Certified Reliability Engineer Exam Preparation

Presented at:

9th Annual Reliability & Maintainability Training Summit

2-3 Nov 2016

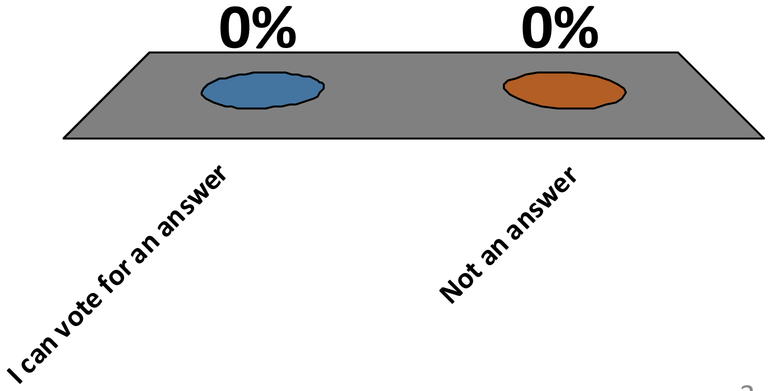
Dr Bill Wessels PE CRE

Introductions

- ➢ Dr Bill Wessels PE CRE, Consulting Reliability Engineer
  - Mechanical Engineer
  - 40-years experience (1975 – present)
- ➢ You
  - Employer
  - Current Field:
  - Engineering Discipline
  - Experience in Years

Test Question

A. I can vote for an answer
B. Not an answer



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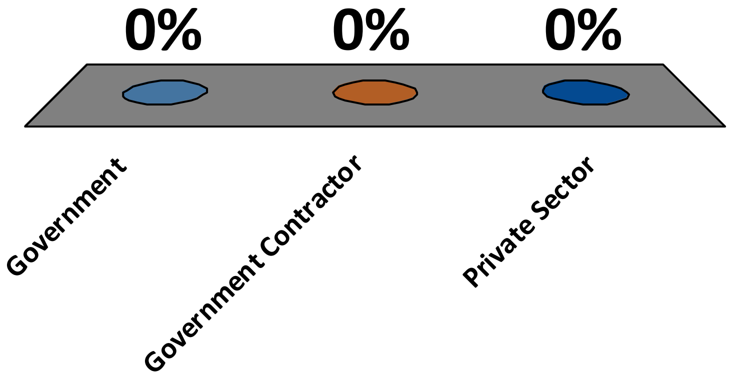
Employer

A. Government
B. Government Contractor
C. Private Sector



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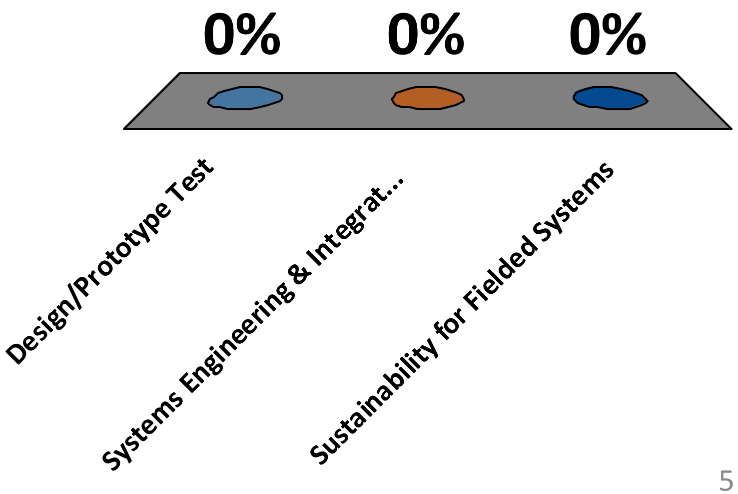
Current Field

A. Design/Prototype Test
B. Systems Engineering & Integration/ Development-Operational Test
C. Sustainability for Fielded Systems



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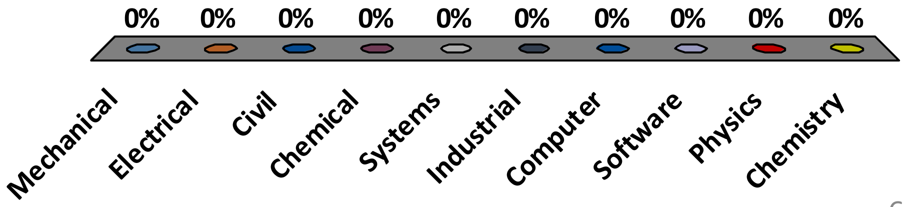
Engineering Discipline (BS Degree)

A. Mechanical
B. Electrical
C. Civil
D. Chemical
E. Systems
F. Industrial
G. Computer
H. Software
I. Physics
J. Chemistry

Answer Now

Response Counter



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Experience (in years since BS)

A. 0 – 4
B. 4 – 8
C. 8 – 12
D. Over 12

Response Counter

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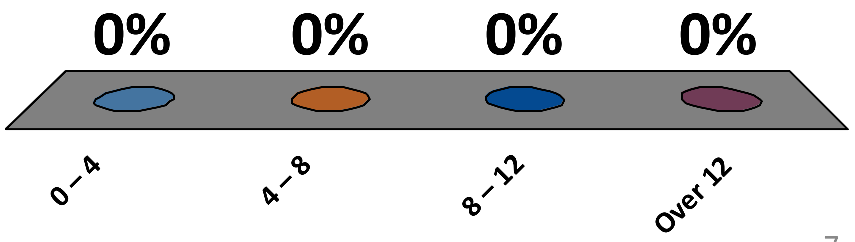

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Goal Daniel Sillivant

- Pass the exam

The Certified Reliability Engineer Exam

- Duration: 5-hrs
- Number of Questions: 150 {that is 120sec / question
- Multiple choice with 4 alternatives
- Coverage: The body of knowledge as described in the Certified Reliability Engineer Primer
- Open Book – Includes the ‘Primer’ without the question sheets and these notes, RAC Reliability Toolbox
- Bring pencils, scratch paper, calculator

American Society for Quality (ASQ) Certified Reliability Engineer (CRE) Preparation Class QM 300 3.6 CEU's
Class dates: 17 Jan thru 4 Apr 2017. Meets Tuesday nights from 5:00 to 8:00 PM at Calhoun Community College (Huntsville Campus).
POC: JENNIFER GEIGER, (256) 306-2584 Email: jennifer.geiger@calhoun.edu May attend in person or online

Instructor/Facilitator: Jim Bartlett, ASQ CRE; Work phone: (256) 313-9075, Cell phone: (256) 341-7167
Email: jbart2718@gmail.com or james.k.bartlett.civ@mail.mil

Cost: $595 Includes CRE Primer and exam practice CD. DEADLINE TO REGISTER: December 20, 2016.

Additional study material will be provided in class. Also, optional references will be suggested to cover specific topics.

Course Summary:
- Introduction and Reliability Management (2 weeks- January 17 & 24)
- Probability and Statistics (3 weeks-January 31, February 7 & 14)
- Reliability in Design (2 weeks-February 21 & 28)
- Modeling & Prediction (1.5 weeks-March 7 & 14)
- Reliability Testing (1.5 weeks-March 14 and 21)
- Maintainability & Availability (1 week-March 28)
- Data Collection (1 week-April 4)

Exams: Several open-book "take-home" exams will be assigned. They do not count toward a class grade, but will help the student plan preparations for the ASQ CRE exam.

Grades: Pass/fail, based solely on attendance.
Attendance : Must attend (either in person or over the phone) 70% of the 12 classes, or no fewer than 9 of 12 classes to pass the class. If you need to miss a class, please notify instructor in advance if possible.

Reliability Engineering Body of Knowledge

- Reliability Management {19 questions
- Probability and Statistics for Reliability {25 questions
- Reliability in Design and Development {25 questions
- Reliability Modeling and Predictions {25 questions
- Reliability Testing {23 questions
- Maintainability and Availability {17 questions
- Data Collection and Use <u>{18 questions</u>
- Sum ??? <u>172 questions</u>

Reliability Management

Strategic Management

Strategic Management

- Demonstrate how reliability engineering improves programs, processes, products and services
- Define and describe quality and reliability and how they relate to each other
- Demonstrate how reliability engineers interact with marketing, safety, product liability, engineering, manufacturing and logistics (analysis methods)
- Explain how reliability integrates with other development activities (program interactions)
- Explain reliability engineering role in determination of failure consequences and liability management (warrantee)
- Determine impact of failures on service and cost throughout product's life-cycle
- Describe how reliability engineering provides feedback to determine customer needs and specify product and service requirements
- Interpret basic project management tools and tools: Gannt, PERT, Critical Path, QFD

Strategic Management

➢Examples of reliability program benefits include:

A. Matching product design with customer's application
B. Applying predictive and preventive maintenance
C. Optimize burn-in time
D. All of the above

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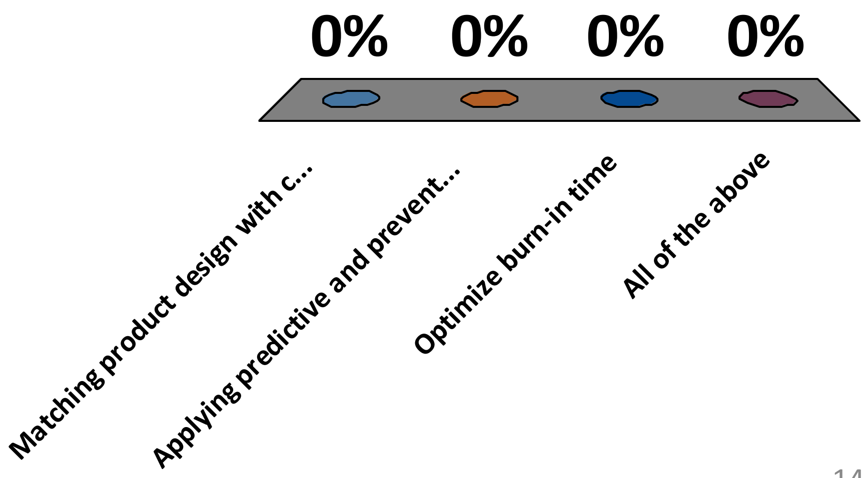

Response Counter

Answer Now

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Examples of reliability program benefits include:

A. Matching product design with customer's application
B. Applying predictive and preventive maintenance
C. Optimize burn-in time
D. <u>All of the above</u>

Key word(s); reliability program benefits

Index lists page II-3

Benefits of Reliability Engineering

Our quality of life depends greatly upon the benefits realized through reliability management and engineering efforts. Each time one turns on a light switch or turns the key in a vehicle's ignition, or boards an airplane our expectations of reliable performance are met and, in turn, grow.

The benefits of effectively managed reliability programs are evident in long term customer and supplier relationships. Firms must strive to offer a superior value to their customers, if they expect to prosper. By understanding and meeting the customer's expectations for long-term, consistent product performance, a firm can achieve a competitive advantage in the marketplace.

Once achieved, a competitive/comparative advantage in the marketplace cannot be taken for granted. Firms must recognize that the level of performance that the customer expects from their product and service offerings will increase over time. Performance that once exceeded the customer's requirements and delighted them, may soon fall short of their expectations. Firms that differentiate themselves from the competition by quantifying reliability performance objectives and cultivating the perception of "high reliability" in their products and services can in turn demand a premium price for their wares.

Advantages and benefits of reliability engineering management may be recognized in many ways. Savings associated with various manufacturing, distribution and quality related processes may be identified and maximized through reliability programs. The impact of product performance in safety critical applications can also be quantified and engineered. Examples of reliability program benefits include:

- Matching the capabilities of product designs with the customer's application environment and performance expectations, thereby optimizing the cost to performance ratio of the product without "over-engineering."

- Avoiding wasted time due to unanticipated failures in products or services through reliability and availability management concepts.

- Applying predictive and preventative maintenance programs that can impact manufacturing expenses by reducing downtime, thereby increasing throughput.

- Optimizing product run-in or burn-in times and conditions may be achieved, thereby reducing inventory carrying costs, tooling costs, and energy requirements.

Reliability Management

Reliability Program Management

Reliability Program Management

- Terminology – reliability definitions
- Element of a reliability program
  - Design-for-Reliability
  - FRACAS
- Product life-cycle costs
  - Life-cycle stages with relationship to reliability
  - Maintenance costs
  - Life expectation
  - Duty cycle
- Design evaluation
- Requirements management
- Reliability training programs

Reliability Program Management

- The usefulness of FMEA as a design tool is largely dependent on the _______________ of the communication within the design process:

A. Existence

B. Quantity

C. Effectiveness

D. Frequency

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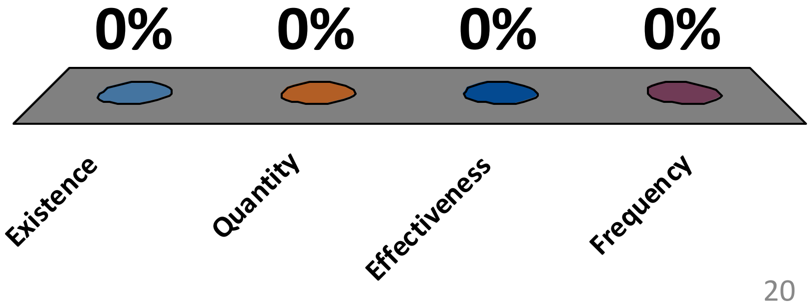
Response Counter

Answer Now



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The usefulness of FMEA as a design tool is largely dependent on the ________________ of the communication within the design process:

A. Existence
B. Quantity
C. Effectiveness
D. Frequency

Key word(s); FMEA, communication, design process

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Product and Process Development (Continued)

Life Cycle Cost Analysis

The design should be optimized with respect to life cycle costs. Production engineering analysis is an integral part of the system engineering process. It includes producibility analyses, production engineering inputs to trade-off studies and life cycle costs. Critical or special producibility requirements are identified as early as possible and are considered in the program risk analysis. Long lead time items, material limitations, transition from development to production, special processes, and manufacturing constraints are considered and documented during the system engineering process. The system engineering process also generates system and item configuration specifications.

The system engineering process includes considerations of production, cost, requirements, and logistics, which required reliability analysis. This process, when followed, produces designs that are cost effective, producible, maintainable, and reliable. Concepts called concurrent engineering build on this process.

Management Uses of FMEAs

FMEA has been used for a number of years as a design tool to improve designs. FMEA was originally defined in MIL-STD-1629 (1980)[47] and has been further refined and strengthened in the QS-9000 (1998)[50] series *Potential Failure Mode and Effects Analysis*, (AIAG, 1995)[5] (Chrysler Corporation, Ford Motor Company, and General Motors Corporation, February, 1995). However, the FMEA process has been of variable success. This is more due to improper application rather than being a poor process.

The usefulness of FMEA as a design tool is largely dependent on the effectiveness of the communication within the design process. FMEA frequently was considered a task of the reliability group and not the design group. Therefore, designers tended to ignore the process and thus not supply the necessary information. This is still the chief problem with FMEAs.

To be effective, FMEAs should be initiated as an integral part of the early design process of system functional assemblies and updated to reflect design changes. A current FMEA should be a major consideration at each design review from preliminary to the final review. In this way, the FMEA can be used to make program decisions regarding the feasibility and adequacy of a design approach.

