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CAVE[3]

Assessment of System Reliability

Sa'd Hamasha, Ph.D.

Assistant Professor
Industrial and Systems Engineering

Phone: 607-372-7431
Email: hamasha@auburn.edu

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AUBURN UNIVERSITY | SAMUEL GINN COLLEGE OF ENGINEERING

Presenter

Sa'd Hamasha, Ph.D.

Assistant Professor of Industrial and Systems Engineering
Auburn University, Auburn, AL

Education:

Ph.D. in Industrial and Systems Engineering
M.S. in Industrial Engineering
B. S. in Mechanical Engineering
Lean Six Sigma Black Belt

Teaching:

Quality Design and Control
Reliability Engineering
Electronics Manufacturing Systems

Research:

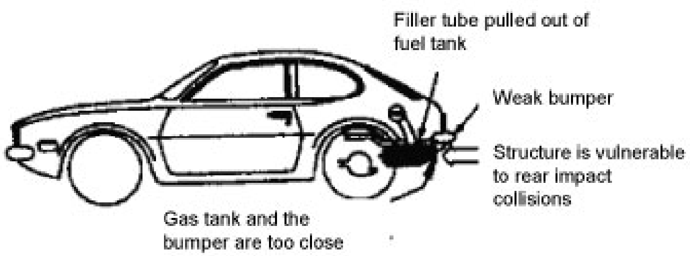
Reliability of Electronic Components and Assemblies

Agenda

- Introduction
- Failure Distributions
  - Constant Failure Rate (Exponential Distribution)
  - Time Dependent Failure Rate (Weibull Distribution)
- Reliability of Serial System
- Reliability of Parallel System
- Reliability of Combined System
- Reliability of Network System

Introduction

- Things Fail!
- 1978 - Ford Pinto: fuel tank fire in rear-end collisions
  - Deaths, lawsuits, and negative publicity (recall then discontinue production)



Introduction

- Things Fail!

- 2016 – Samsung Note 7
  - Battery catches fire

- Southwest Airlines (Louisville to Baltimore)
  - Burn the plane’s carpet and caused some damage to its subfloor

- Reliability engineers attempt to study, characterize, measure, and analyze the failure in order to eliminate the likelihood of failures

Are Failures Random?

- Common approach taken in reliability is to treat failures as random or probabilistic occurrences
- In theory, if we were able to comprehend the exact physics and chemistry of a failure process, failures could be predicted with certainty
- With incomplete knowledge of the physical/chemical processes which cause failures, failures will appear to occur at random over time
- This random process may exhibit a pattern which can be modeled by some probability distribution (i.e. Weibull)

