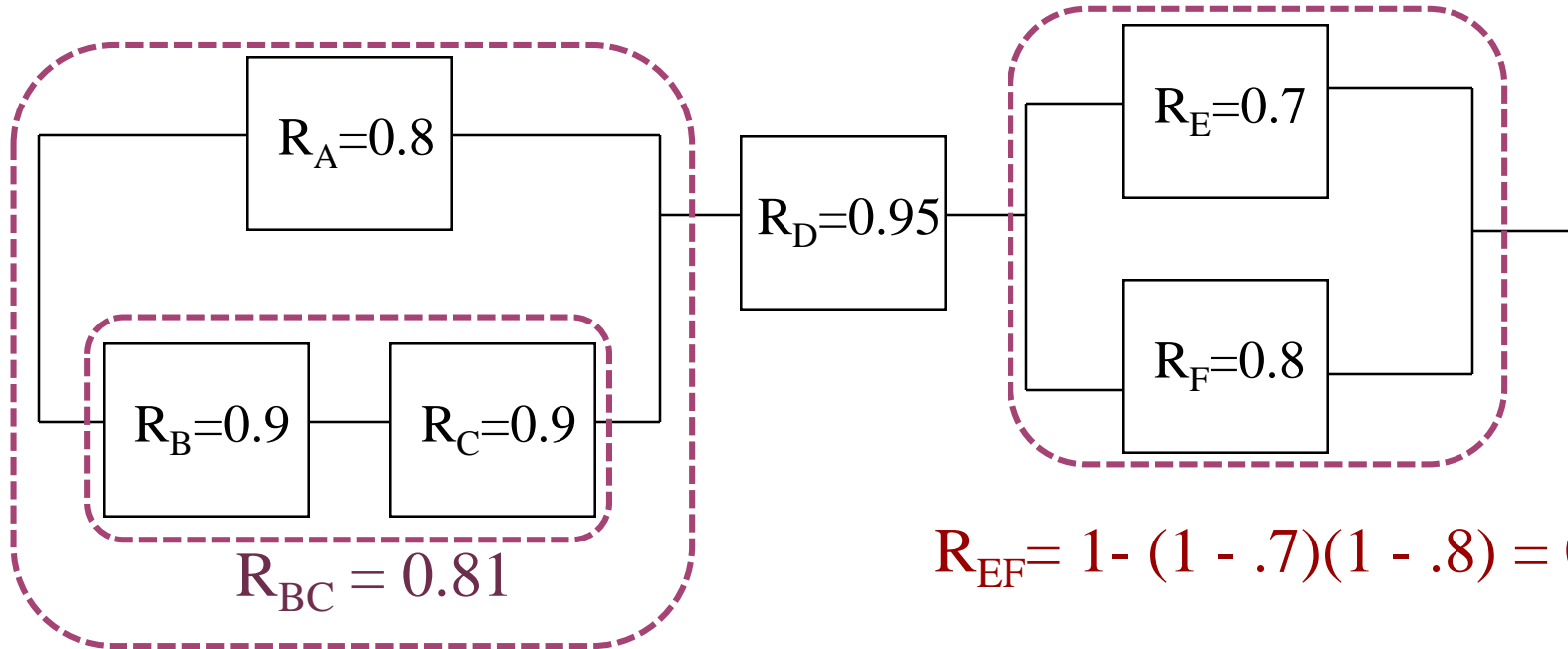
Exercise



Help! Can you calculate the reliability of this block diagram?