Reliability Management

Product Safety and Liability

Product Safety and Liability

- ➢ Role of reliability engineering
- ➢ Ethical issues
- ➢ System safety program
  - Risk assessment tools
    - FMECA
    - PRAT
    - FTA

Product Safety and Liability

- Reliability engineering applied failure consequences analysis to address:

A. Product failure expectation
B. Personal injury concerns
C. Training requirements
D. Product testing requirements

Reliability engineering applied failure consequences analysis to address:

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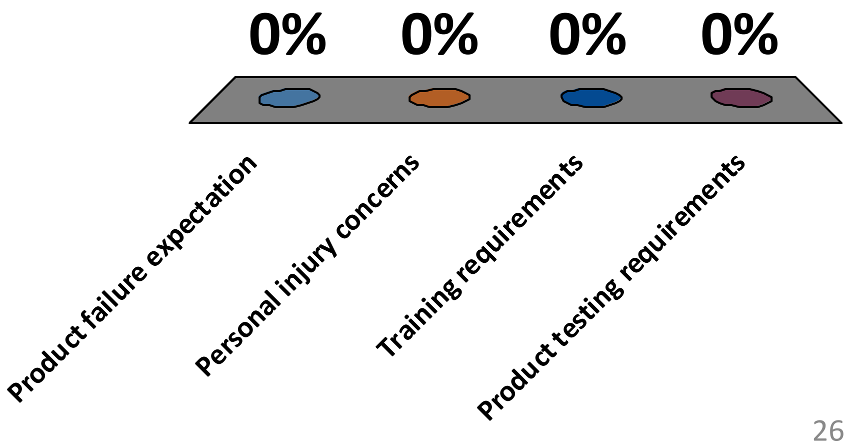

Response Counter

Answer Now

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Product Safety and Liability

- Reliability engineering applied failure consequences analysis to address:

A. Product failure expectation
B. <u>Personal injury concerns</u>
C. Training requirements
D. Product testing requirements

Key word(s); Product safety, consequences

Index lists page II-17

Personal Injury Concerns

One area of concern is that products can impact human health and safety. Design errors can lead to large personal injury claims. Strategies for dealing with this type of liability include:

- Paying attention to reliability and quality in product development and testing
- Establishing mechanisms for immediately notifying customers of any hazards, such as safety recalls and replacement upgrades
- Selecting a product liability insurance policy that includes provisions for defense of any lawsuit and for payments of settlements or judgments
- Allocating the risk of personal injury claims or liability with a customer, in exchange for price concessions
- Including legal counsel in contract negotiations to be sure the company has adequate protection

Financial Harm to a Business

Potential liability can accrue from a product's design and function. Strategies for dealing with this type of liability include:

- Paying attention to reliability and quality in product development and testing
- Releasing products that are well tested and meet requirements
- Arranging a quick replacement of defective units, when a critical problem is found
- Negotiating end-user agreements that have a limitation on all damages, and that disclaim liability for consequential damages
- Considering product liability insurance coverage for this type of business loss

Additional discussion of risk management and assessment may be found later in this Section in the System Safety Program element.

Probability and Statistics for Reliability

Basic Concepts

Basic Concepts

- Terminology – statistical definitions
- Basic probability concepts
  - Independence
  - Mutually exclusive
  - Complementary
  - Conditional
  - Joint occurrence
  - Expected value
- Discrete and continuous distributions
  - Binomial
  - Poisson (homogeneous & non-homogeneous)
  - Exponential
  - Lognormal
  - Weibull
  - Normal
  - Bathtub curve
- Statistical Process Control
  - Terminology – SPC definitions
  - Relationship to reliability

Reliability Terminology

- Table of contents and index have entry for “Reliability Terminology”
- Primer provides a multi-page table that includes all of the terms and definitions

Reliability Terminology (Continued)

Derating	(a) Using an item in such a way that applied stresses are below rated values or (b) The lowering of the rating of an item in one stress field to allow an increase in another stress field.
Direct Maintenance Man Hours Per Maintenance Action (DMMH/MA)	A measure of the maintainability parameter related to item demand for maintenance manpower: The sum of direct maintenance man hours, divided by the total number of maintenance actions (preventative and corrective) during a stated period of time.
Direct Maintenance Man Hours Per Maintenance Event (DMMH/ME)	A measure of the maintainability parameter related to item demand for maintenance manpower: The sum of direct maintenance man hours, divided by the total number of maintenance events (preventative and corrective) during a stated period of time.
Disassemble	Opening an item and removing a number of parts or subassemblies to make the item that is to be replaced accessible to removal. This does not include the actual removal of the item to be replaced.
Dormant	The state wherein an item is able to function but is not required to function. See Not Operating
Downing Event	The event which causes an item to become unavailable to initiate its mission (the transition from Up-Time to Down-Time)
Durability	A measure of useful life (a special case of reliability)
Environment	The aggregate of all external and internal conditions (such as temperature, humidity, radiation, magnetic and electric fields, shock vibration, etc.) either natural or man made, or self-induced, that influences the form, performance, reliability or survival of an item.
Environmental Stress Screening (ESS)	A series of tests conducted under environmental stresses to disclose weak parts and workmanship defects for correction

Basic Concepts

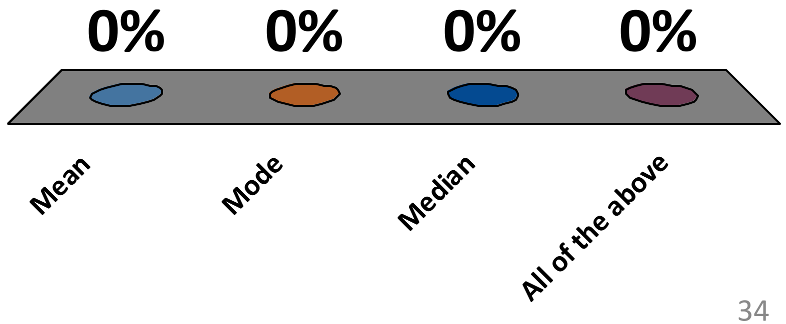
➢ Distinction between:
  - Mean
  - Point estimate of the mean
  - Measure of central tendency

The measure of central tendency is:

A. Mean
B. Mode
C. Median
D. All of the above

Response Counter

Answer Now



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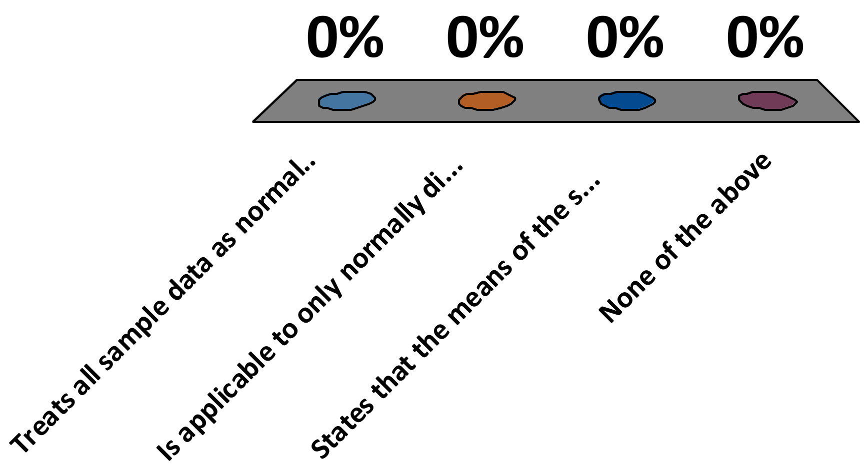
The Central Limit Theory

A. Treats all sample data as normally distributed

B. Is applicable to only normally distributed data

C. States that the means of the samples are normally distributed

D. None of the above



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The Central Limit Theorem

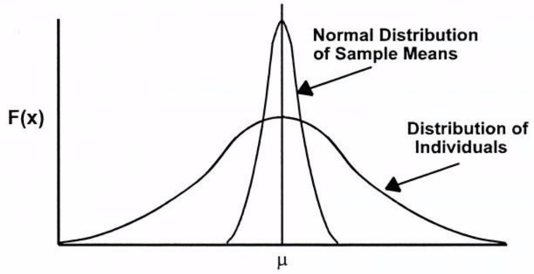


Figure 3.2 Graphical Illustration of the Central Limit Theorem

The Central Limit Theorem States:

- The sample means ($\overline{X}$s) will be more normally distributed around μ than individual readings (Xs). The distribution of sample means approaches normal regardless of the shape of the parent population. This is the underlying reason that $\overline{X}$ - R control charts work.
- For normal distributions the spread in sample means ($\overline{X}$s) is less than Xs with the standard deviation of $\overline{X}$s equal to the standard deviation of the population (individuals) divided by the square root of the sample size:

$$s_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$s_{\bar{x}}$ is referred to as the standard error of the mean.

Example 3.4: Assume the following are weight variation results:

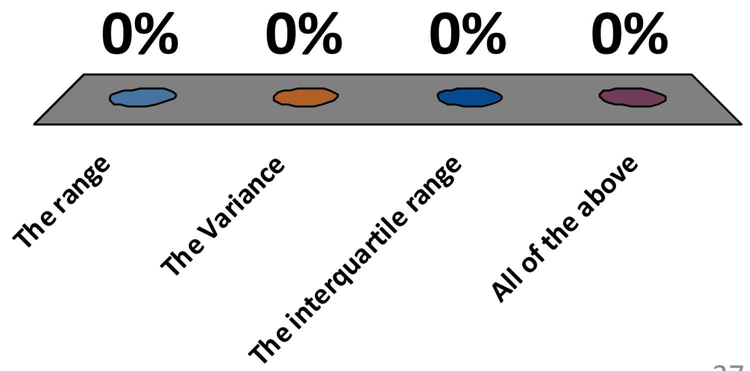
$\overline{X}$ = 20 grams σ = 0.124 grams

Estimate $s_{\bar{x}}$ for a sample size of 4.

Solution: $s_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{0.124}{\sqrt{4}} = 0.062$ grams

The measure of dispersion is:

A. The range
B. The Variance
C. The interquartile range
D. All of the above



Response Counter

Answer Now

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The measure of dispersion is:

A. The range
B. The variance
C. The interquartile range
D. <u>All of the above</u>

Primer only mentions range and variance by finding page in index for "measure of dispersion".

But right answer is either one of the first three or D, yet A and B are correct, therefore the answer must be D.

Shortcut Formula for Standard Deviation

- Given:
  - ΣX
  - ΣX^2
  - N
- Then:
  - Var = $[\Sigma X^2 - (\Sigma X)^2/n]/(n-1)$
  - Std Dev = sqrt(VAR)

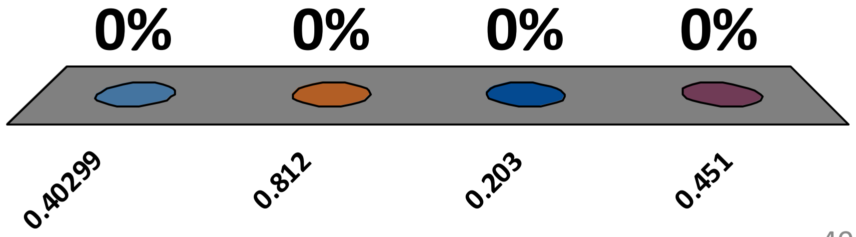
Find Standard Deviation

Given $\Sigma X = 35.8$, $\Sigma X^2 = 257.14$, $n = 5$

A. 0.40299
B. 0.812
C. 0.203
D. 0.451

Response Counter

Answer Now



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Find Standard Deviation
Given $\Sigma X = 35.8$, $\Sigma X^2 = 257.14$, $n = 5$

- VAR = $[\Sigma X^2 - (\Sigma X)^2/n]/(n-1)$
- VAR = $[257.14 - (35.8)^2/5]/(5-1) = 0.812/4 = 0.203$
- Std Dev = sqrt(0.203) = 0.45056

A. 0.40299 – divided VAR by n = 5, not n-1 = 4

B. 0.812 – did not divide numerator by n-1

C. 0.203 – did not take sqrt of VAR

D. <u>0.451</u>

Probability Concepts

- Union of events – $\cup$ – stated as events A or B occur - Addition
- Intersection of events – $\cap$ – stated as events A and B occur – Multiplication
- Additive Rule $P(A \cup B) = P(A \text{ and } B) = P(A) + P(B) – P(A \text{ and } B)$
- Multiplication Rule $P(A \cap B) = P(A \text{ or } B) = P(A)P(B)$

Counting

- Permutation – number of ways that n items can be arranged taking them r at time where order matters
  ${}_nP_r = n!/(n-r)!$ – NOTE: $0! = 1$
  - A B C can be arrange as ABC, ACB, BAC, BCA, CAB, CBA, AB, BA, AC, CA, BC, CB, A, B, C
  - Permutation of n = 3 at r = 3: ${}_3P_3 = 3!/(3-3)! = (3)(2)(1)/1 = 6$
  - Permutation of n = 3 at r = 2: ${}_3P_2 = 3!/(3-2)! = (3)(2)(1)/1 = 6$
  - Permutation of n = 3 at r = 1: ${}_3P_1 = 3!/(3-1)! = (3)(2)(1)/(2)(1) = 3$

Counting

- Combination– number of ways that n items can be arranged taking them r at time where order does not matter
  ${}_nC_r$ = n!/r!(n-r)! – NOTE: 0! = 1
  - A B C can be arrange as ABC, AB, AC, BC, A, B, C
  - Combination of n = 3 at r = 3: ${}_3C_3$ = 3!/3!(3-3)! = (3)(2)(1)/(3)(2)(1) = 1
  - Combination of n = 3 at r = 2: ${}_3C_2$ = 3!/2!(3-2)! = (3)(2)(1)/(2)(1) = 3
  - Combination of n = 3 at r = 1: ${}_3C_1$ = 3!/1!(3-1)! = (3)(2)(1)/(2)(1) = 3

Advanced Concepts

- Statistical interval estimates
  - Confidence intervals – {UCI & LCI, LCL, UCL – NOTE: treatment of α
  - Tolerance intervals – design variance
- Hypothesis testing
  - Means
  - Variances
  - Proportions
  - Type I & II error
- Bayesian technique

Distributions

Statistical distributions fall into two categories; modeling distributions and sampling distributions. Modeling distributions are used to describe data sets, and are divided into two classes; continuous distributions and discrete distributions. Sampling distributions are used to construct confidence intervals and test hypotheses.