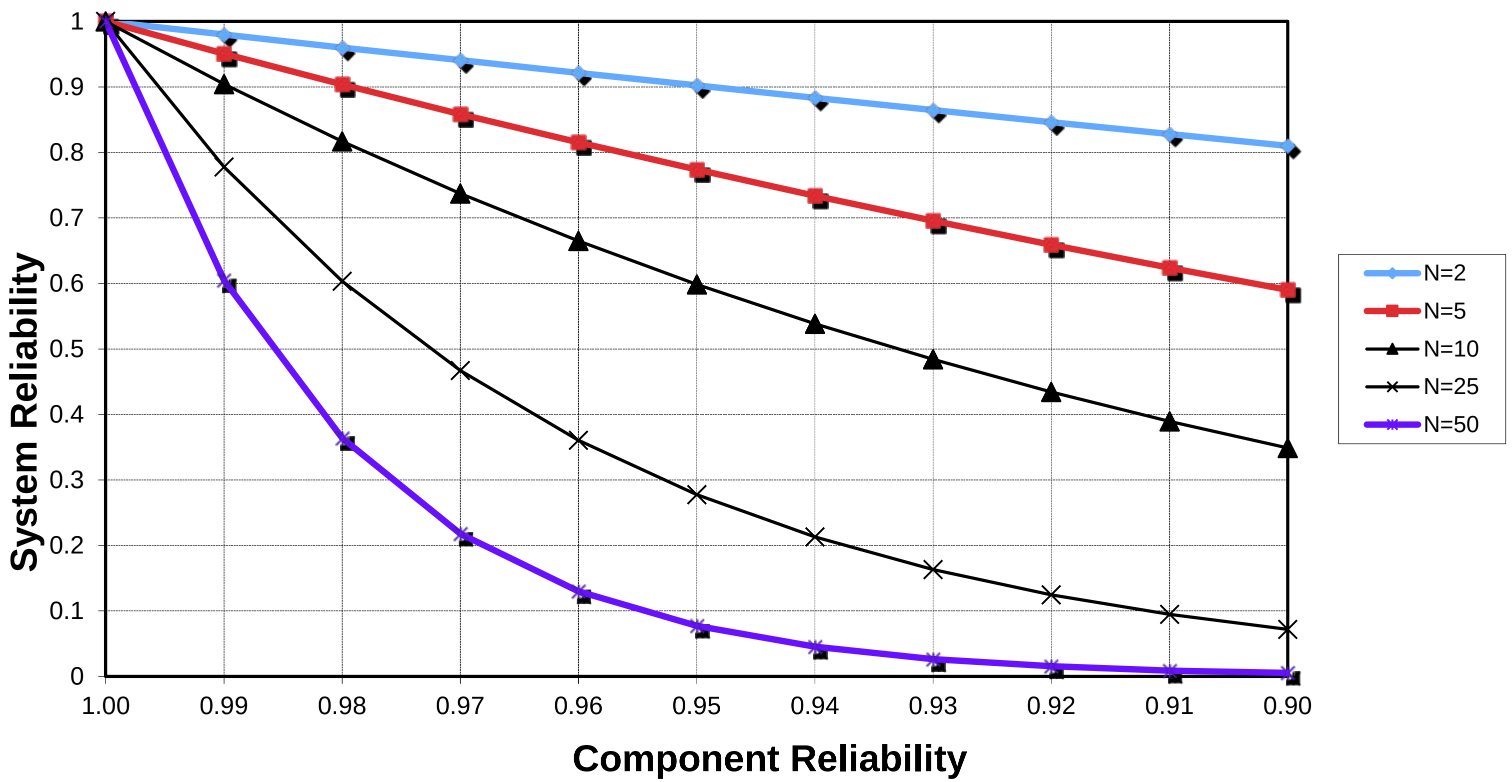
Reliability?

- Reliability is the probability that a component or system will perform a required function for a given period of time when used under stated operating conditions
- R(t) = It is the probability of non-failure

- More focus on reliability
  - System complexity
  - Cost of failures
  - Public awareness of product quality and reliability
  - New regulations concerning product liability

Complexity and Reliability

For serial System



The Reliability Function, R(t)

- Reliability is defined as the probability that a system (component) will function over some time period t

- Let T = a random variable, the time to failure of a component
- R(t) is the probability that the time to failure is greater than or equal to t

$$R(t) = Pr\{T \geq t\}$$

$$\text{where } R(t) \geq 0\ ,\ R(0) = 1, \text{and}$$

$$\lim_{t \to \infty} R(t) = 0$$

Often called the SURVIVAL FUNCTION

The Failure Function, F(t)

- F(t) is the probability that a failure occurs before time t

$$F(t) = 1 - R(t) = Pr\{T < t\}$$

$$\text{where } F(0) = 0 \text{ and } \lim_{t \to \infty} F(t) = 1$$

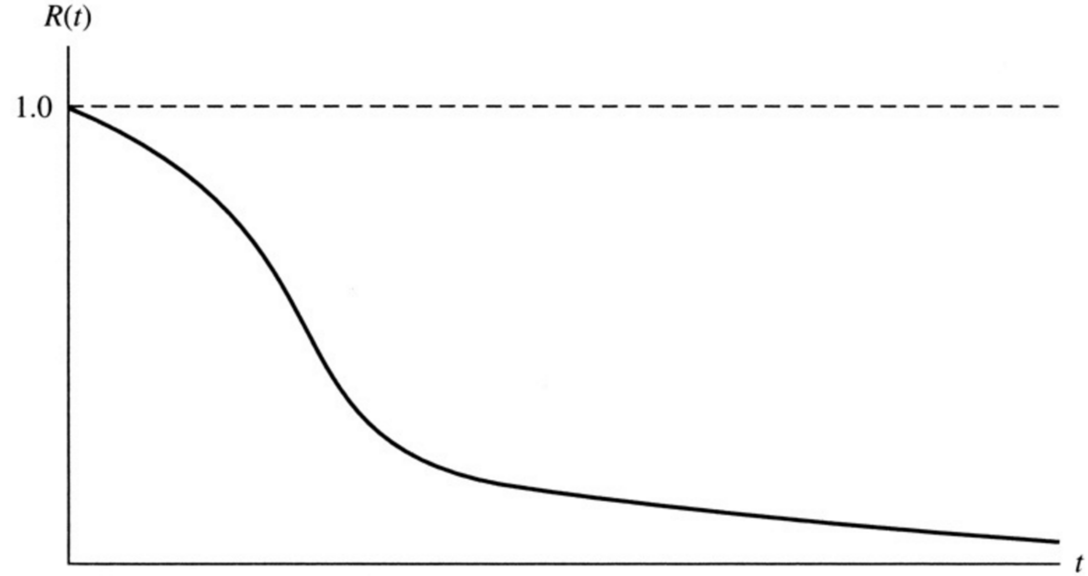
- It is the cumulative distribution function (CDF) of the failure distribution

