Stress-Strength Risk-Based Reliability Fatigue Model

Huntsville Society of Reliability Engineers

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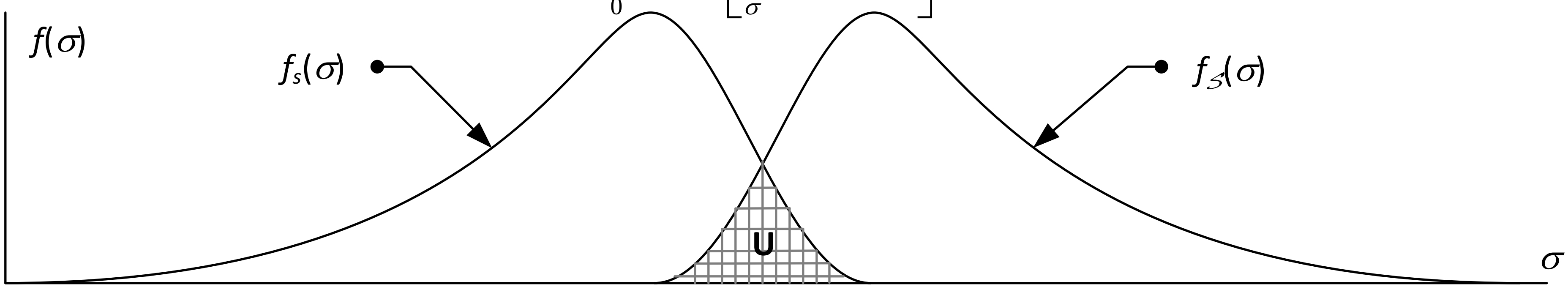
Bivariate Hazard Function, $h(\sigma,n)$

- Characterizes probabilistic risk of failure
  $h(x) = f(x)/[1-F(x)]$ $h_{exp}(t) = \lambda$
- Application of interference theory to structural reliability over operating life degradation
- Failure modes – damage done by failure mechanisms
  - Change in shape – elongation strain
  - Change in geometry – crack initiation
  - Change in material property – strength reduction
- Introduces organization's risk threshold, r
- Emphasis on hazard function rather than MTBF/FR
- GOAL: <u>FAILURE AVOIDANCE</u>

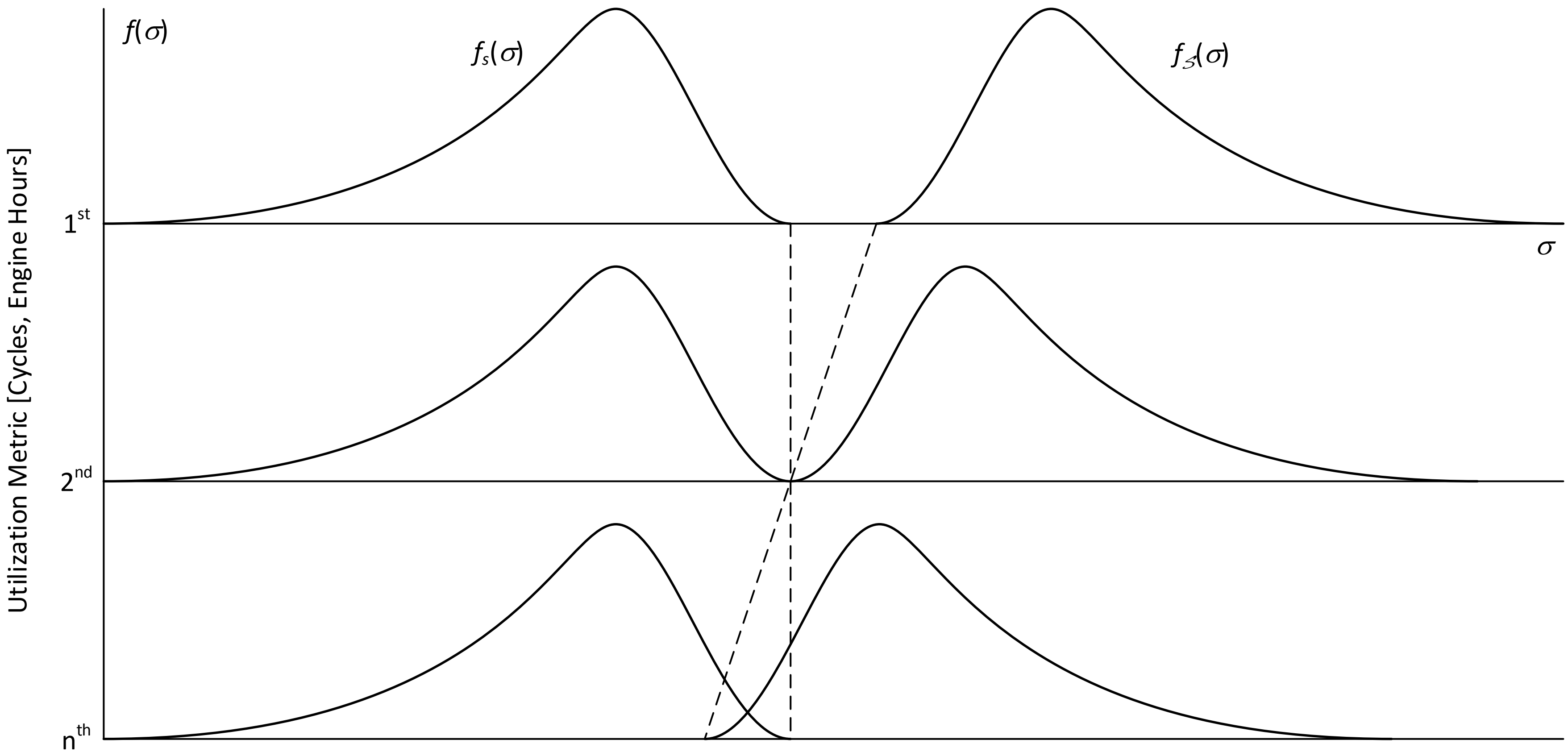
Structural Interference Theory
Stress-Strength Reliability Model