Common Continuous Modeling Distributions

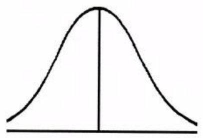
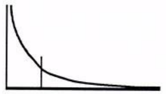
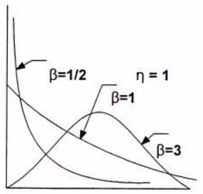
	NORMAL (GAUSSIAN)	EXPONENTIAL	WEIBULL
SHAPE			
FORMULAS	$P_{(x)} = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(X-\mu)^2}{2\sigma^2}}$ μ = Mean σ = Standard deviation e = 2.718	$P_{(x)} = \frac{1}{\mu} e^{-\frac{x}{\mu}}$ or $P_{(x)} = \lambda e^{-\lambda x}$ **μ = θ = Mean** **X = X axis reading** **λ = failure rate**	$P_{(x)} = \frac{\beta}{\eta}(X-\gamma)^{\beta-1} e^{\frac{-(x-\gamma)^{\beta}}{\eta}}$ η = Scale parameter β = Shape parameter γ = Location parameter
APPLICATION	**Numerous applications. Useful when it is equally likely that readings will fall above or below the average. Can be used along with other distributions, to determine the beginning of the wearout period.**	**Describes constant failure rate conditions. Applies for the useful life cycle of many products. Often, time(t) is used for X.**	**Used for many reliability applications. Can test for the end of the infant mortality period. Can also describe the normal and exponential distributions.**

Figure 3.16 Continuous Modeling Distributions

Normal Confidence Intervals

- Given: Confidence – C% = 90%, Sample size – n = 30
  Sample mean – $\bar{X}$ = 100, Standard deviation, s = 10
- Find confidence intervals about the sample mean
  - Find Standard error of the mean, s.e. = $\dfrac{s}{\sqrt{n}}$

$$UCI = \overline{X} + z_{1-\alpha/2} s.e.$$

$$LCI = \overline{X} - z_{\alpha/2} s.e.$$

- Find confidence intervals for the sample

$$UCI = \overline{X} + z_{1-\alpha/2}(s)$$

$$LCI = \overline{X} - z_{\alpha/2}(s)$$

Normal Confidence Limits

- Given: Confidence – C% = 90%, Sample size – n = 30
  Sample mean – $\bar{X}$ = 100, Standard deviation, s = 10
- Find appropriate confidence limit about the sample mean
  - Find Standard error of the mean, s.e. = $\dfrac{s}{\sqrt{n}}$

$$UCI = \bar{X} + z_{1-\alpha} s.e.$$

$$LCI = \bar{X} - z_{\alpha} s.e.$$

- Find confidence intervals for the sample

$$UCI = \bar{X} + z_{1-\alpha}(s)$$

$$LCI = \bar{X} - z_{\alpha}(s)$$

Confidence Intervals and Limits, z Sampling Statistic

mean =	100								
s =	10								
n =	30								
se =	1.83	Mean				Sample			
a =	0.1	LCL	UCL	LCI	UCI	LCL	UCL	LCI	UCI
z_{α} =	-1.282	97.66				87.18			
$z_{1-\alpha}$ =	1.282		102.34				112.82		
$z_{\alpha/2}$ =	-1.645			97.00				83.55	
$z_{1-\alpha/2}$ =	1.645				103.00				116.45

Confidence Intervals and limits, small sample size, n < 30

- Given: Confidence – C% = 90%, Sample size – n = 15
  Sample mean – $\bar{X}$ = 100, Standard deviation, s = 10
- Use Student-t rather than z_α
  - $t_{\alpha,\nu}$ …where ν = degrees of freedom = n - 1

Confidence Intervals and Limits, t Sampling Statistic

mean =	100								
s =	10								
n =	15								
se =	2.58	Mean				Sample			
a =	0.1	LCL	UCL	LCI	UCI	LCL	UCL	LCI	UCI
$t_{\alpha,\nu}$ =	-1.345	96.53				86.55			
$t_{1-\alpha,\nu}$ =	1.345		103.47				113.45		
$t_{\alpha/2,\nu}$ =	-1.761			95.45				82.39	
$t_{1-\alpha/2,\nu}$ =	1.761				104.55				117.61

Normal Distribution (Continued)

The normal probability density function is:

$$f(x) = \frac{1}{\sigma\sqrt{2\Pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}, -\infty < x < \infty$$

Where μ is the mean and σ is the standard deviation.

The normal probability density function is not skewed, and is shown in Figure 3.23.

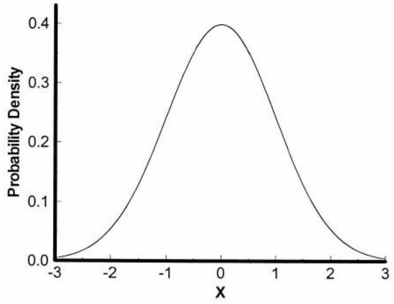


Figure 3.23 The Standard Normal Probability Density Function

The density function shown in Figure 3.23 is the standard normal probability density function. The standard normal probability density function has a mean of 0 and a standard deviation of 1. The normal probability density function cannot be integrated implicitly. Because of this, a transformation to the standard normal distribution is made, and the normal cumulative distribution function or reliability function is read from a table. Table III in the Appendix gives the standard normal reliability function. If x is a normal random variable, it can be transformed to standard normal using the expression:

$$z = \frac{x-\mu}{\sigma}$$

Exponential Distribution

The exponential distribution is used to model items with a constant failure rate, usually electronics. The exponential distribution is closely related to the Poisson distribution. If a random variable, x, is exponentially distributed, then the reciprocal of x, y = 1/x follows a Poisson distribution. Likewise, if x is Poisson distributed, then y = 1/x is exponentially distributed. Because of this behavior, the exponential distribution is usually used to model the mean time between occurrences, such as arrivals or failures, and the Poisson distribution is used to model occurrences per interval, such as arrivals, failures or defects.

The exponential probability density function is:

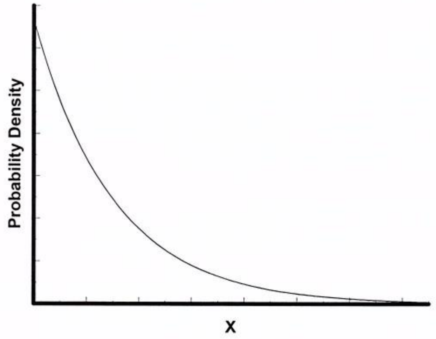
$$f(x) = \frac{1}{\theta} e^{-\frac{x}{\theta}} = \lambda e^{-\lambda x}, x \geq 0$$

Where: λ is the failure rate
θ is the mean

From the equations above, it can be seen that $\lambda = 1/\theta$. The variance of the exponential distribution is equal to the mean squared.

$$\sigma^2 = \theta^2 = \frac{1}{\lambda^2}$$

The exponential probability density function is shown in Figure 3.32.



N
W
L
E
B
P

Figure 3.32 Exponential Probability Density Function

Exponential Distribution (Continued)

The mean of the exponential distribution is estimated using the expression:

$$\theta = \frac{T}{r}$$

Where T represents the total test time for all items, both failed and unfailed, and r is the number failed, a (1-α)% confidence limit for θ is:

$$\frac{2T}{\chi^2_{\alpha,2r+2}} \leq \theta \leq \frac{2T}{\chi^2_{1-\alpha,2r}}$$

The equation above assumes time censoring. If the data is failure censored, the critical value of the chi-square statistic has 2r degrees of freedom instead of 2r+2.

$$\frac{2T}{\chi^2_{\frac{\alpha}{2},2r}} \leq \theta \leq \frac{2T}{\chi^2_{1-\frac{\alpha}{2},2r}}$$

Confidence limits for reliability can be found by substituting the lower and upper limits for θ into the exponential reliability function.

Example 3.37: Five items were tested with failures after 43 hours, 57 hours and 80 hours. The remaining two items were removed from testing after 80 hours without failing. Find the 90% confidence interval for reliability at 130 hours.

Solution: Since the items that did not fail were removed from testing at the same time as the last failure, this is failure censoring. To determine the confidence limits for reliability, the confidence limits for the mean must be determined first. The critical value of the chi-square distribution with α=0.05 and 6 degrees of freedom is 12.592. The critical value of the chi-square distribution with α=0.95 and 6 degrees of freedom is 1.635. The 90% confidence interval for the mean is:

$$\frac{2(43 + 57 + 80 + 80 + 80)}{12.592} \leq \theta \leq \frac{2(43 + 57 + 80 + 80 + 80)}{1.635}$$

$$54.00 \leq \theta \leq 415.9$$

The 90% confidence interval for reliability is:

$$e^{-\frac{130}{54.00}} \leq e^{-\frac{130}{415.9}}$$

$$0.0900 \leq 0.7316$$

Censored MTBF Confidence Limits

T =	2750
r =	3
a =	0.10

$$\theta = \frac{2T}{\chi^2_{\alpha,\nu}}$$

Time Censored				Failure Censored			
LCL		UCL		LCL		UCL	
$c^2_{\alpha,2r+2}$ =	13.36	$c^2_{1-\alpha,2r}$ =	2.20	$c^2_{\alpha,2r+2}$ =	12.59	$c^2_{1-\alpha,2r}$ =	1.64
$\theta >$	411.6	$\theta <$	2495.3	$\theta >$	436.8	$\theta <$	3363.1

Exponential Distribution (Continued)

The exponential distribution is characterized by its hazard function which is constant. Because of this, the exponential distribution exhibits a lack of memory. That is, the probability of survival for a time interval, given survival to the beginning of the interval, is dependent ONLY on the length of the interval, and not on the time of the start of the interval.

Example 3.35: Consider an item that has a mean time to fail of 150 hours that is exponentially distributed. What is the probability of surviving through the interval 0 to 20 hours?

Solution:

$$R(20) = e^{-\frac{20}{150}} = 0.8751$$

Example 3.36: Refer to the prior example. What is the probability of surviving the interval 100 to 120 hours?

Solution:

$$R(120, \text{given survival to } t = 100) = \frac{R(120)}{R(100)} = \frac{e^{-\frac{120}{150}}}{e^{-\frac{100}{150}}} = \frac{0.4493}{0.5134} = 0.8751$$

Exponential Hazard Function

$$h(t) = \frac{f(t)}{R(t)} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda$$

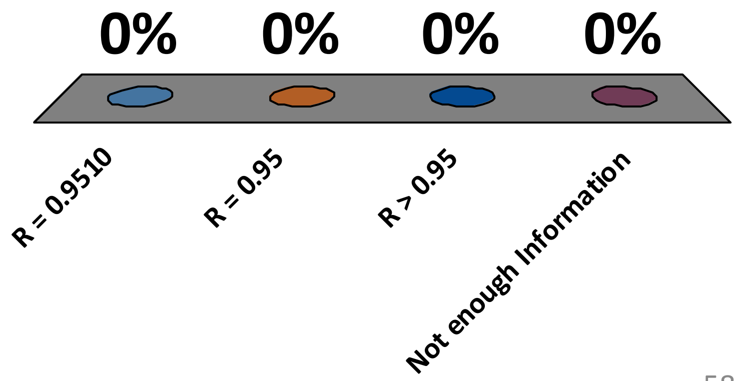
Given: Reliability of a part with an exponential reliability calculated to be 0.95 for a 10-hr mission duration. Find the reliability of the part for another mission given that the part has accumulated 100 hours (10 missions)? R(10|t = 100)

A. $R = 0.95^{10}$

B. $R = 0.95$

C. $R > 0.95$

D. Not enough Information



Response Counter

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Discrete Modeling Distributions

The Poisson, binomial, hypergeometric and geometric distributions are used to model discrete data. Time to fail is continuous data, but some situations call for discrete data, such as; the number of missiles required to destroy a target or the number of defects in a lot of 1000 items.

Common Discrete Modeling Distributions

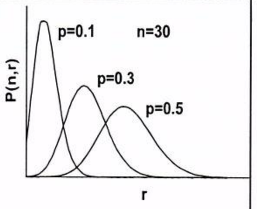
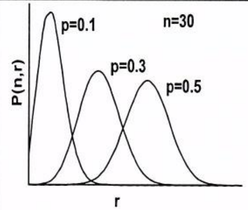
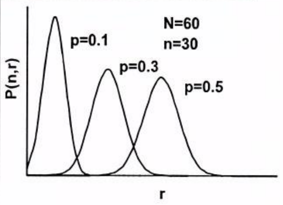
	POISSON	BINOMIAL	HYPERGEOMETRIC
SHAPE			
FORMULAS	$P_{(r)} = \dfrac{(np)^r e^{-np}}{r!}$ n = Sample size r = Number of occurrences p = Probability np = μ = average	$P_{(r)} = \dfrac{n!}{r!(n-r)!} p^r q^{n-r}$ n = Sample size r = Number of occurrences p = Probability q = 1 - p	$P_{(r)} = \dfrac{\binom{d}{r}\binom{N-d}{n-r}}{\binom{N}{n}}$ n = Sample size r = Number of occurrences d = Occurrences in population N = Population size
APPLICATION	**The Poisson is used as a distribution for defect counts and can be used as an approximation to the binomial. For np<5 the binomial is better approximated by the Poisson than the Normal.**	**The binomial is an approximation to the hypergeometric. Sampling is with replacement. The sample size is less than 10% of N (n<10% of N). The normal distribution approximates the binomial when np ≥ 5.**	**Used when the number of defects (d) is known. Sampling is without replacement. The population size (N) is frequently small. Applied when the sample (n) is a relatively large proportion of the population (n>10% of N).**

Figure 3.38 Discrete Modeling Distributions

t Distribution (Continued)

From a random sample of n items, the probability that

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$$

falls between any two specified values is equal to the area under the t probability density function between the corresponding values on the x-axis with n–1 degrees of freedom.

Example 3.49: The burst strength of 15 randomly selected seals is given below. What is the probability that the burst strength of the population is greater than 500?

480	489	491	508	501
500	486	499	479	496
499	504	501	496	498

Solution: The mean of these 15 data points is 495.13. The sample standard deviation of these 15 data points is 8.467. The probability that the population mean is greater than 500 is equal to the area under the t probability density function, with 14 degrees of freedom, to the left of

$$t = \frac{495.13 - 500}{8.467 / \sqrt{15}} = -2.227$$

From Table VII of the Appendix, the area under the t probability density function, with 14 degrees of freedom, to the left of -2.227 is 0.0214. This value must be interpolated using Table VII (2.227 falls between the 0.025 value of 2.145 and the 0.010 value of 2.624), but can be computed directly using electronic spreadsheets, or calculators. Simply stated, making an inference from the sample of 15 data points, there is a 2.14% possibility that the true population mean is greater than 500.

F Distribution (Continued)

The F probability density function is shown in Figure 3.44.

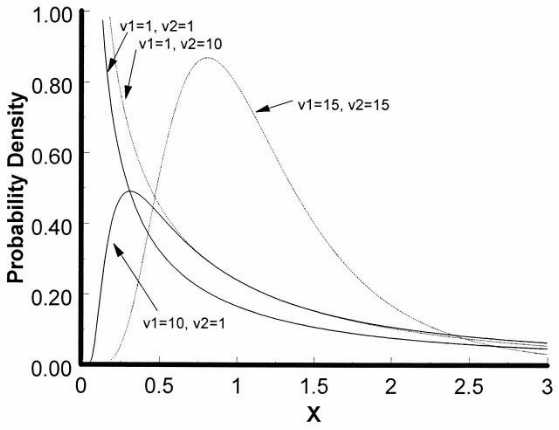


Figure 3.44 The F Probability Density Function

The F cumulative distribution function is given in Tables VIII & IX of the Appendix. Both the lower and upper tails are given in this table, but most texts only give one tail, and require the other tail to be computed using the expression:

$$F_{1-\alpha,\nu_2,\nu_1} = \frac{1}{F_{\alpha,\nu_1,\nu_2}}$$

Example 3.50: Given that $F_{0.05} = 3.07$ with $\nu_1 = 8$ and $\nu_2 = 10$, find the value of $F_{0.95}$ with $\nu_1 = 10$ and $\nu_2 = 8$.

Solution:

$$F_{0.95,10,8} = \frac{1}{F_{0.05,8,10}} = \frac{1}{3.07} = 0.326$$

Other applications of the F test are illustrated in Section IV on Statistical Inference.

Types of Charts

There are many variations of possible control charts. The two primary types are:

1. Control Charts for Variables

   Plots specific measurements of a process characteristic (temperature, size, weight, sales volume, shipments, etc.).

   Types:
   - $\bar{X}$ - R Charts (when data is readily available)
   - Run Charts (limited single-point data)
   - M$\bar{X}$ - MR Charts (limited data - moving average/moving range)
   - X - MR Charts (limited data, I - MR, individual moving range)
   - $\bar{X}$ - S Charts (when sigma is readily available)
   - Median Charts
   - Short Run Charts

2. Control Charts for Attributes

   Plots general measurement of the total process (the number of complaints per order, number of orders on time, absenteeism frequency, number of errors per letter, etc.).

   Types:
   - p Charts (for defectives - sample size varies)
   - np Charts (for defectives - sample size fixed)
   - c Charts (for defects - sample size fixed)
   - u Charts (for defects - sample size varies)
   - Short Run Varieties of p, np, c and u Charts

Charts for variables are generally most costly since each separate variable (thought to be important) must have data gathered and analyzed. In some cases, the relatively larger sample sizes associated with attribute charts can prove to be more expensive. Often, variable charts are the most valuable and useful because the specific measurement values are known. Variable charts are reviewed first in this Primer Section.

The student should be advised that only examples of $\bar{X}$ - R and p charts are reviewed here. The *CQT Primer* (Wortman, 1998)[10] contains a large number of additional control chart types.