Reliability

Reliability function

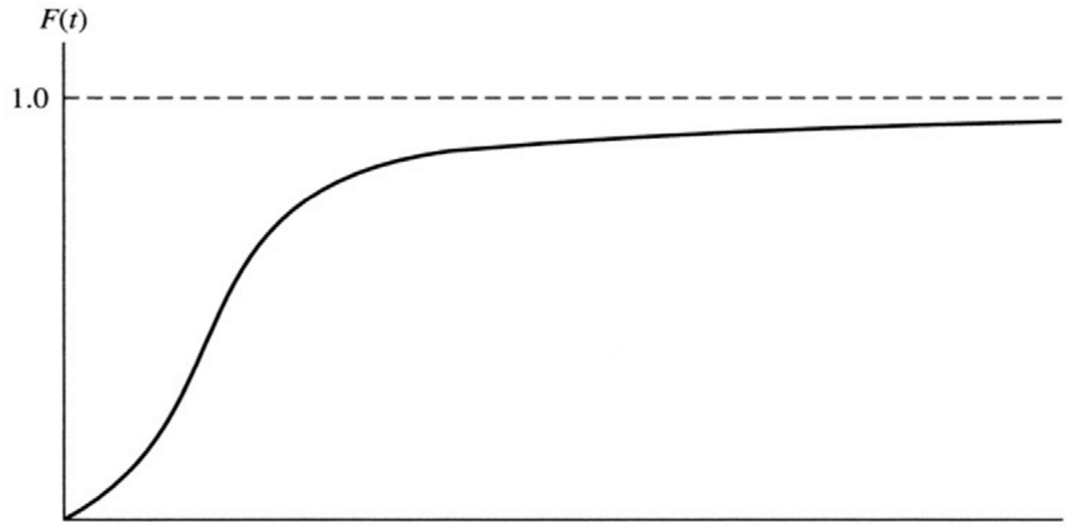
$$R(t) = \int_t^{\infty} f(t')\,dt'$$

$f(t)$ is Probability Density Function



Failure function

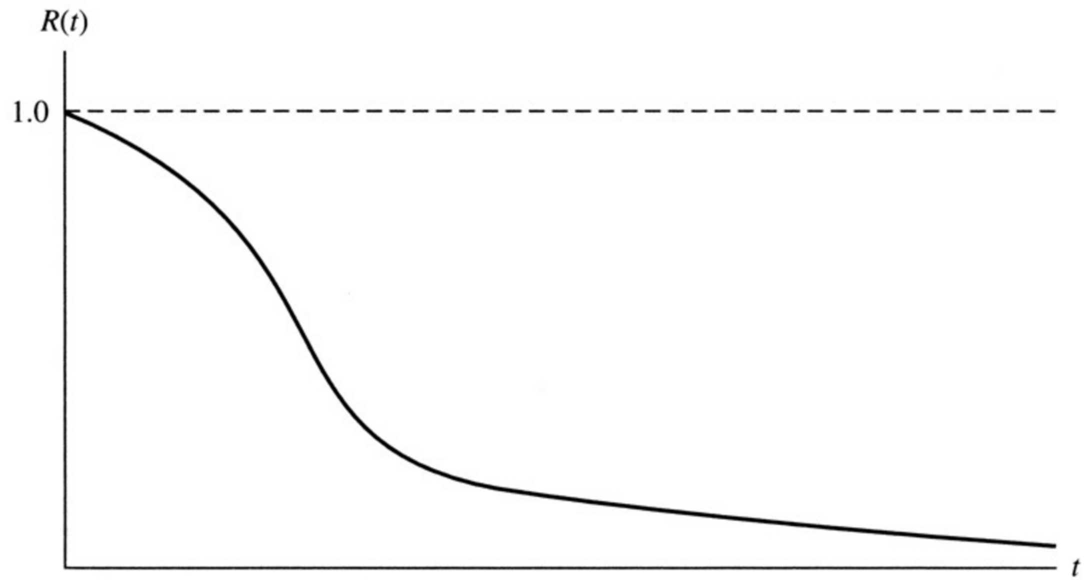
$$F(t) = \int_0^{t} f(t')\,dt'$$



Mean Time to Failure

- It is the average time of survival

$$MTTF = \int_0^\infty R(t)dt$$



(a)