$$R = \int_0^\infty f_s(\sigma)\left[\int_\sigma^\infty f_S(\sigma)\,d\sigma\right]d\sigma$$

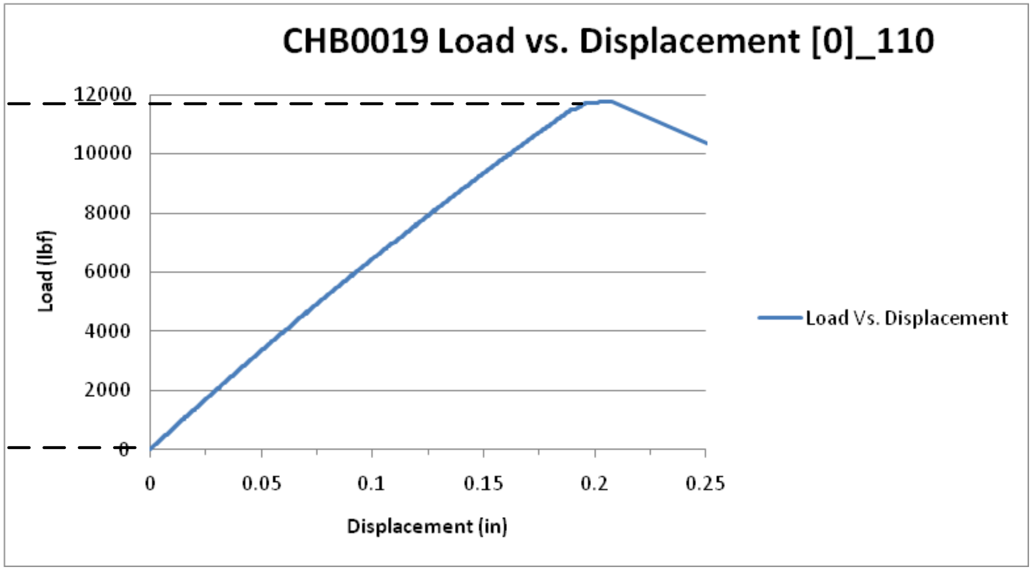


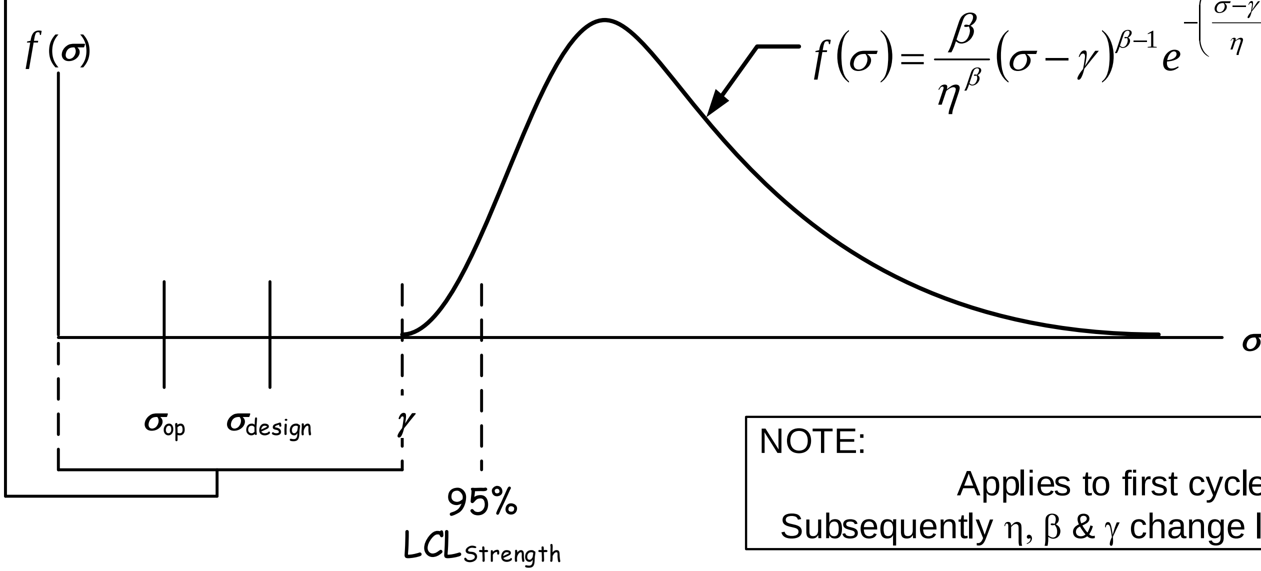
R = 1 - U

Stress-Strength Degradation



Structural Reliability Failure Model





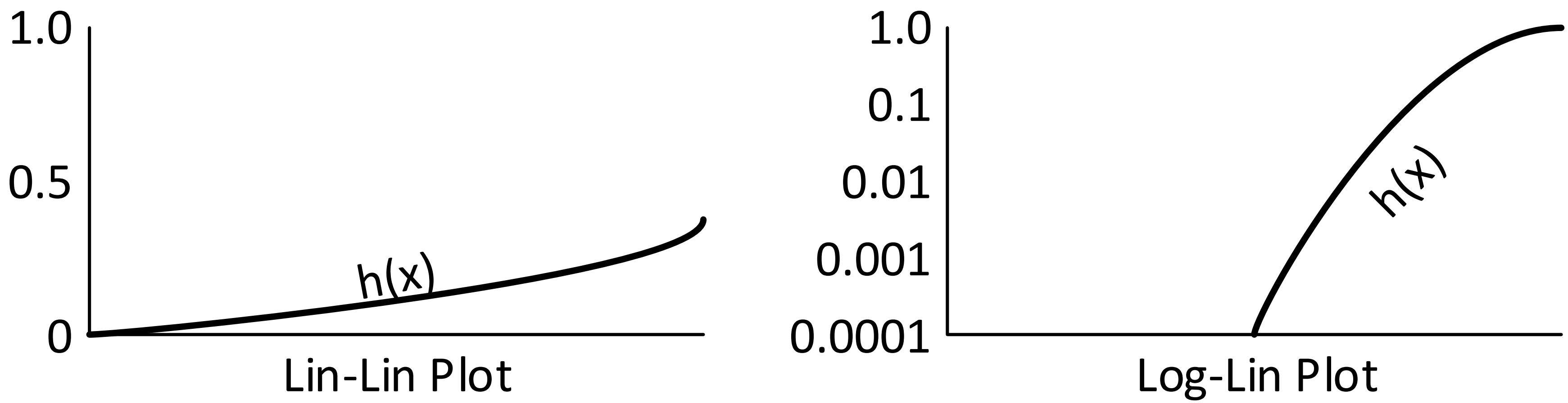
$$f(\sigma) = \frac{\beta}{\eta^{\beta}}(\sigma - \gamma)^{\beta - 1} e^{-\left(\frac{\sigma - \gamma}{\eta}\right)^{\beta}}$$

$$R = \int_{\sigma_{design}}^{\infty} f(\sigma) d\sigma$$

R = 1.000 IF $\gamma > \sigma_{design}$

NOTE:
Applies to first cycle of use.
Subsequently η, β & γ change leading to $\gamma < \sigma_{design}$