Steps for Constructing $\bar{X}$ - R Charts

1. Determine the sample size (n = 3, 4, or 5) and the frequency of sampling.
2. Collect 20 to 25 sets of time - sequenced samples (60 to 125 individual data points.)
3. Calculate the average for each set of samples, equals $\bar{X}$.
4. Calculate the range for each set of samples, equals R.
5. Calculate $\bar{\bar{X}}$ (the average of all the $\bar{X}$'s). This is the center line of the $\bar{X}$ chart.
6. Calculate $\bar{R}$ (the average of all the R's). This is the center line of the R chart.
7. Calculate the control limits:

   $\bar{X}$ CHART: $UCL_{\bar{X}} = \bar{\bar{X}} + A_2\bar{R}$*

   $LCL_{\bar{X}} = \bar{\bar{X}} - A_2\bar{R}$*

   R CHART: $UCL_R = D_4\bar{R}$

   $LCL_R = D_3\bar{R}$

8. Plot the data and interpret the chart for special or assignable causes.

n	A_2	D_3	D_4	d_2
2	1.88	0	3.27	1.13
3	1.02	0	2.57	1.69
4	0.73	0	2.28	2.06
5	0.58	0	2.11	2.33
6	0.48	0	2.00	2.53

Table 3.46 Common Factors used for $\bar{X}$ - R Control Limits

* Note: $A_2\bar{R}$ can be shown to be identical in value to $3S_{\bar{X}}$.

Probability and Statistics for Reliability

Statistical Inference

MTBF Point Estimation (Continued)

Noncensored Data

When units are tested to failure, either repairable or non-repairable (MTTF), the estimate of the MTBF (θ) is simply the ratio of total test time divided by the number of failures during the test.

$$\hat{\theta} = \frac{T}{r}$$

Where: T = the total amount of time on test for all units (failed and unfailed)
r = total number of failures that occurred

Example 4.2: Total test time on several units was 1760 hours. During the test, 13 failures were noted. Investigation confirmed that the failures were not infant mortality, nor due to wearout. What is the estimate of the MTBF?

Solution: $$\hat{\theta} = \frac{1760}{13} = 135.4 \text{ hours}$$

If an estimate of the failure rate (λ) was called for, the calculation would have been:

$$\lambda = \frac{13}{1760} = 0.0074 \text{ failures / hour}$$

Censored Data

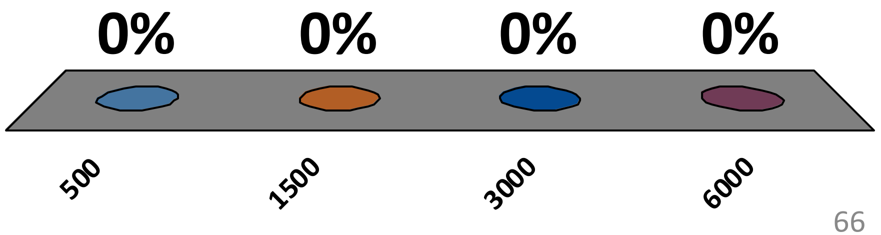
Tests are censored (stopped) at either a pre-planned number of hours or cycles or a pre-planned number of failures. Censoring at a predetermined amount of time allows for scheduling when the test will be completed and is called Type I censoring. Censoring at a predetermined number of failures allows for planning the maximum number of units that will be required for testing and is referred to as Type II censoring.

Six test articles were put on test for 1000-hrs each with 2 failures with replacement. What is the point estimate of the MTBF.

A. 500
B. 1500
C. 3000
D. 6000

Response Counter

Answer Now



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Six test articles were put on test for 1000-hrs each with 2 failures with replacement. What is the point estimate of the MTBF.

A. 500
B. 1500
C. <u>3000</u>
D. 6000

MTBF = nT/r = (6)(1000)/2 = 3000

Time-censored
with replacement

MTBF Point Estimation (Continued)

Type I Censoring

Type I censoring is generally performed in the following manner allowing the testing to be completed at a predetermined time.

A total of n items are placed on test at t = 0. The test is set to run a predetermined number of hours or cycles. As individual items fail, they are replaced at the time of failure and are to run to the prescribed time or cycles still remaining. At the predetermined time or cycles the test is terminated.

The formula for estimating MTBF is:

$$\hat{\theta} = \frac{nT}{r}$$

Where: $\hat{\theta}$ = the estimate of MTBF, MTTF, or MCBF
n = the number of items originally placed on test
T = total test time or cycles
r = number of failures during the test

Example 4.3: 10 items are placed on test for 240 hours (a cumulative test time of 2400 hours). As units failed they were replaced with new units. During the test, 4 failures occurred. What is the estimate of the MTBF?

Solution:

$$\hat{\theta} = \frac{10(240)}{4} = 600 \text{ hours}$$

Type II Censoring

Type II censoring is performed in a manner that terminates the test at a predetermined number of failures. Type II censored tests are performed for a variety of reasons, among those are: A) to determine what types of failures can be expected to occur and if necessary take action to eliminate the mechanism that results in the particular failure mode, B) test to a certain number of failures resulting in a narrow enough confidence interval for the MTBF to have meaning. It will be shown later that the confidence interval for the MTBF is directly related to the number of failures occurring during the test. The more failures, the more we know and the narrower the interval.

MTBF Point Estimation (Continued)

Type II Censoring (Continued)

To estimate $\hat{\theta}$ in a Type II censoring test,

$$\hat{\theta} = \frac{\sum_{i=1}^{r} y_i + (n-r)\,y_r}{r}$$

Where: y_i is the time to failure of the ith item.
y_r is the time to failure of the unit terminating the test.

Example 4.4: Ten units are placed on test. The test is to be terminated (truncated) when the second failure occurs. The failures occur at the following times:

Failure 1 $y_1 = 260$ hours
Failure 2 $y_2 = 310$ hours

$\Sigma y_i = 570$ hours

The test is terminated at 310 hours.

Solution: $$\hat{\theta} = \frac{570 + 8\,(310)}{2} = 1525 \text{ hours}$$

Confidence Intervals for the Mean

Continuous Data - Large Samples

Use the normal distribution to calculate the confidence interval for the mean.

$$\bar{X} \pm Z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$$

Where:
$\bar{X}$ = the sample average
σ = the population standard deviation
n = the sample size
$Z_{\frac{\alpha}{2}}$ = the normal distribution value for a desired confidence level

Example 4.5: The average of 100 samples is 18 with a population standard deviation of 6. Calculate the 95% confidence interval for the population mean.

$$\mu = 18 \pm 1.96 \frac{(6)}{\sqrt{100}} = 18 \pm 1.176$$

$$16.82 \leq \mu \leq 19.18$$

Continuous Data - Small Samples

If a relatively small sample is used (<30) then the t distribution must be used.

$$\bar{X} \pm t_{\frac{\alpha}{2}} \frac{S}{\sqrt{n}}$$

Where:
$\bar{X}$ = the sample average
S = the sample standard deviation
n = the sample size
$t_{\frac{\alpha}{2}}$ = the t distribution value for a desired confidence level and (n - 1) degrees of freedom

Example 4.6: Use the same values as in the prior example except that the sample size is 25.

$$\mu = 18 \pm 2.064 \frac{(6)}{\sqrt{25}} = 18 \pm 2.48$$

$$15.52 \leq \mu \leq 20.48$$

The sample MTBF is 200 hrs with a standard deviation of 24 hrs. Sample size is 8 test articles. Find the 90% lower confidence limit for the MTBF given $t_{\alpha,\nu} = 1.4$

A. 166.04

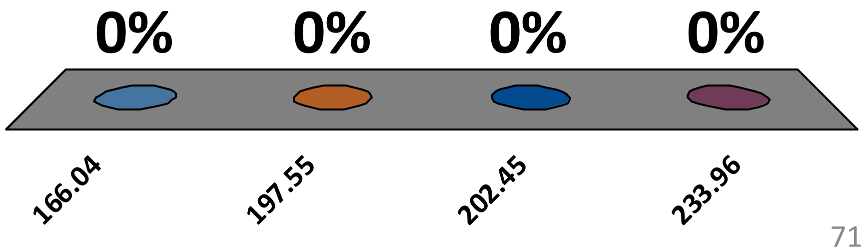
B. 188.8

C. 202.45

D. 233.96

Response Counter

Answer Now



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The sample MTBF is 200 hrs with a standard deviation of 24 hrs. Sample size is 9 test articles. Find the 90% lower confidence limit for the MTBF given $t_{\alpha,\nu} = 1.4$

A. 166.4

B. <u>188.8</u>

C. 202.425

D. 233.6

➢ LCL = 200 – (1.4)(24)/SQRT(9) = 188.8

Confidence Intervals for Variation

The confidence intervals for the mean were symmetrical about the average. This is not true for the variance, since it is based on the Chi-Square distribution. The formula is:

$$\frac{(n-1)\,S^2}{X^2_{\frac{\alpha}{2},\,n-1}} \le \sigma^2 \le \frac{(n-1)\,S^2}{X^2_{1-\frac{\alpha}{2},\,n-1}}$$

Where: n = the sample size
S^2 = point estimate of variance
$X^2_{\frac{\alpha}{2}}$ and $X^2_{1-\frac{\alpha}{2}}$ = are the table values for (n - 1) degrees of freedom

Example 4.7: The sample variance for a set of 25 samples was found to be 36. Calculate the 90% confidence interval for the population variance.

$$\frac{(24)\,(36)}{36.42} \le \sigma^2 \le \frac{(24)\,(36)}{13.85}$$

$$23.72 \le \sigma^2 \le 63.38$$

Confidence Intervals for Proportion

For large sample sizes, with n(p) and n(1-p) greater than or equal to 4 or 5, the normal distribution can be used to calculate a confidence interval for proportion. The following formula is used:

$$p \pm Z_{\frac{\alpha}{2}} \sqrt{\frac{p\,(1-p)}{n}}$$

Where: p = the population proportion estimate
n = the sample size
$Z_{\frac{\alpha}{2}}$ = the appropriate confidence level from a Z table

Example 4.8: If 16 defectives were found in a sample size of 200 units, calculate the 90% confidence interval for the proportion.

$$0.08 \pm 1.645 \sqrt{\frac{(0.08)(0.92)}{200}} = 0.08 \pm 0.032$$

$$0.048 \le p \le 0.112$$

MTBF Confidence Intervals (Continued)

Type I Censoring

In Type I censoring situations, it is possible to terminate a test at a predetermined time without witnessing any failures. If that were the case, the previous formulas would be inadequate for giving a lower limit on the MTBF. If no failures occur, a point estimate as well as an upper confidence limit on MTBF is not available from the test data.

For a one sided lower confidence limit in this type of test, calculate:

$$\frac{2T}{X^2_{(\alpha,2r+2)}} \le \theta$$

Note that the only change is in the degrees of freedom being 2r+2. This change is also noted in the lower confidence limit of the two sided interval.

For the two sided confidence interval, the formula is:

$$\frac{2T}{X^2_{(\alpha/2,2r+2)}} \le \theta \le \frac{2T}{X^2_{(1-\alpha/2,2r)}}$$

Example 4.10: A test was set to run a total of 300 hours. During the 300 hour test, 3 failures occurred. Calculate the lower one sided limit as well as the two sided confidence interval for MTBF at 90% confidence (α = 0.10).

Solution for one sided limit:

$$\frac{2(300)}{13.362} \le \theta$$

$$44.90 \le \theta \text{ at } 90\% \text{ confidence}$$

Two sided interval:

$$\frac{2(300)}{15.507} \le \theta \le \frac{2(300)}{1.635}$$

$$38.69 \le \theta \le 366.97$$

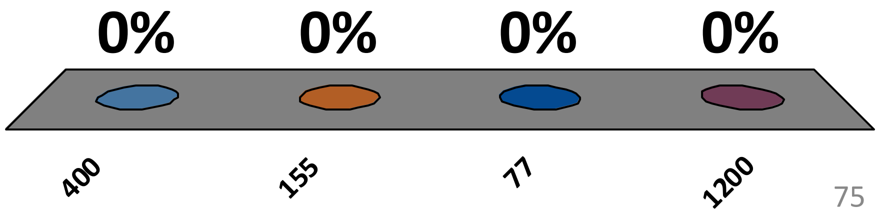
The 90% two sided confidence interval is from 38.69 to 366.97 hours.

Find the lower confidence limit for the MTBF for three test article failures for 1200 hours total test time.

A. 400
B. 155
C. 77
D. 1200

Response Counter

Answer Now



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Find the 95% lower confidence limit for the MTBF for three test article failures for 1200 hours total test time.

A. 400
B. <u>155</u>
C. 77
D. 1200

Chi-square from the table for a = 0.05 and 8 degrees of freedom (2r+2) is 15.5. MTBF = $2T/\chi^2_{\alpha,2r+2}$ = (2)(1200)/15.5 = 154.84 ~ 155.

MTBF Confidence Intervals (Continued)

Type II Censoring (Continued)

The two sided confidence interval is given as;

$$\frac{2T}{X^2_{(\alpha/2,\,2r)}} \le \theta \le \frac{2T}{X^2_{(1-\alpha/2,\,2r)}}$$

In the one sided case, what is being stated is that, at the given level of confidence, it is likely that the true MTBF is greater than the calculated value. In the two sided case, it is likely that the true MTBF is between the two calculated values.

Example 4.9: Continuing with the prior example, ten units were placed on test (without replacement of failed items), and the test was truncated at the second failure. The total amount of time on test for the ten units was 3,050 hours, and there were two failures. The point estimate for the MTBF was calculated as 1525 hours. Calculate the one-sided lower confidence limit as well as the two sided confidence interval, both at 90% confidence ($\alpha = 0.10$).

Solution for one sided limit:

$$\frac{2(3050)}{7.779} = 784.16 \text{ hours}$$

$$784.16 \le \theta$$

Although the point estimate was 1525 hours, the 90% one sided confidence limit is only about 784 hours.

Two sided interval:

$$\frac{2(3050)}{9.488} \le \theta \le \frac{2(3050)}{0.711}$$

$$642.92 \le \theta \le 8579.47$$

Again, although the point estimate for MTBF was 1525 hours, the 90% confidence interval ranges from a low of about 643 hours to a high of about 8579 hours.

Hypothesis Tests for Means (Continued)

Student's t Test (Continued)

Example 4.14: A new spark plug design is tested for wear. A sample of six plugs yielded: 0.0058, 0.0049, 0.0052, 0.0044, 0.0050 and 0.0047 inches of wear. The current design has historically produced an average wear of 0.0055". Is the new design better?

Example 4.15: A very expensive experiment has been conducted to evaluate the manufacture of synthetic diamonds by a new technique. Five diamonds have been generated with recorded weights of 0.46, 0.61, 0.52, 0.57 and 0.54 carats. An average diamond weight equal to or greater than 0.50 carats must be realized for the venture to be profitable. What is your recommendation?

Conclusions

STEPS	EXAMPLE 4.14	EXAMPLE 4.15
Step 1: Establish the null hypothesis: [there is no difference between the target value and the sample average]	H_0: $\mu_1 \geq 0.0055"$ H_1: $\mu_1 < 0.0055"$	H_0: $\mu_2 \leq 0.50$ H_1: $\mu_2 > 0.50$
Step 2: Determine the critical value of t for a 95% confidence level from the t distribution	DF = 5 from (n - 1) Left Tail -2.015	DF = 4 from (n - 1) Right Tail 2.132
Step 3: Calculate the t statistic: $t = \dfrac{\bar{X} - \mu}{s/\sqrt{n}}$	$t = \dfrac{0.0050 - 0.0055}{0.00048/\sqrt{6}}$ $t = -2.551$	$t = \dfrac{0.54 - 0.50}{0.056/\sqrt{5}}$ $t = 1.597$
Step 4: Can we reject the null hypothesis?	Since the value of calculated t is to the left of -2.015, the null hypothesis is rejected. The wear is less for the new plug design.	Since the value of calculated t (1.597) is not to the right of the critical t (2.132), the null hypothesis can't be rejected. Insufficient evidence exists for the new technique to be profitable.