Exercise



$$R_{EF} = 1 - (1 - .7)(1 - .8) = 0.94$$

$$R_{ABC} = 1 - (1 - .81)(1 - .8) = 0.962$$

$$R_s = (0.962) (0.95) (0.94) = 0.859$$



k-out-of-n Redundancy

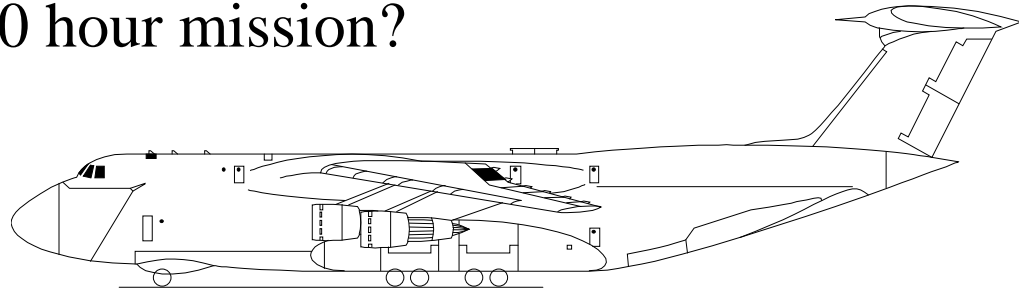
- Let n = the number of redundant, identical and independent components each having a reliability of R
- Let k = the number of components that must operate for the system to operate
- The reliability of the system (from binomial distribution):

$$R_s = \sum_{x=k}^n \binom{n}{x} R^x (1 - R)^{n-x},$$

$$\text{where } \binom{n}{x} = \frac{n!}{x!(n-x)!}$$

A Very Good Example

Out of the 12 identical AC generators on the C-5 aircraft, at least 9 of them must be operating in order for the aircraft to complete its mission. Failures are known to follow an exponential distribution with a failure rate of 0.01 failure per hour. What is the reliability of the generator system over a 10 hour mission?



For exponential distribution:

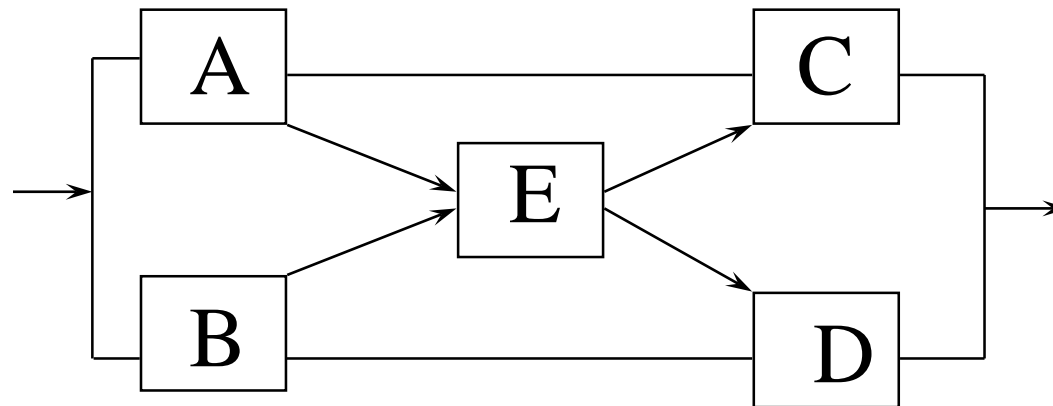
$$R(t) = \exp(-\lambda t)$$

$$R(10) = \exp(-0.01 * 10) = 0.905$$

$$R_s = \sum_{x=k}^n \binom{n}{x} R^x (1 - R)^{n-x} = \sum_{x=9}^{12} \binom{12}{x} .905^x (1 - .905)^{12-x} = 0.978$$

Reliability of Complex Configurations

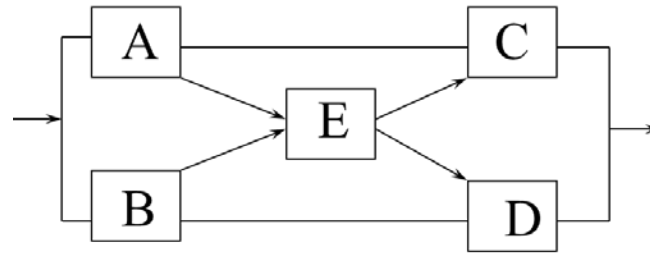
- Network:



- Two approaches:
 1. Decomposition Approach
 2. Enumeration Method

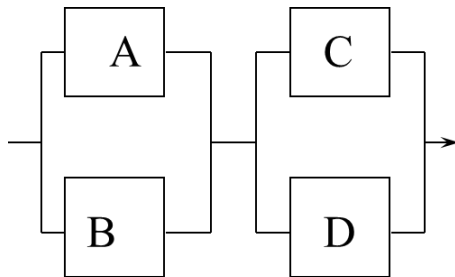
Decomposition Approach

- Decompose the network to combined (parallel and serial) system



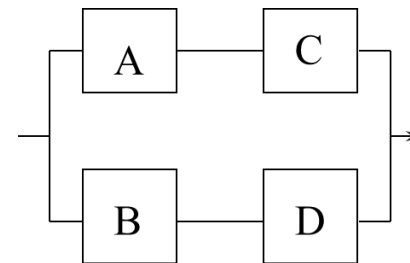
Case I: If E does not fail

Probability = R_E



Case II: If E fails

Probability = $1 - R_E$

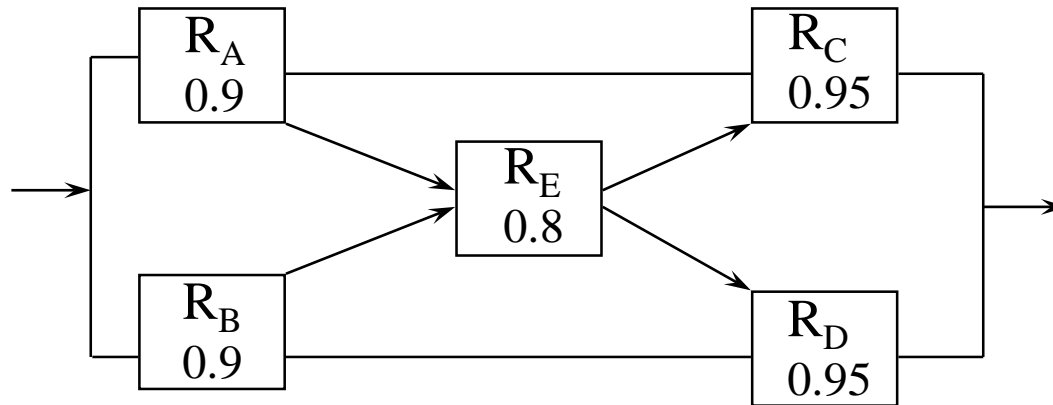


System Reliability = Σ (R of each Case x Case Probability)

$$R_s = R_{\text{Case I}} \times R_E + R_{\text{case II}} \times (1 - R_E)$$

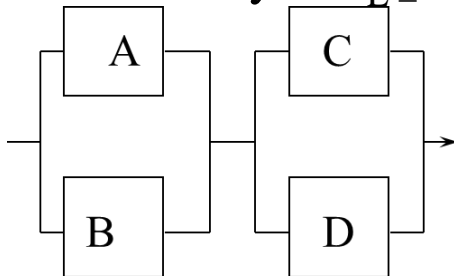
Decomposition Approach

- Example: Calculate the reliability of this system:



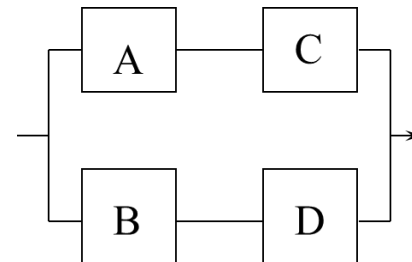
Case I: If E does not fail

Probability = $R_E = 0.8$



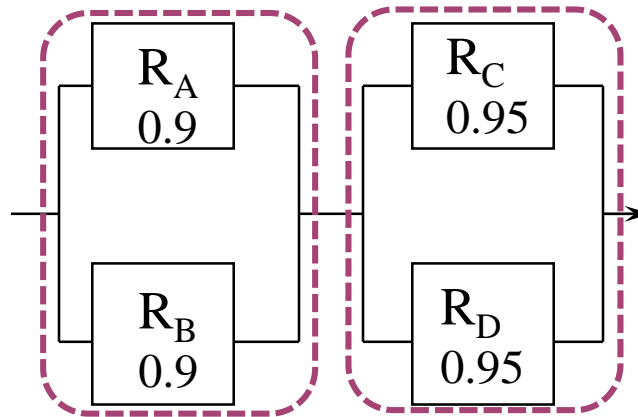
Case II: If E fails

Probability = $1 - R_E = 0.2$



Decomposition Approach

- Case I:
Probability = 0.8



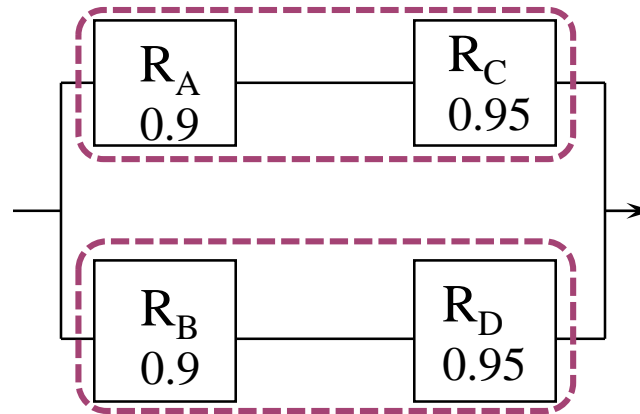
$$R_{\text{CaseI}} = [1 - (1 - R_A)(1 - R_B)] \times [1 - (1 - R_C)(1 - R_D)]$$

$$R_{\text{CaseI}} = [1 - (1 - 0.9)(1 - 0.9)] \times [1 - (1 - 0.95)(1 - 0.95)]$$

$$R_{\text{CaseI}} = 0.9875$$

Decomposition Approach

- Case II:
Probability = 0.2



$$R_{\text{CaseII}} = 1 - (1 - R_A R_C) (1 - R_B R_D)$$

$$R_{\text{CaseII}} = 1 - (1 - 0.9 \times 0.95) (1 - 0.9 \times 0.95)$$

$$R_{\text{CaseII}} = 0.979$$

- System Reliability:

System Reliability = Σ (R of each Case x Case Probability)

$$R_S = 0.8 \times 0.9875 + 0.2 \times 0.979$$

$$R_S = 0.9858$$

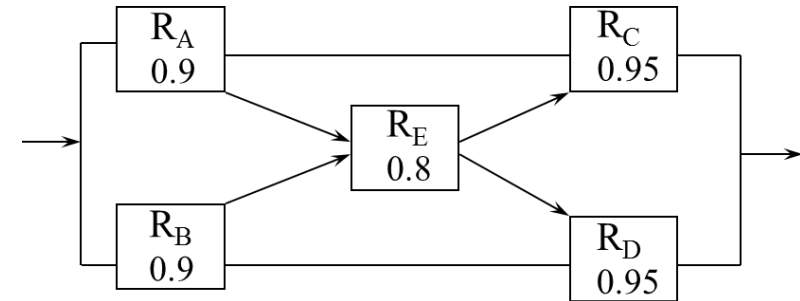


Enumeration Method

- Identify all possible combinations of success (S) or failure (F) of each component and the resulting success or failure of the system
- Calculate the probability of intersection of each possible combination of component successes or failures that leads to system success
- System reliability is the sum of the success probabilities

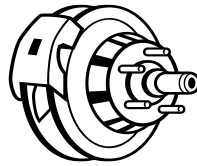
Enumeration Method

S = success, F = failure



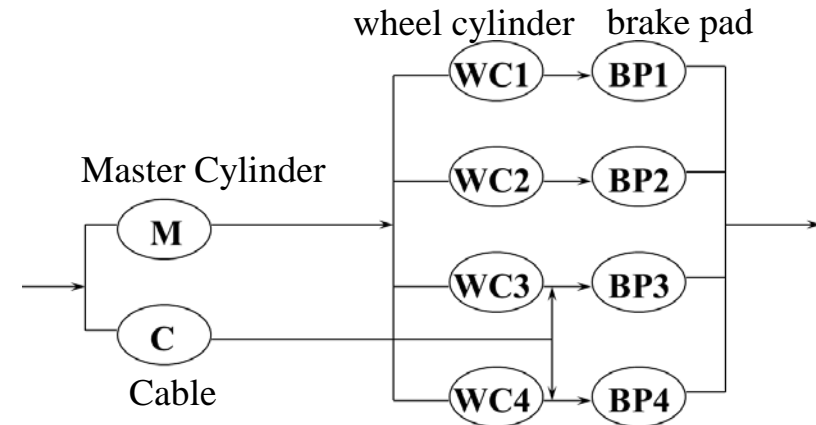
A	B	C	D	E	System	Probability
S	S	S	S	S	S	0.584820
F	S	S	S	S	S	0.064980
S	F	S	S	S	S	0.064980
S	S	F	S	S	S	0.030780
S	S	S	F	S	S	0.030780
S	S	S	S	F	S	0.146205
F	F	S	S	S	F	
S	F	F	S	S	S	0.003420
S	S	F	F	S	F	
⋮	⋮	⋮	⋮	⋮	⋮	
F	F	F	S	F	F	
F	F	F	F	F	F	
Total						0.985800