Structural Reliability Hazard Function – h(x)

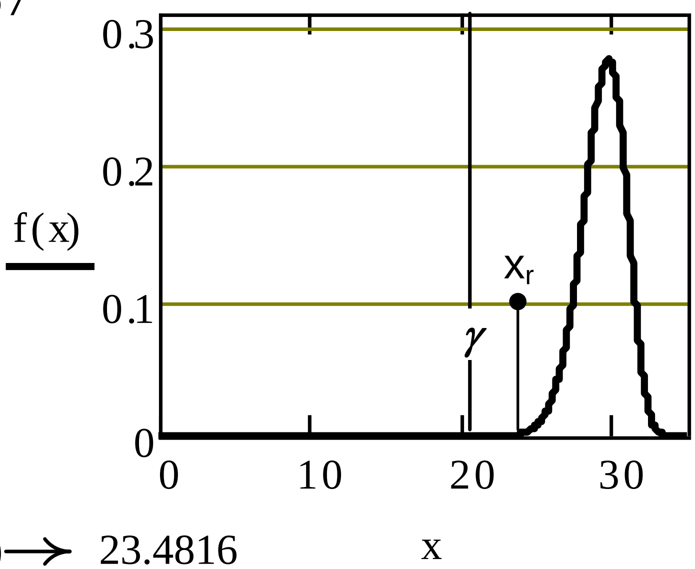


Risk-based Structural Hazard Function – $h_r(\sigma)$

$\beta := 6.667$

$\eta := 9$

$\gamma := 21$



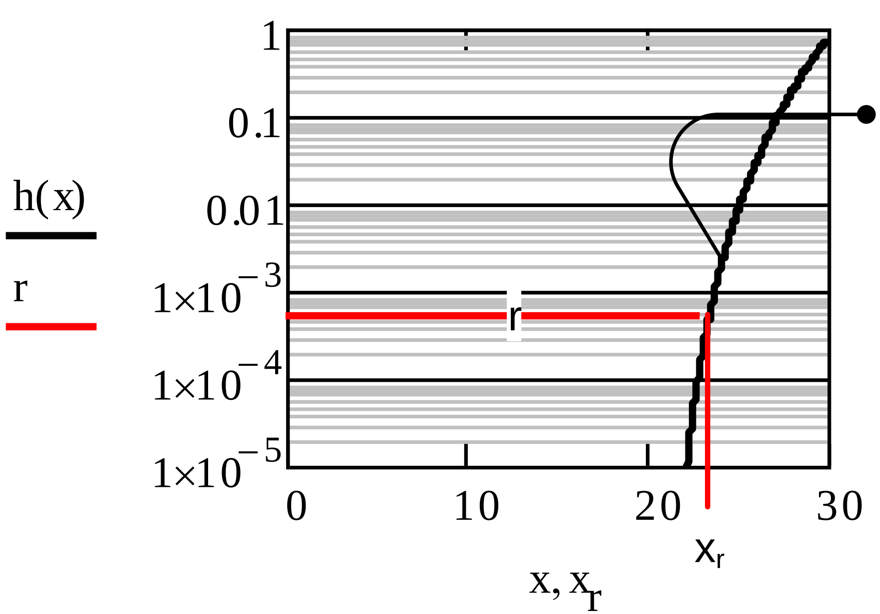
$$f(x) := \begin{cases} 0 & \text{if } 0 \le x < \gamma \\ \left(\frac{\beta}{\eta^{\beta}}\right)\cdot(x-\gamma)^{\beta-1}\cdot e^{-\left(\frac{x-\gamma}{\eta}\right)^{\beta}} & \text{if } x \ge \gamma \end{cases}$$

Given

r:= 0.0033

h(t) = r

x_r := Find(t) → 23.4816



$$h(x) := \left(\frac{\beta}{\eta^{\beta}}\right)\cdot(x-\gamma)^{\beta-1}$$

r = 3300 failures per million utilization events

Bivariate Hazard Function – $h_r(\sigma,n)$