Table 4.5 A Matrix Review of Two Student's t Tests

One underlying assumption is that the sampled population has a normal probability distribution. This is a restrictive assumption since the distribution of the sample is unknown. The t distribution works well, however, for distributions that are bell shaped.

Paired-Comparison Hypothesis Tests

2 Mean, Equal Variance, t Test

The paired-comparison hypothesis tests for 2 mean, equal variance, t test, tests the difference between 2 sample means ($\bar{X}_1$ vs $\bar{X}_2$) when σ_1 and σ_2 are unknown but considered equal.

$$H_0: \mu_1 = \mu_2 \qquad H_1: \mu_1 \neq \mu_2$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Where: s_p = Pooled standard deviation

$$s_p = \sqrt{\frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}}$$

$$DF = n_1 + n_2 - 2$$

Example 4.18: Compare product weight data from two machines.

Machine 1	Machine 2
3.125	3.110
3.120	3.095
3.135	3.115
3.130	3.120
3.125	3.125

$\bar{X}_1 = 3.127$, $s_1 = 0.0057$ $\qquad$ $\bar{X}_2 = 3.113$, $s_2 = 0.0115$

$$s_p = \sqrt{\frac{4(0.0057)^2 + 4(0.0115)^2}{8}} = 0.0091$$

$$t = \frac{3.127 - 3.113}{0.0091\sqrt{\frac{1}{5} + \frac{1}{5}}} = 2.43 \qquad DF = 5 + 5 - 2 = 8$$

The critical value for $t_{0.025,\,8} = 2.306$ (a 2 sided test for $\alpha = 0.05$)
The null hypothesis H_0 is rejected.

Paired-Comparison Hypothesis Tests (Continued)

2 Mean, Unequal Variance, t Test

The paired-comparison hypothesis tests for 2 mean, unequal variance, t test, tests the difference between 2 sample means ($\bar{X}_1$ vs $\bar{X}_2$) when σ_1 and σ_2 are unknown, but are not considered equal.

$$H_0: \mu_1 = \mu_2 \qquad H_1: \mu_1 \neq \mu_2$$

$$t = \frac{\bar{X}_1 - \bar{X}_2}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

$$DF = \frac{1}{\frac{\left(\frac{\frac{s_1^2}{n_1}}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)^2}{n_1 - 1} + \frac{\left(\frac{\frac{s_2^2}{n_2}}{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}\right)^2}{n_2 - 1}}$$

Example 4.19: Use data from the prior example to perform an unequal variance t test.

$$DF = 5.83 = 5$$

Note, if DF is rounded off conservatively, the effect is to increase the confidence level rather than to reduce it.

$$t = \frac{3.127 - 3.113}{\sqrt{\frac{(0.0057)^2}{5} + \frac{(0.0115)^2}{5}}} = 2.440$$

The critical value for $t_{0.025,\,5} = 2.571$ (two sided test for $\alpha = 0.05$). The null hypothesis H_0 cannot be rejected (although it is fairly close).

Reliability in Design and Development

Reliability Design Techniques

Reliability Design Techniques

- Use factors
  - Environmental
    - Temperature
    - Humidity
    - Vibration
    - Corrosive
    - Pollutants
  - Stresses
    - Static & dynamic loads
    - Severity of service
    - Electrostatic discharge
    - Radio frequency interference
- Stress-strength analysis
- FMEA in design
- FTA in design
- Tolerance and worst-case analyses
- Robust design approaches (DOE)
  - Independent and dependent variable
  - Factors and levels
  - Error
  - Replication
  - Experimental design – full & fractional factorials
- Human factors reliability

Reliability in Design and Development

Parts and Systems Management

Parts and Systems Management

- Parts Selection
- Materials selection and control
- Derating methods and principles
  - S-N diagram
  - Stress-life relationship
- Establishing specifications

Stress-Strength Analysis

In the most basic terms, an item fails when the applied stress exceeds the strength of the item. In general, designers design for a normal strength and a nominal stress that will be applied to an item. One must also be aware of the variability about the stress and strength nominals.

In Figure 5.2, the distribution curves for stress and strength are far enough apart that there is little probability that a high stress level would interfere with an item that is on the low end of the strength distribution, and as a result we do not expect a failure to occur from overstressing the item.

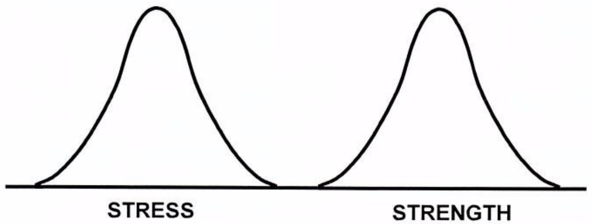


Figure 5.2 Stress-Strength Separation

In Figure 5.3, there is too much variability for the proximity of the means for stress and strength and there is an increased likelihood of failure which is represented by the overlapping shaded area.

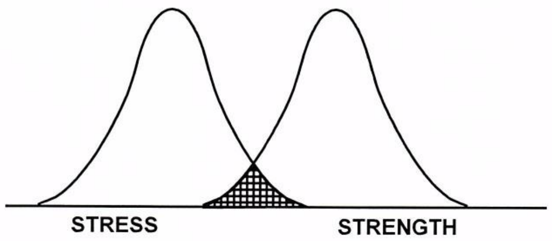


Figure 5.3 Stress - Strength Overlap

Stress-Strength Analysis (Continued)

When the stress distribution and strength distribution are independent of each other, the following relationships apply:

$$\mu_{x-y} = \mu_x - \mu_y$$

$$\sigma_{x-y} = \sqrt{\sigma_x^2 + \sigma_y^2}$$

To calculate the probability of a failure from stress-strength interference, the standard normal distribution and Z tables are normally utilized. If the distributions are non-normal, other techniques, such as Monte Carlo simulation, may be used to determine the probability of failure.

$$Z = \frac{\mu_x - \mu_y}{\sqrt{\sigma_x^2 + \sigma_y^2}}$$

Example 5.1: If in Figure 5.3, the stress distribution has a mean stress of 1500 watts with a standard deviation of 20 watts and the unit is designed to handle 1600 watts with a standard deviation of 30 watts, determine the probability of failure:

$\mu_x = 1600$ $\quad \mu_y = 1500$
$\sigma_x = 30$ $\quad \sigma_y = 20$

Solution:

Calculating Z to get the probability of failure:

$$Z = \frac{1600 - 1500}{\sqrt{30^2 + 20^2}}$$

$$Z = \frac{100}{\sqrt{1300}} = 2.77$$

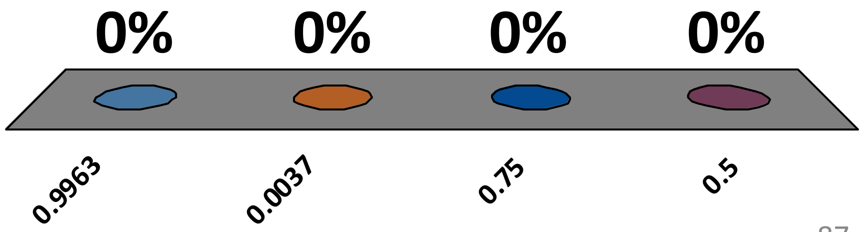
Using the table of Z values from a standard normal distribution, the area above a Z value of 2.77 (2.77 standard deviations) is 0.0028. The probability of failure is 0.28%.

Analysis finds part strength has a mean of 1200 with a std dev of 50. Mean stress is 900 with a std dev of 100. Find the reliability.

A. 0.9963
B. 0.0037
C. 0.75
D. 0.5

Response Counter

Answer Now



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Analysis finds part strength has a mean of 1200 with a std dev of 50. Mean stress is 900 with a std dev of 100. Find the reliability.

A. 0.9963
B. 0.0037 - unreliability
C. 0.75 =900/1200
D. 0.5 =50/100

$z = (1200 – 900)/\mathrm{sqrt}(50^2 + 100^2) = 2.68$

U = 0.0037 from the standard normal table

$R = 1 – U = 0.9963$

Reliability Modeling and Predictions

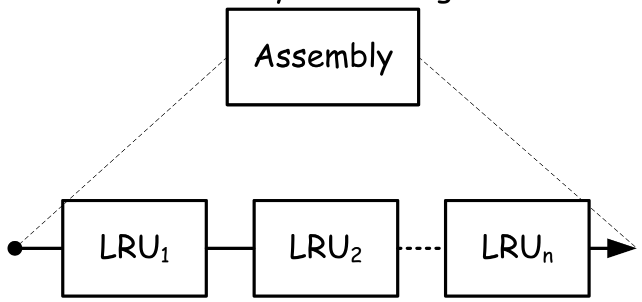
Reliability Modeling

Reliability Modeling

- Sources of reliability data
- Reliability block diagrams and models
- Simulation techniques

Serial Assembly – Reliability Calculation

Reliability Block Diagram



$$R_{ASSY}(t) = \prod_{i=1}^{n} R_{LRU_i}(t) = \left[R_{LRU_1}(t)\right]\left[R_{LRU_2}(t)\right]\cdots\left[R_{LRU_n}(t)\right]$$

Let:

$R_1(t) = 0.90$
$R_2(t) = 0.95$
$R3(t) = 0.99$

Then:

$R_{ASSY}(t) = (0.90)(0.95)(0.99) = R_1(t)R_2(t)R_3(t) = 0.8465$

If:

$R_1(t) = R_2(t) = R_3(t) = R(t) = 0.95$

Then:

$R_{ASSY}(t) = (R(t))^3 = (0.95)^3 = 0.8574$

Serial Design Configuration
Reliability Rule-of-Thumb

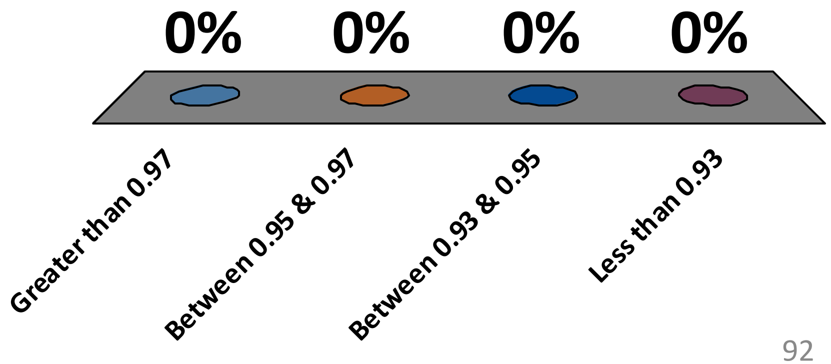
$R_{ASSY}(t) \leq MIN\{R_i(t)\}$

Given the following RBD, the assembly reliability is…

A. Greater than 0.97
B. Between 0.95 & 0.97
C. Between 0.93 & 0.95
D. Less than 0.93

Response Counter

Answer Now



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Given the following RBD, the assembly reliability is…

A. Greater than 0.97

B. Between 0.95 & 0.97

C. Between 0.93 & 0.95

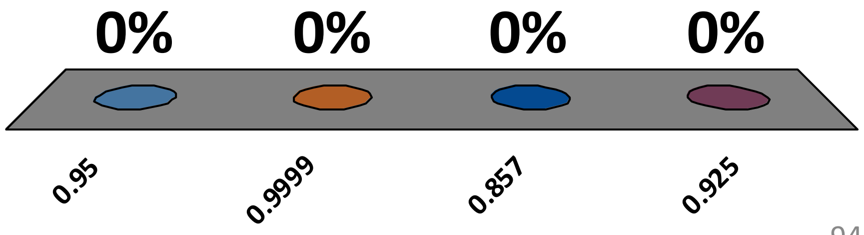
D. Less than 0.93

Given the following RBD, the assembly reliability is…

A. 0.95
B. 0.9999
C. 0.857
D. 0.925

Response Counter

Answer Now



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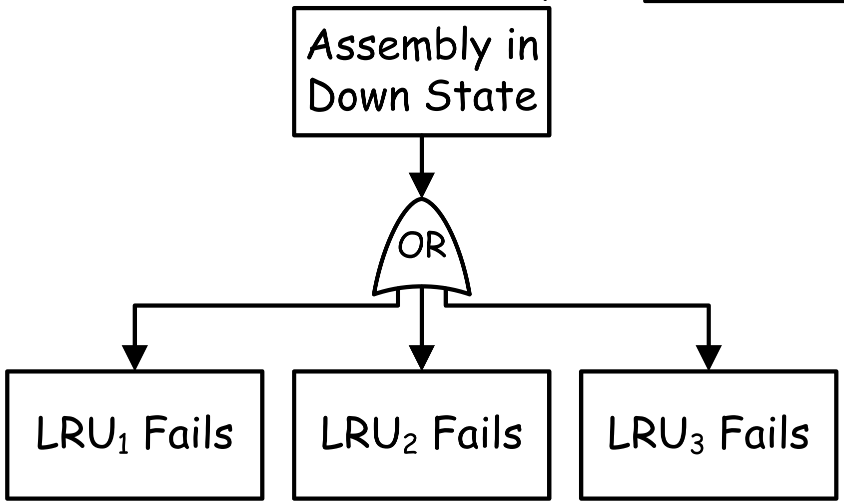
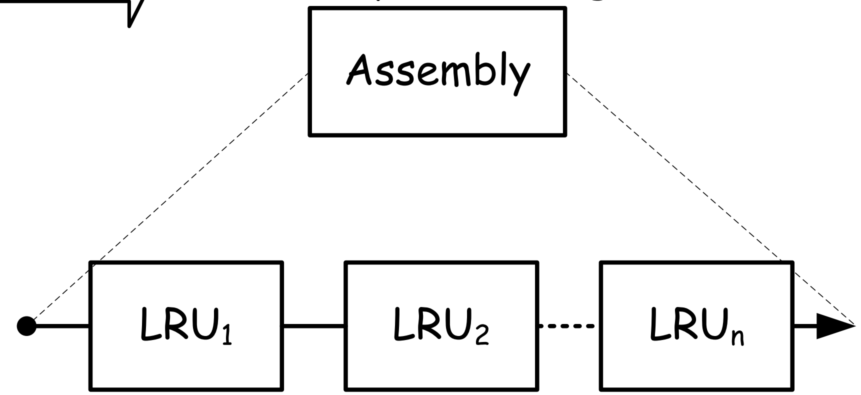
Given the following RBD, the assembly reliability is…

A. 0.95
B. 0.9999
C. 0.857
D. 0.925

R = (0.97)(0.95)(0.93) = 0.857

Serial Assembly – Equal Reliability Allocation





$$R_{ASSY}(t) = \prod_{i=1}^{n} R_{LRU_i}(t)$$

$$R_{LRU_i}(t) = \sqrt[n]{R_{ASSY}(t)}$$

Given:

$R_{ASSY}(t) = 0.99884$
$n_{LRU} = 3$

Then:

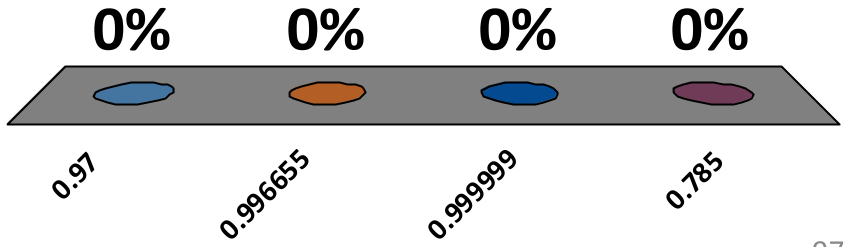
$$R_{LRU}(t) = \sqrt[3]{0.99884} = 0.999628$$

Find the reliability allocation for B, C & D in a serial design configuration given that the reliability requirement for A is 99%

A. 0.97
B. 0.996655
C. 0.999999
D. 0.785

Response Counter

Answer Now



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Find the reliability allocation for B, C & D in a serial design configuration given that the reliability requirement for A is 99%