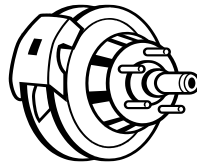
of combinations: $5^2 = 32$



Case Study: Automotive Braking System

- An automobile braking system consists of a fluid braking subsystem and a mechanical braking subsystem (parking brake)
- Both subsystems must fail in order for the system to fail
- The fluid braking subsystem will fail if the Master cylinder fails (M) (which includes the hydraulic lines) or all four wheel braking units fail
- A wheel braking unit will fail if either the wheel cylinder fails (WC1, WC2, WC3, WC4) or the brake pad assembly fails (BP1, BP2, BP3, BP4)
- The mechanical braking system will fail if the cable system fails (event C) or both rear brake pad assemblies fail (BP3, BP4)



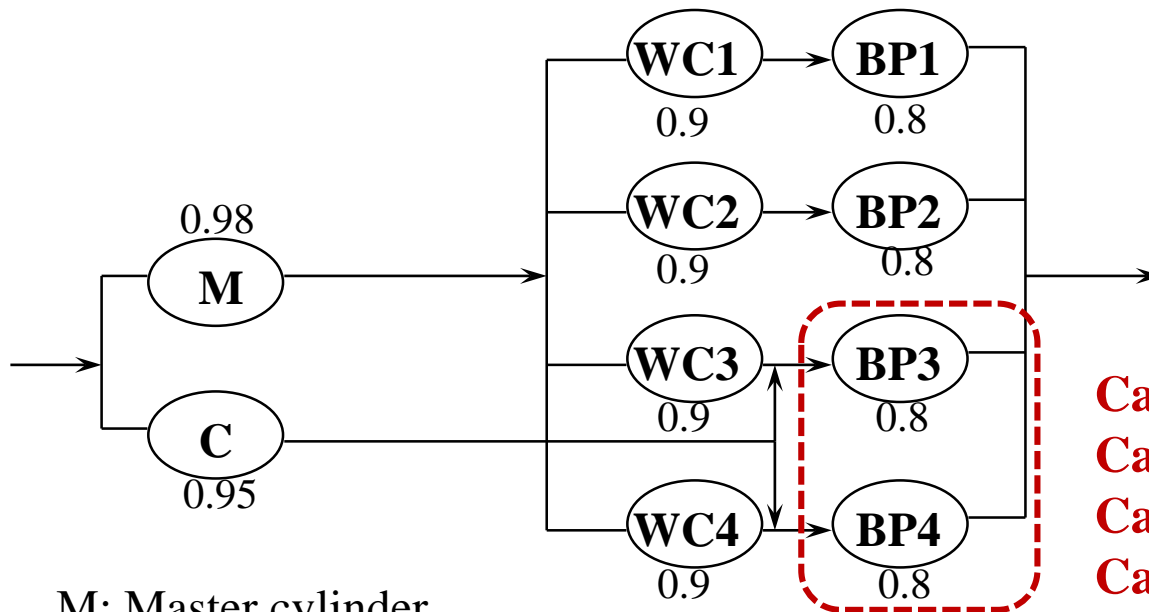


Case Study: Automotive Braking System

The reliability of driving 5k miles without brake maintenance:

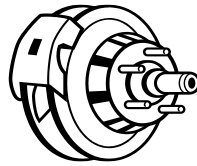
$$R_M = 0.98, R_C = 0.95, R_{WC} = 0.9, R_{BP} = 0.8$$

What is the reliability of the brake system?