A. 0.97
B. <u>0.996655</u>
C. 0.999999
D. 0.785

R Allocation = $R_A^{(1/n)} = 0.99^{(1/3)} = 0.996655$

Note: $R_A = R_B R_C R_D$ = R Allocation$^3 = 0.996655^3 = 0.99$

Serial System – Equal Reliability Allocation

$$R_{sys} := 0.99$$

$$n_{subsys} := 3$$

$$R_{subsys} := \sqrt[n_{subsys}]{R_{sys}} = 0.996655$$

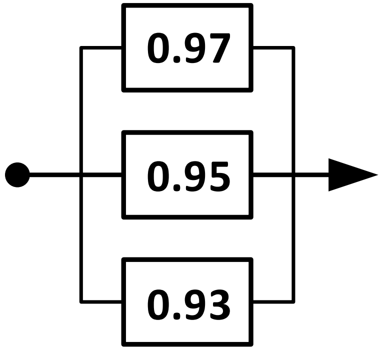
$$n_{assy} := 3$$

$$R_{assy} := \sqrt[n_{assy}]{R_{subsys}} = 0.998884$$

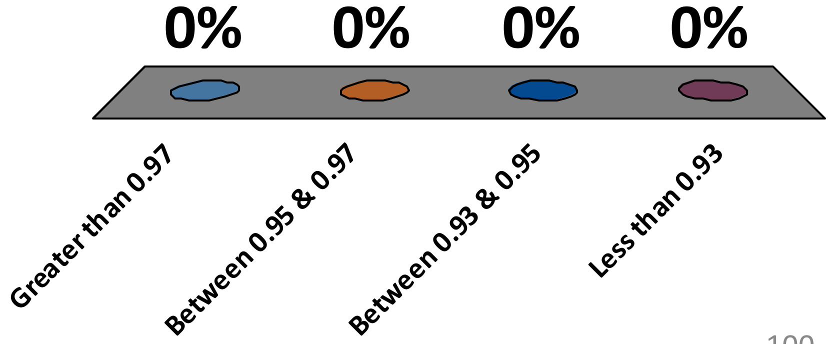
$$n_{LRU} := 3$$

$$R_{LRU} := \sqrt[n_{LRU}]{R_{assy}} = 0.999628$$

Given the following RBD, the assembly reliability is…



A. Greater than 0.97
B. Between 0.95 & 0.97
C. Between 0.93 & 0.95
D. Less than 0.93

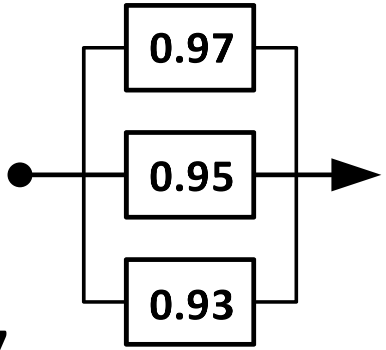


Response Counter

Answer Now

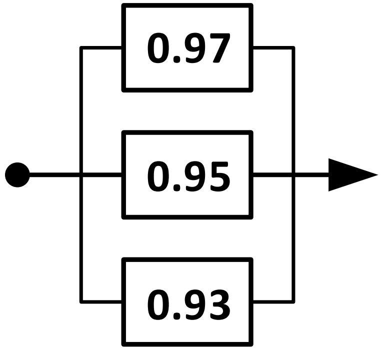
30

Given the following RBD, the assembly reliability is…

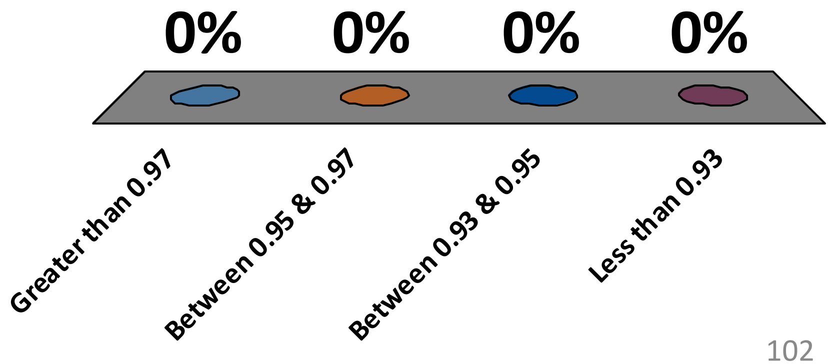


A. <u>Greater than 0.97</u>
B. Between 0.95 & 0.97
C. Between 0.93 & 0.95
D. Less than 0.93

Given the following RBD, the assembly reliability is…



A. Greater than 0.97
B. Between 0.95 & 0.97
C. Between 0.93 & 0.95
D. Less than 0.93

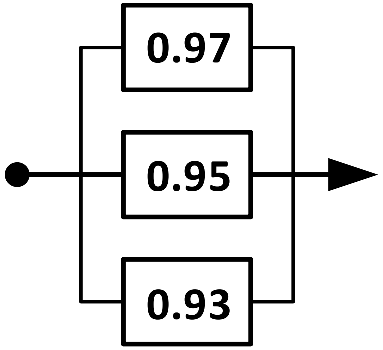


Response Counter

Answer Now

30

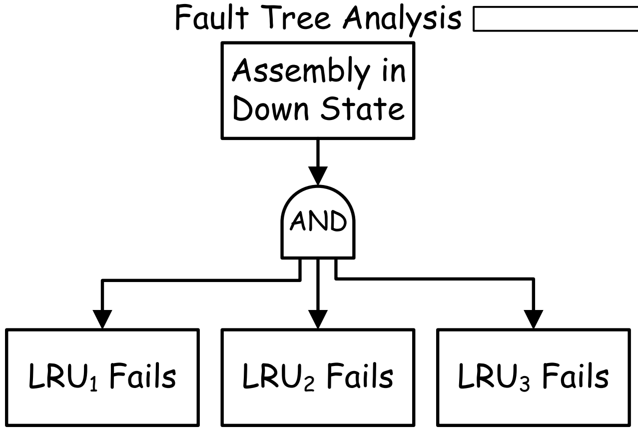
Given the following RBD, the assembly reliability is…

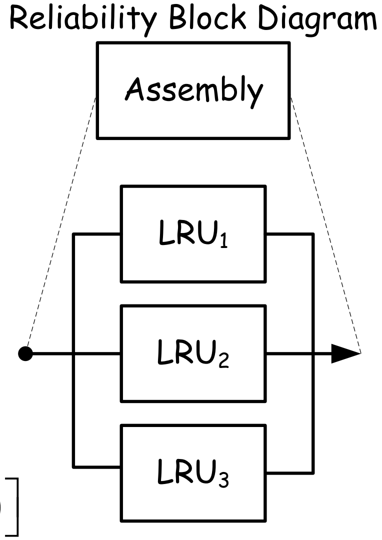


A. 0.95
B. <u>0.9999</u>
C. 0.857
D. 0.925

R = 1 - (0.03)(0.05)(0.07) = 0.9999

Parallel Assembly Reliability Calculation





$$U_{ASSY}(t) = \prod_{i=1}^{3} U_{LRU_i}(t) = \left[U_{LRU_1}(t)\right]\left[U_{LRU_2}(t)\right]\left[U_{LRU_3}(t)\right]$$

$$R_{ASSY}(t) = 1 - U_{ASSY}(t) = 1 - \left[U_{LRU_1}(t)\right]\left[U_{LRU_2}(t)\right]\left[U_{LRU_3}(t)\right]$$

$$U_{LRU}(t) = 1 - R_{LRU}(t)$$

$$R_{ASSY}(t) = 1 - \left(1 - R_{LRU_1}(t)\right)\left(1 - R_{LRU_2}(t)\right)\left(1 - R_{LRU_3}(t)\right)$$

Let:

$R_1(t) = 0.90$ $U_1(t) = 0.10$
$R_2(t) = 0.95$ $U_2(t) = 0.05$
$R_3(t) = 0.99$ $U_3(t) = 0.01$

Then:

$U_{ASSY}(t) = (0.10)(0.05)(0.01) = 0.00005$
$R_{ASSY}(t) = 1 - U_{ASSY}(t) = 0.99995$

If:

$R_1(t) = R_2(t) = R(t) = 0.95$

Then:

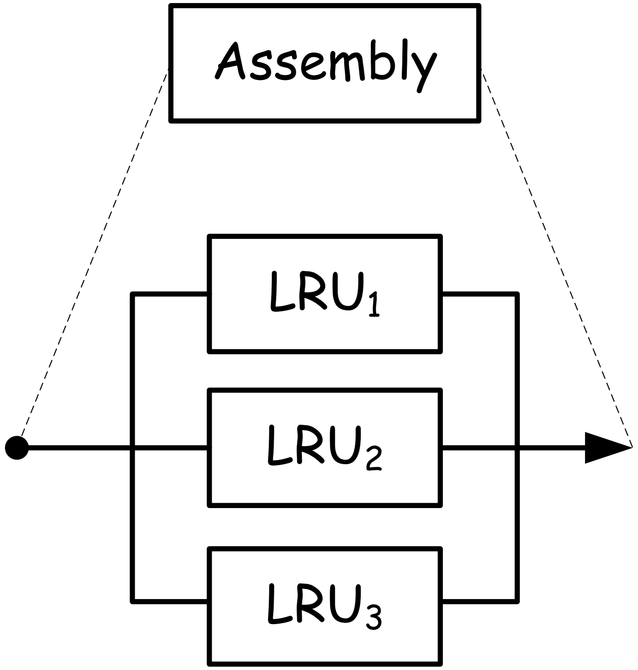
$U_{ASSY}(t) = (U(t))^3 = (0.05)^3 = 0.000125$
$R_{ASSY}(t) = 1 - U_{ASSY}(t) = 0.999875$

Parallel Design Configuration Reliability Rule-of-Thumb:

$R_{ASSY}(t) \geq MAX\{R_i(t)\}$

Parallel Assembly Reliability Allocation

Reliability Block Diagram



Let:

$R_{assy}(t) = 0.99 \qquad U_{assy}(t) = 0.01$

Then: $R_{assy} := 0.99$

$$U_{assy} := 1 - R_{assy} = 0.01$$

$$n_{LRU} := 3$$

$$R_{LRU} := 1 - \sqrt[n_{LRU}]{U_{assy}} = 0.785$$

OR

$$U_{LRU} := \sqrt[n_{LRU}]{U_{assy}} = 0.215$$

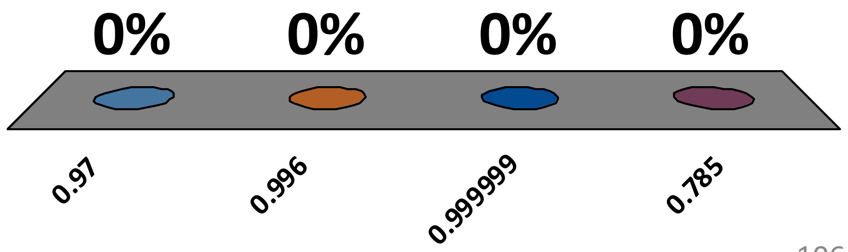
$$R_{LRU} := 1 - U_{LRU} = 0.785$$

Find the reliability allocation for B, C & D in a parallel design configuration given that the reliability requirement for A is 99%

A. 0.97
B. 0.996
C. 0.999999
D. 0.785

Response Counter

Answer Now



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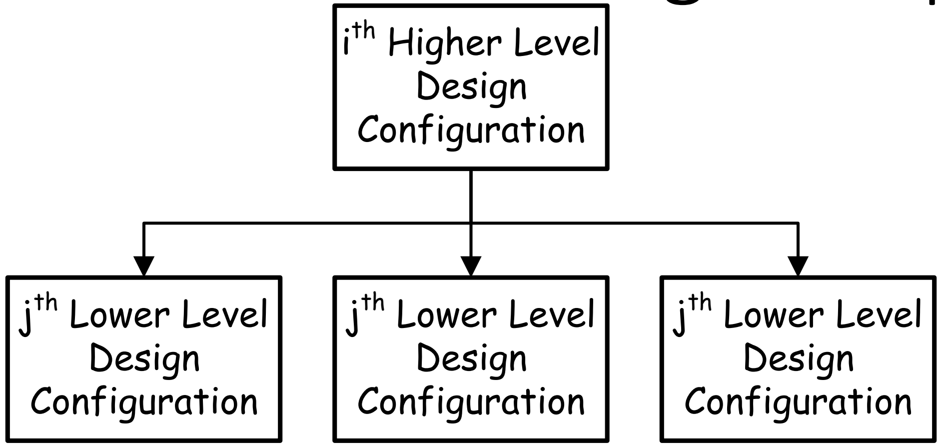
Find the reliability allocation for B, C & D in a parallel design configuration given that the reliability requirement for A is 99%

A. 0.97
B. 0.996
C. 0.999999
D. <u>0.785</u>

R Allocation = $1\text{-}U_A^{(1/n)} = 1 - (1\text{-}0.99)^{(1/3)} = 0.785$

NOTE: $R_A = 1 - (1 - \text{R Allocation})^3 = 1 - (1 - 0.785)^3 = 1 - (0.215)^3 = 0.99$

Parallel Design – Equal Reliability Allocation



$$R_i(t) = 1 - \prod_{j}^{n}\left(1 - R_j(t)\right)$$

$$Assume: \quad R_1(t) = R_2(t) = \cdots R_n(t)$$

$$Then: \quad R_i(t) = 1 - \left(1 - R_j(t)\right)^n$$

$$U_j(t) = \left(1 - R_j(t)\right)$$

$$R_i(t) = 1 - \left(U_j(t)\right)^n$$

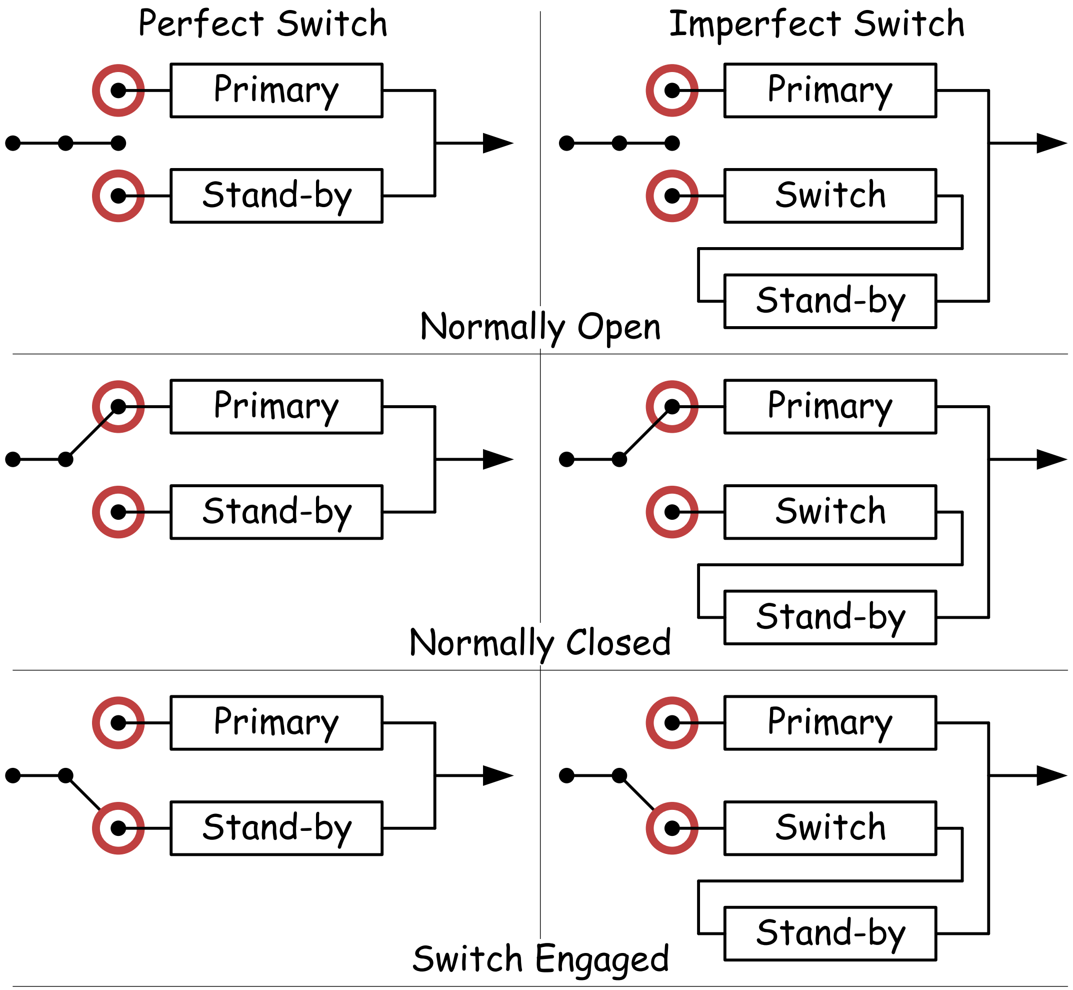
$$\left(U_j(t)\right)^n = 1 - R_i(t)$$

$$\sqrt[n]{\left(U_j(t)\right)^n} = \sqrt[n]{1 - R_i(t)} = \sqrt[n]{U_i(t)}$$

$$U_j(t) = \sqrt[n]{1 - R_i(t)} = \sqrt[n]{U_i(t)}$$

$$R_j(t) = 1 - U_j(t)$$

Stand-by Design Logic



- Fail Open
- Fail Closed
- Fail in Operation

- Mechanical – typically Weibull
- Digital – typically exponential

- Mechanical failure mode - evident to operator/maintainer
- Digital failure mode- hidden to operator/ maintainer

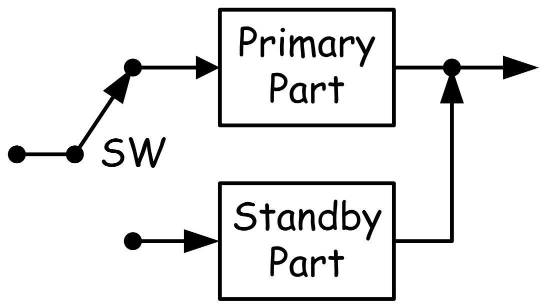
- Mechanical FD/FI - less potential for BIT/ BITE
- Digital FD/FI- high potential for BIT/ BITE

- Mechanical & digital subject to significant pre-Log downtime
- Mechanical subject to significant post-Log downtime

Equal Failure Rates – Perfect Switch

Exponential Failure Model

Equal Failure Rates
Perfect Switch



$$Rassy(\tau) = R_p(\tau)\left(1 + \lambda_p \tau\right)$$

$R_p(\tau\|t)$ =	0.990050

λ_p =	0.000125

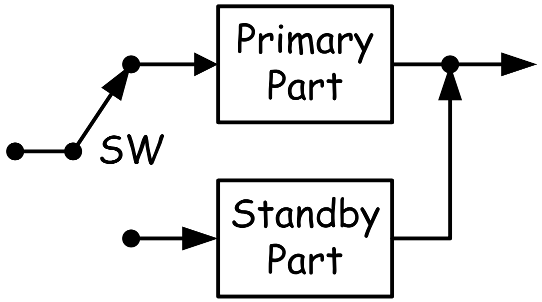
$\lambda_p\tau$ =	0.010000
τ =	80

1]	Equal Failure Rate, Perfect Switch ($\lambda_P = \lambda_S$)	
	$R(\tau\|t)$ =	$R_p(\tau\|t)(1 + \lambda_p\tau)$
		0.999950

Unequal Failure Rates – Perfect Switch

Exponential Failure Model

Unequal Failure Rates
Perfect Switch



$$R_{assy}(\tau) = R_p(\tau) + \left(\frac{\lambda_p}{\lambda_s - \lambda_p} \right) \left(R_p(\tau) - R_s(\tau) \right)$$

$R_p(\tau\|t)$ =	0.990050
$R_s(\tau\|t)$ =	0.956571

λ_p =	0.000125
λ_s =	0.000555

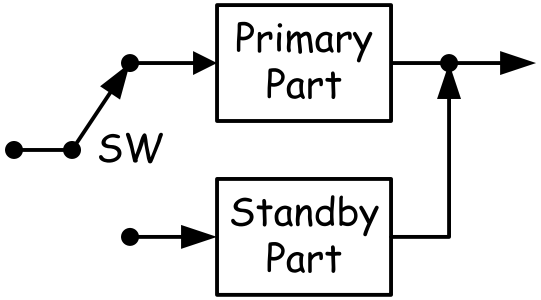
$\lambda_p/(\lambda_s-\lambda_p)$ =	0.290698
τ =	80

2]	Unequal Failure Rate, Perfect Switch ($\lambda_P < \lambda_S$)	
	$R(\tau\|t)$ =	$R_p(\tau\|t) + [(\lambda_p/(\lambda_s - \lambda_p)][R_p(\tau\|t) - R_s(\tau\|t)]$
		0.999782

Equal Failure Rates – Imperfect Switch

Exponential Failure Model

Equal Failure Rates Imperfect Switch



$$R_{assy}(\tau) = R_p(\tau)\left(1 + R_{sw}(\tau)\lambda_p t\right)$$

$R_p(\tau \mid t) =$	0.990050
$R_{sw}(\tau \mid t) =$	0.941765

$\lambda_p =$	0.000125

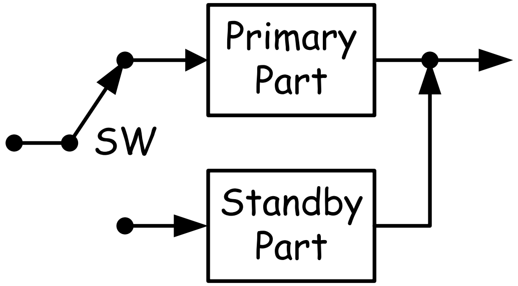
$\lambda_p\tau =$	0.010000
$\tau =$	80

3]	Equal Failure Rate, Imperfect Switch ($\lambda_P = \lambda_S$)	
	$R(\tau \mid t) =$	$R_p(\tau \mid t)[1 + R_{sw}(\tau \mid t)\lambda_p\tau]$
		0.999374

Unequal Failure Rates – Imperfect Switch

Exponential Failure Model

Unequal Failure Rates
Imperfect Switch



$$R_{assy}(\tau) = R_p(\tau) + R_{sw}(\tau)\left(\frac{\lambda_p}{\lambda_s - \lambda_p}\right)\left(R_p(\tau) - R_s(\tau)\right)$$

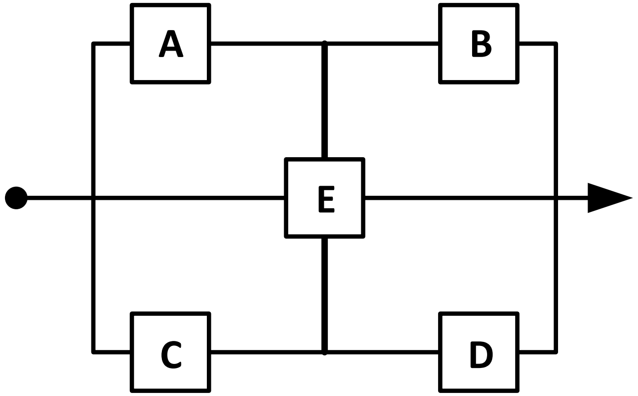
$R_p(\tau\|t)$ =	0.990050
$R_s(\tau\|t)$ =	0.956571
$R_{sw}(\tau\|t)$ =	0.941765

λ_p =	0.000125
λ_s =	0.000555

$\lambda_p/(\lambda_s-\lambda_p)$ =	0.290698
τ =	80

4]	Unequal Failure Rate, Imperfect Switch ($\lambda_P < \lambda_S$)	
	$R(\tau\|t)$ =	$R_p(\tau\|t) + R_{sw}(\tau\|t)[\lambda_p/(\lambda_s - \lambda_p)][R_p(\tau\|t) - R_s(\tau\|t)]$
		0.999215

Complex RBD – Cut Set Method



Cut Set	A	B	C	D	E	Path
Terms	0.96	0.95	0.94	0.93	0.97	R/U
AB	0.96	0.95				0.91
CD			0.94	0.93		0.87
AED	0.96			0.93	0.97	0.87
CEB		0.95	0.94		0.97	0.87
-ABCD	0.96	0.95	0.94	0.93		-0.80
-ABDE	0.96	0.95		0.93	0.97	-0.82
-ABCE	0.96	0.95	0.94		0.97	-0.83
-ACDE	0.96		0.94	0.93	0.97	-0.81
-BCDE		0.95	0.94	0.93	0.97	-0.81
-(2)ABCDE	0.96	0.95	0.94	0.93	0.97	-1.55
+(4)ABCDE	0.96	0.95	0.94	0.93	0.97	3.09
				Reliability =		0.994

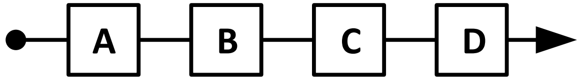
Reliability Modeling and Predictions

Reliability Predictions

Reliability Predictions

- Part count and part stress analysis
- Advantages and limitations
- Reliability prediction methods
- Reliability apportionment and allocation

Parts Count Method for MTBF & MTTR



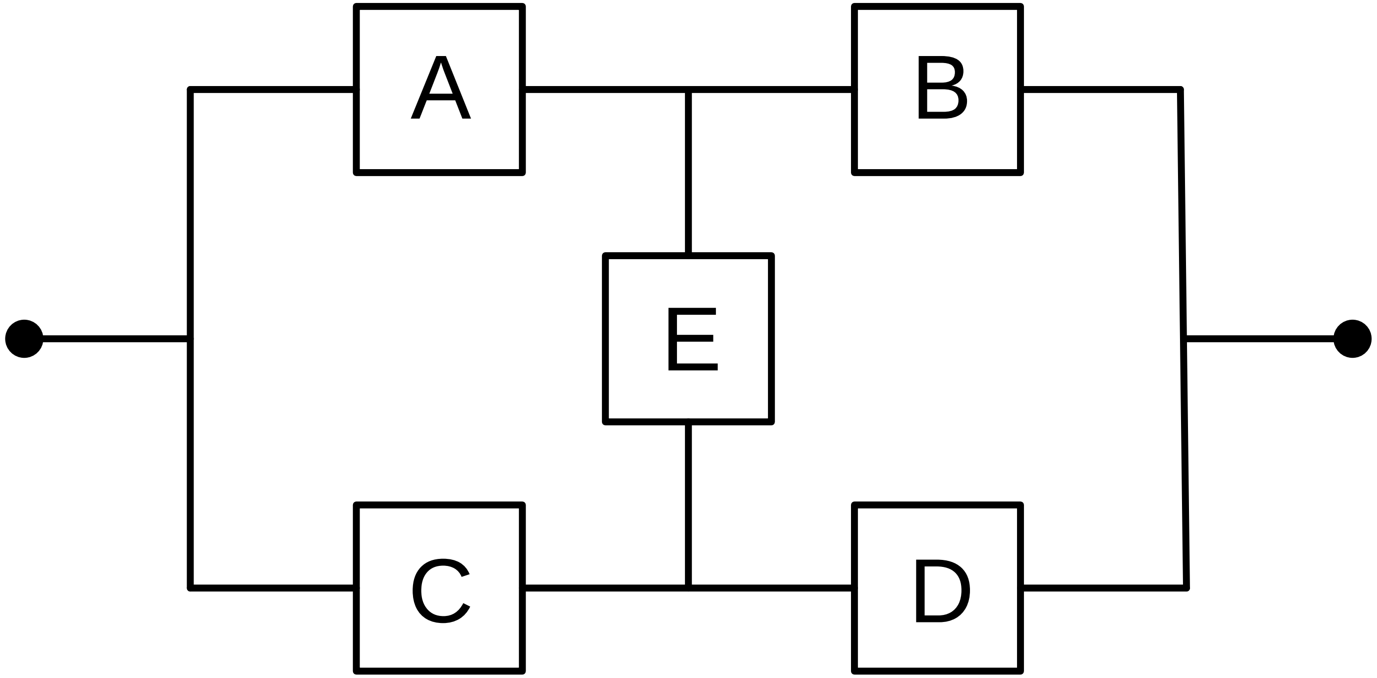
Assembly							
MTBF by Parts Count Method						MTTR	
Part	MTBF	FR	n	nFR		MTTR	FR*MTTR
A	1000	0.0010	1	0.0010		2.5	0.0025
B	300	0.0033	2	0.0067		3	0.0100
C	500	0.0020	4	0.0080		5	0.0100
D	200	0.0050	3	0.0150		1	0.0050
	ΣFR =	0.0113	ΣnFR =	0.0307		ΣFR*MTTR =	0.0275
			MTBF =	32.61		MTTR =	2.4265

Keynote Component Model

FIND: Reliability of the keynote component, R

$R = 1 - P_F$ where P_F = Unreliability, U

$P_F = U_{ABCD}R_E + U_{ABCD}U_E$



GIVEN:	
R_A =	0.96079
R_B =	0.95123
R_C =	0.94176
R_D =	0.93239
R_E =	0.97045

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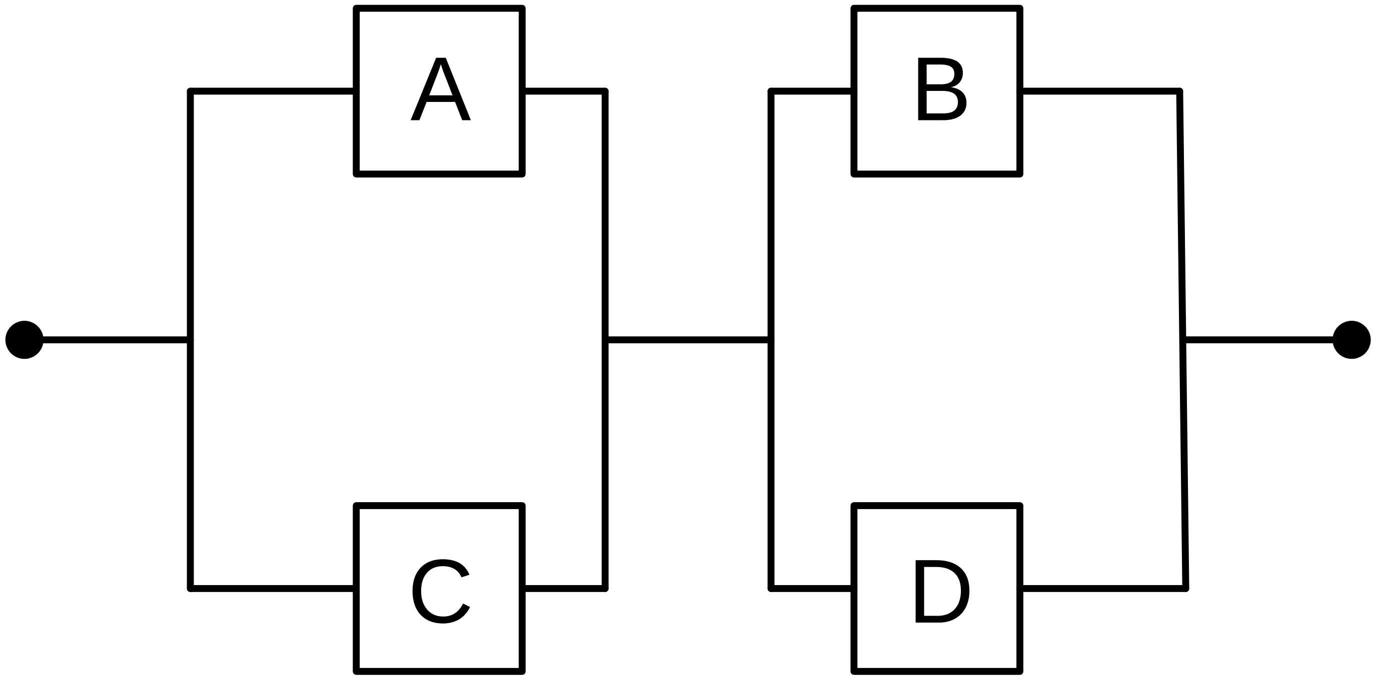
STEP 1