Case I = BP3 fails & BP4 works
Case II = BP3 works & BP4 fails
Case III = both BP3 & BP4 fail
Case IV = both BP3 & BP4 work

M: Master cylinder
C: Cable
WC: Wheel Cylinder
BP: Brake Pad

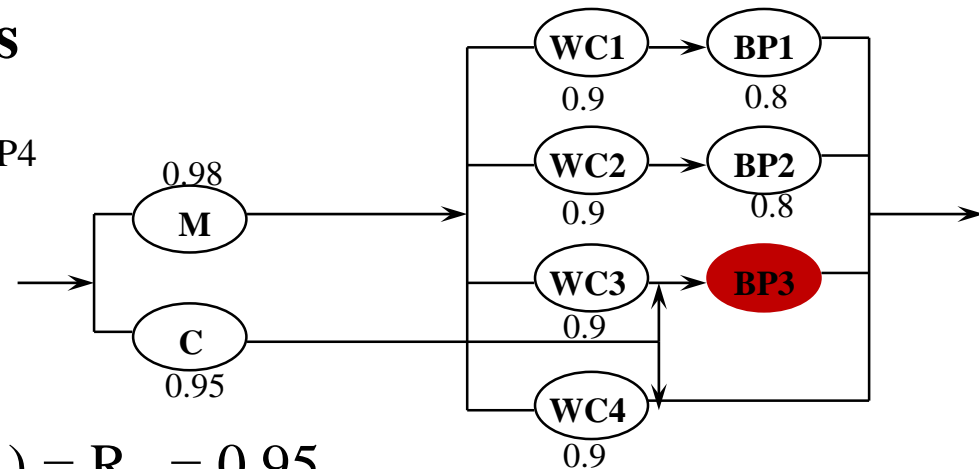


Case Study: Automotive Braking System

Case I = BP3 fails & BP4 works

$$\text{Probability } (P_I) = (1 - R_{BP3}) R_{BP4}$$

$$P_I = (1 - 0.8) 0.8 = 0.16$$



Reliability of parking brake (R_{park}) = $R_C = 0.95$

Reliability of hydraulic brake (R_H):

$$R_H = R_M [1 - (1 - R_{WC} R_{BP})^2 (1 - R_{WC})]$$

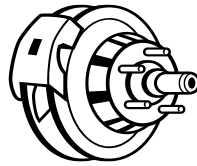
$$R_H = 0.98 [1 - (1 - 0.9 \times 0.8)^2 (1 - 0.9)]$$

$$R_H = 0.972$$

These two subsystems operate in parallel, therefore:

$$R_I = 1 - (1 - R_H) (1 - R_{\text{park}})$$

$$R_I = 1 - (1 - 0.972) (1 - 0.95) = 0.9986$$

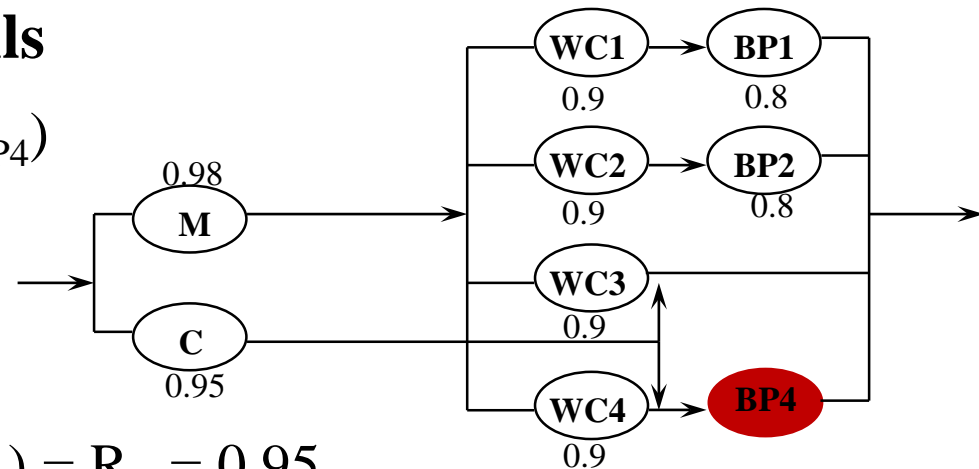


Case Study: Automotive Braking System

Case II = BP3 works & BP4 fails

$$\text{Probability } (P_{II}) = R_{BP3} (1 - R_{BP4})$$

$$P_{II} = 0.8 (1 - 0.8) = 0.16$$



Reliability of parking brake (R_{park}) = $R_C = 0.95$

Reliability of hydraulic brake (R_H):

$$R_H = R_M [1 - (1 - R_{WC} R_{BP})^2 (1 - R_{WC})]$$

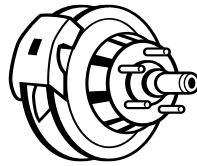
$$R_H = 0.98 [1 - (1 - 0.9 \times 0.8)^2 (1 - 0.9)]$$

$$R_H = 0.972$$

These two subsystems operate in parallel, therefore:

$$R_{II} = 1 - (1 - R_H) (1 - R_{\text{park}})$$

$$R_{II} = 1 - (1 - 0.972) (1 - 0.95) = 0.9986$$

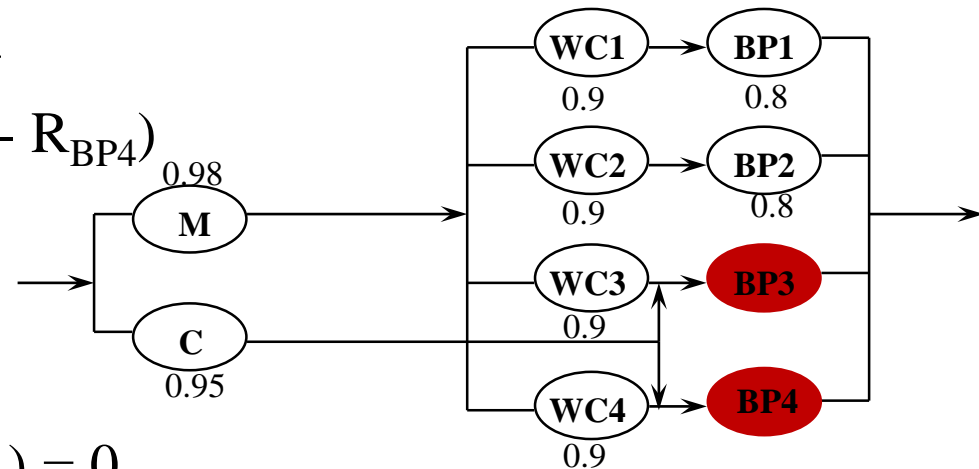


Case Study: Automotive Braking System

Case III = both BP3 & BP4 fail

$$\text{Probability } (P_{\text{III}}) = (1 - R_{\text{BP3}}) (1 - R_{\text{BP4}})$$

$$P_{\text{III}} = (1 - 0.8) (1 - 0.8) = 0.04$$



Reliability of parking brake ($R_{\text{park}} = 0$)

Reliability of hydraulic brake (R_H):

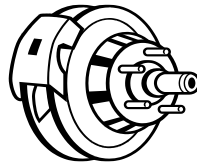
$$R_H = R_M [1 - (1 - R_{\text{WC}} R_{\text{BP}})^2]$$

$$R_H = 0.98 [1 - (1 - 0.9 \times 0.8)^2]$$

$$R_H = 0.903$$

The parking brake is not operating, therefore:

$$R_{\text{III}} = R_H = 0.903$$

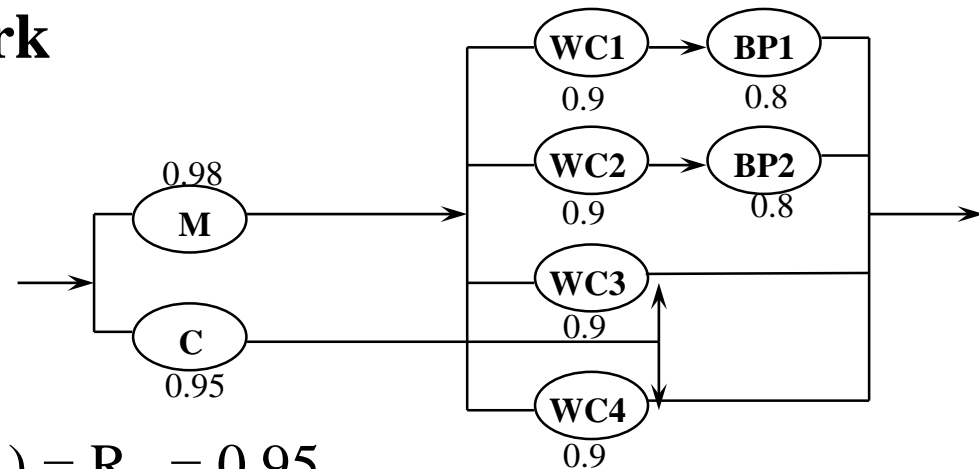


Case Study: Automotive Braking System

Case IV = both BP3 & BP4 work

$$\text{Probability } (P_{IV}) = R_{BP3} \times R_{BP4}$$

$$P_{IV} = 0.8 \times 0.8 = 0.64$$



$$\text{Reliability of parking brake } (R_{\text{park}}) = R_C = 0.95$$

Reliability of hydraulic brake (R_H):

$$R_H = R_M [1 - (1 - R_{WC} R_{BP})^2 (1 - R_{WC})^2]$$

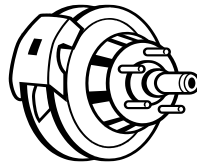
$$R_H = 0.98 [1 - (1 - 0.9 \times 0.8)^2 (1 - 0.9)^2]$$

$$R_H = 0.979$$

These two subsystems operate in parallel, therefore:

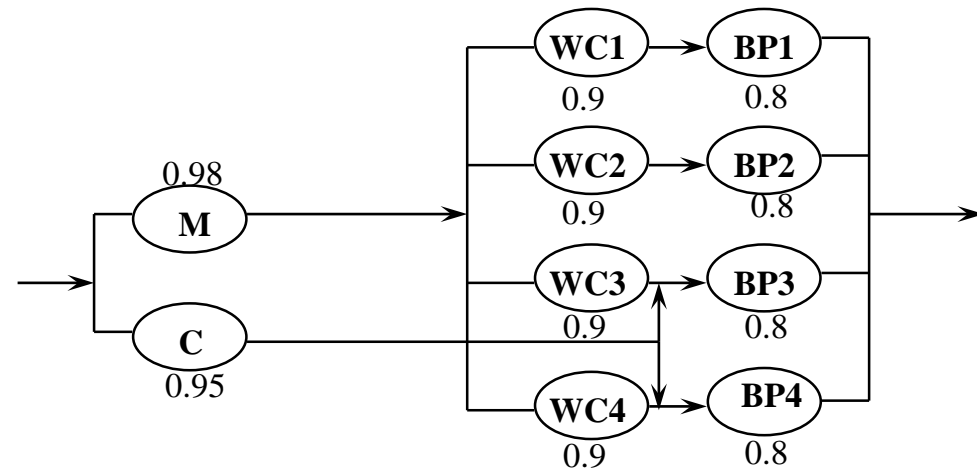
$$R_{IV} = 1 - (1 - R_H) (1 - R_{\text{park}})$$

$$R_{IV} = 1 - (1 - 0.979) (1 - 0.95) = 0.999$$



Case Study: Automotive Braking System

Reliability of the System:



System Reliability = Σ (R of each Case x Case Probability)

$$R_S = R_I P_I + R_{II} P_{II} + R_{III} P_{III} + R_{IV} P_{IV}$$

$$R_S = (0.16 \times 0.9986 \times 2 + 0.04 \times 0.903 + 0.64 \times 0.999)$$

$$R_S = 0.995$$



Summary

- Failure Distributions
 - Exponential Distribution
 - Weibull Distribution
- Series Configuration
- Parallel Configuration
- Combined Series-Parallel Configuration
- K out-of-n Redundancy
- Complex Configurations – linked networks

System Reliability

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