FIND:
$P_F = U_{ABCD}R_E$

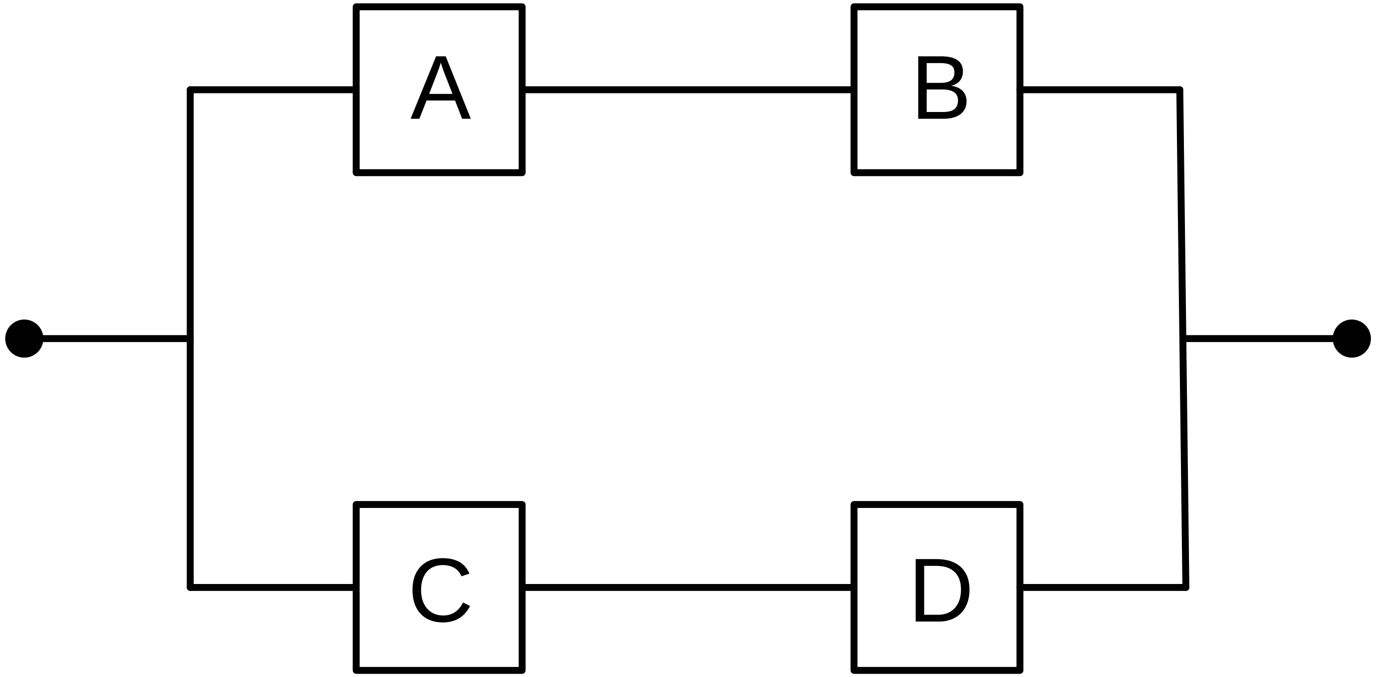


STEP 1			
R_{AC} =	0.99772		$1 - (1-R_A)(1-R_C)$
R_{BD} =	0.99670		$1 - (1-R_B)(1-R_D)$
R_{ABCD} =	0.99443		$(R_{AC})(R_{BD})$
U_{ABCD} =	0.00557		$1-R_{ABCD}$
$U_{ABCD}R_E$ =	0.00541		

STEP 2

FIND:

$P_F = U_{ABCD}U_E$



STEP 2			
$R_{AB} =$	0.91393		$(R_A)(R_B)$
$R_{CD} =$	0.87809		$(R_C)(R_D)$
$R_{ABCD} =$	0.98951		$1 - (1-R_{AB})(R_{CD})$
$U_{ABCD} =$	0.01049		$1-R_{ABCD}$
$U_{ABCD}U_E =$	0.00031		

STEP 3 & 4

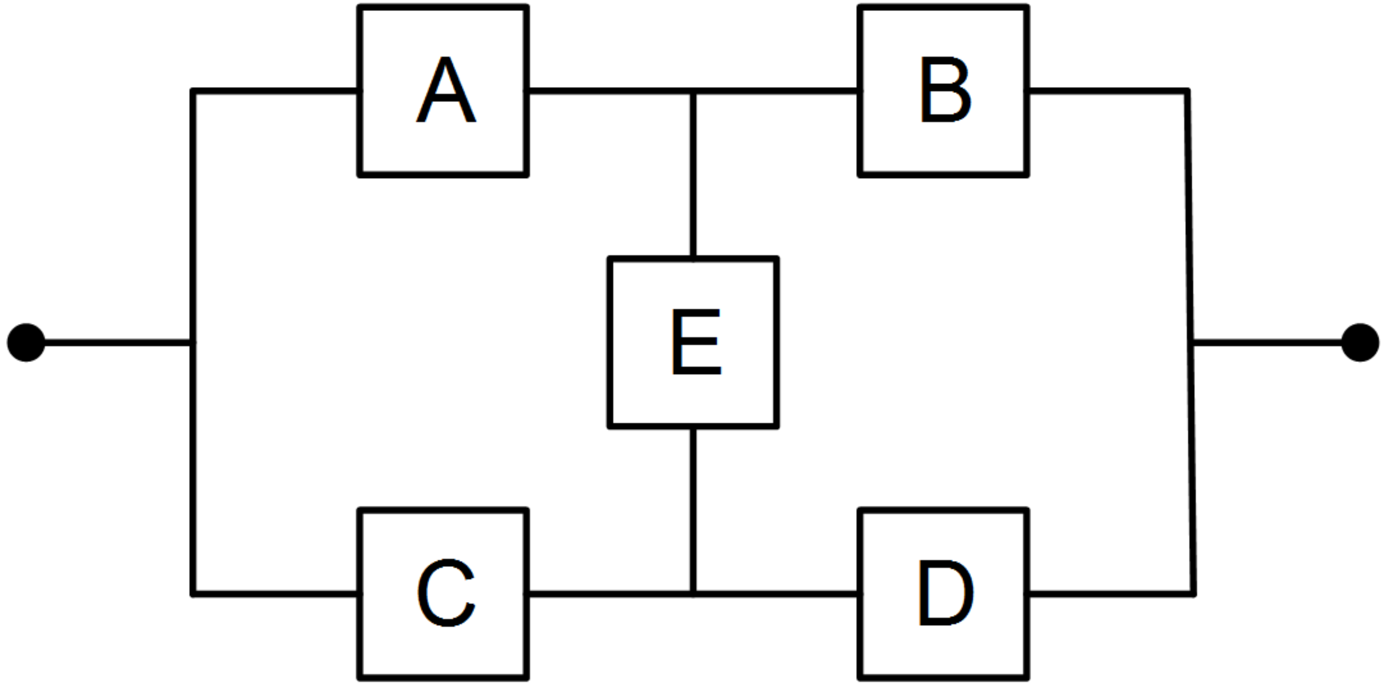
SOLVE: Reliability of the keynote component, R

STEP 3: $P_F = U_{ABCD}R_E + U_{ABCD}U_E$

STEP 4: $R = 1 – P_F$

STEP 3		
U_{SYS} =	0.00572	$U_{ABCD}R_E + U_{ABCD}U_E$
STEP 4		
R_{SYS} =	0.99428	$1 - U_{SYS}$

Keynote Component Model Problem



GIVEN:	
R_A =	0.95
R_B =	0.94
R_C =	0.98
R_D =	0.99
R_E =	0.90

Keynote Component Model Solution

A. 0.95
B. 0.99999
C. 0.998
D. 0.76

Keynote Component Model Solution

STEP 1			STEP 2			STEP 3		
R_{AC} =	0.99900	$1 - (1-R_A)(1-R_C)$	R_{AB} =	0.89300	$(R_A)(R_B)$	U_{SYS} =	0.00176	$U_{ABCD}R_E + U_{ABCD}U_E$
R_{BD} =	0.99940	$1 - (1-R_B)(1-R_D)$	R_{CD} =	0.97020	$(R_C)(R_D)$			
R_{ABCD} =	0.99840	$(R_{AC})(R_{BD})$	R_{ABCD} =	0.99681	$1 - (1-R_{AB})(R_{CD})$	STEP 4		
						R_{SYS} =	0.99824	$1 - U_{SYS}$
U_{ABCD} =	0.00160	$1-R_{ABCD}$	U_{ABCD} =	0.00319	$1-R_{ABCD}$			
$U_{ABCD}R_E$ =	0.00144		$U_{ABCD}U_E$ =	0.00032				

Shared Load Parallel Model

Reliability Testing

Reliability Test Planning

Reliability Test Planning

- Elements of a reliability test plan
- Types and applications of reliability testing
- Test environment considerations

Reliability Testing Introduction (Continued)

For each program or product, an individual or project manager should have responsibility and authority for carrying out the objectives. An inherent conflict that arises is that uncovered failures will slow the completion of the program. However, failures are required in order to detect weaknesses in the design or in the production process. An ideal test program will produce the discovery of every failure mode.

Reliability Test Planning

Reliability Test Planning is reviewed in the following topic areas:

- Elements of a Reliability Test Plan
- Types and Applications of Reliability Testing
- Test Environment Considerations

Elements of a Reliability Test Plan

The project manager (reliability engineer or coordinator) in charge of the program must develop a plan and schedule. The project manager must also consider the following factors in the design:

- How critical is the product?
- Are safety and reliability a concern?
- Does the customer need a certain level of reliability?
- How mature is the design?
- Are new technologies or processes involved?
- How complex is the product?
- What are the environmental extremes involved?
- What is the budget for testing?
- Are the equipment or facilities able to perform test conditions?
- How many items are available for testing?
- What is the existing design reliability? (*Reliability Toolkit*, 1993)[37]

Elements of a Reliability Test Plan (Continued)

The planning should start as early as possible in the design process. It would be ideal to have test equipment available for the first prototypes with built in test points or other test points already designed in. Information from the design phase concerning reliability prediction, FMEAs, stress analysis, parameter variation analysis, fault tree analysis, etc., will aid in preparing the reliability test plan. Use of existing information can focus the test plan in detecting those failure modes or concerns. Unexpected failure modes will come from the manufacturing process itself.

O'Connor (1996)[34] states that it is rare to test on fewer than four items. A typical sample size of items to test on would be from 5 to 20. Of course, cost will be a major factor for large, complex, expensive, one-onlys, such as power stations, space shuttles, or Voyagers. Generally, mechanical devices are subject to wear, fatigue, or corrosion, so accelerated testing would be a good choice. Electronic components will enter the constant hazard rate, after the early burn-in period, so long duration testing is not a good choice for these types of products.

In planning the testing program, the project manager will have additional information available to process. Ireson (1996)[11] lists them as:

- Contract or customer requirements
- Drawings, specifications
- Test plans, procedures
- Tests and inspections
- Cost analysis
- Tolerance studies, funneling
- Spare parts lists
- Critical parts lists
- Field service test plans
- Test facility and capability lists
- Plant layout

A tentative test plan can be developed, but revised at frequent intervals due to changes in hardware design concepts, production techniques, failure feedback, or new failure modes uncovered. The test plan and schedule should be charted as a Gantt or PERT chart. The test plan should include:

- A test plan for each item (characteristics to be tested, specifications required, quantity to be tested, environmental requirements, document numbers for the plans and procedures)

Reliability Testing

Developmental Testing

Development Testing

- Accelerated life test – ALT
  - Single stress
  - Multiple stress
  - Sequential stress
- Step stress testing – HALT
- Reliability growth testing
  - AMSAA
  - Duane
  - TAAF
- Software testing
  - White box
  - Fault injection

Reliability Testing

Product Testing

Product Testing

- Assess purpose, advantage and limitations
  - Qualification/demonstration testing
    - Sequential
    - Fixed length
  - Product reliability acceptance test – PRAT
  - Stress screening
    - ESS
    - HASS
    - Burn-in
  - Attribute testing
    - Binomial
    - Hypergeometric
  - Degradation testing
  - Software testing

Comparison of Test Plans

Sequential Test Plans

Sequential tests have the advantage of having the lowest overall test costs as the number of failures found in the tests is lower on the average than in the other plans. These plans also have the advantage of the least time (on average) to come to a decision. The truncation points limit the maximums in terms of hours to perform the test as well as number of failures that will occur prior to terminating the test, thus giving an estimate of the maximum test costs.

Disadvantages of sequential tests include variable test costs. When compared to time truncated tests, the maximum number of failures and thus test time can be about three times the time truncated tests. Scheduling of tests is imprecise due to the variability in the times to decision.

Time Terminated Tests

Time terminated tests have the advantage of knowing when the test will be complete as well as the maximum number of items that will be required for the test. The maximum time to a decision is the least for a time truncated test. Though this feature can be placed in a sequential test, the value will likely be higher in a sequential test.

Some of the disadvantages include the variable number of failures up to some fixed value as well as a higher expected number of failures and waiting time than with a sequential test. The average time is higher though the maximum time is less.

Failure Terminated Tests

Some of the advantages listed for these tests include a known maximum number of items required for the test even when testing with replacement. Failures must occur in a test of this type which allows for an analysis of failure modes as well as the mechanisms of failure.

Some disadvantages of these tests include an average waiting time that is higher than the sequential tests as well as a higher expected number of failures.

Testing Summary

On the average, PRST provides the minimum overall costs and time to reach a decision. This is especially true when a lot is very good or very bad as compared to the requirements.

Maintainability and Availability

Management Strategies

Management Strategies

- Maintainability and availability planning
- Maintenance strategies
  - Corrective maintenance – reactive
  - Preventive maintenance – proactive
    - RCM
- Maintainability apportionment/allocation
- Availability tradeoffs

Availability

- A = Uptime/(Uptime + Downtime)

$$A_i = \frac{MTBF}{MTBF + MTTR}$$

Achieved

$$A_O = \frac{MTBM}{MTBM + MTTR + ALDT}$$

$$A_O = \frac{MTBM}{MTBM + MMT + ALDT}$$

Prediction

$$A_a = \frac{MTBM}{MTBM + MDT}$$

Maintainability and Availability

Analyses

Analyses

- Maintenance time distributions
  - Lognormal
  - Weibull
- Preventive maintenance (PM) analysis
  - Tasks
  - Intervals
  - Non-applicability
- Corrective maintenance (CM) analysis
  - Fault isolation time – AKA FD/FI
  - Repair/replace time
  - Skill level
- Testability
- Spare parts strategy

Data Collection and Use

Data Collection

Data Collection

- Types of data
  - Attribute (discrete) v continuous
  - Compete v censored
- Data sources
- Collection methods
- Data management
  - Acquisition
  - Media
  - Database

Data Collection and Use

Data Use

Data Use

- Analysis
  - Algorithmic
  - Graphic

  FOR
  - Math Modeling
  - Trend Analysis
  - ANOVA
- Measures of effectiveness

Data Collection and Use

Data and Failure Analysis Tools

Data and Failure Analysis Tools

- FMEA
- Criticality analysis
  - Hazards analysis
  - Severity analysis
  - Consequences analysis
- FTA
- FRACAS