Failure Rate Function, $\lambda(t)$

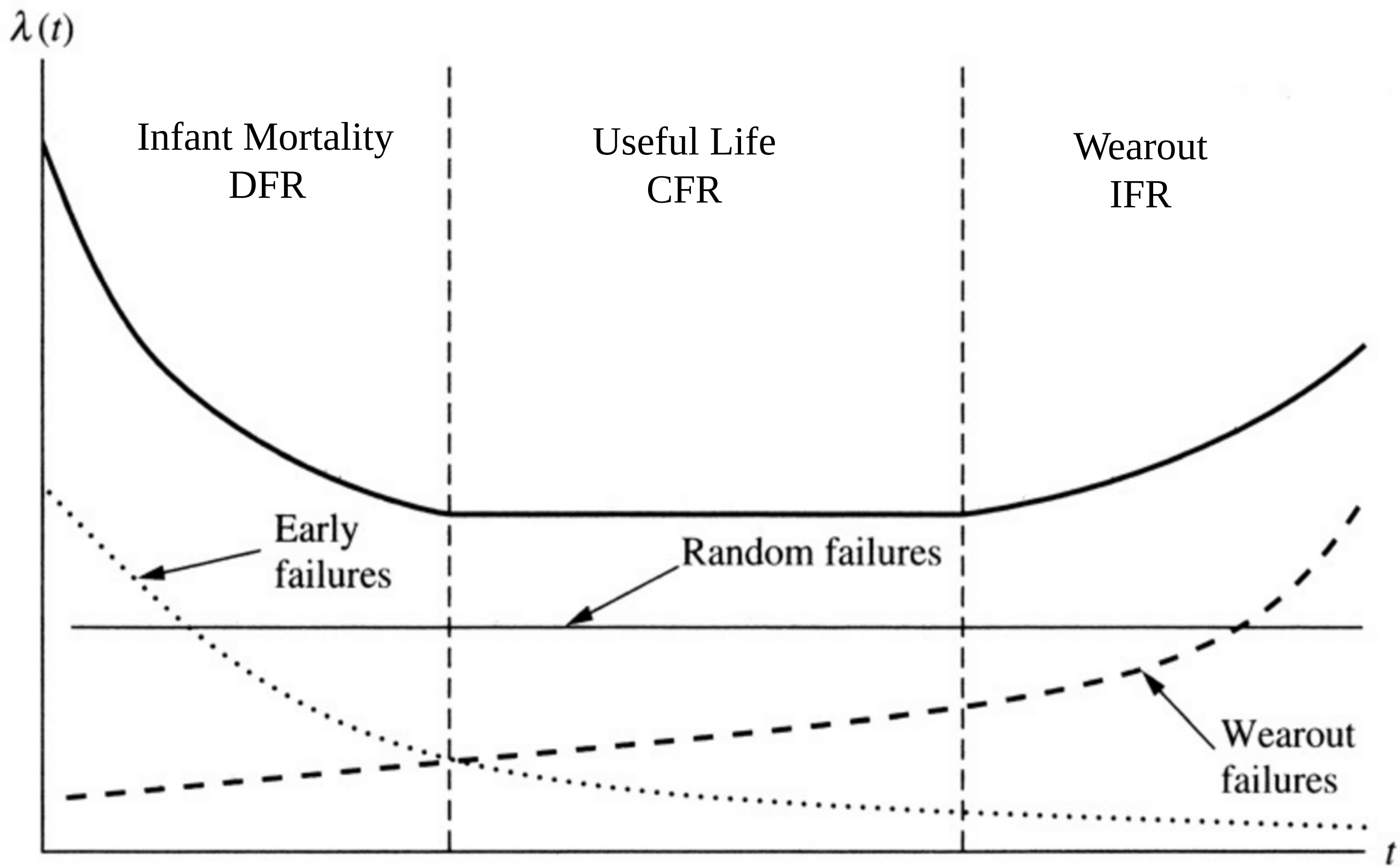
- Failure rate is expressed as a function of time
- Mathematically, failure rate equals probability density function divided by reliability function:

$$\lambda(t) = \frac{f(t)}{R(t)}$$

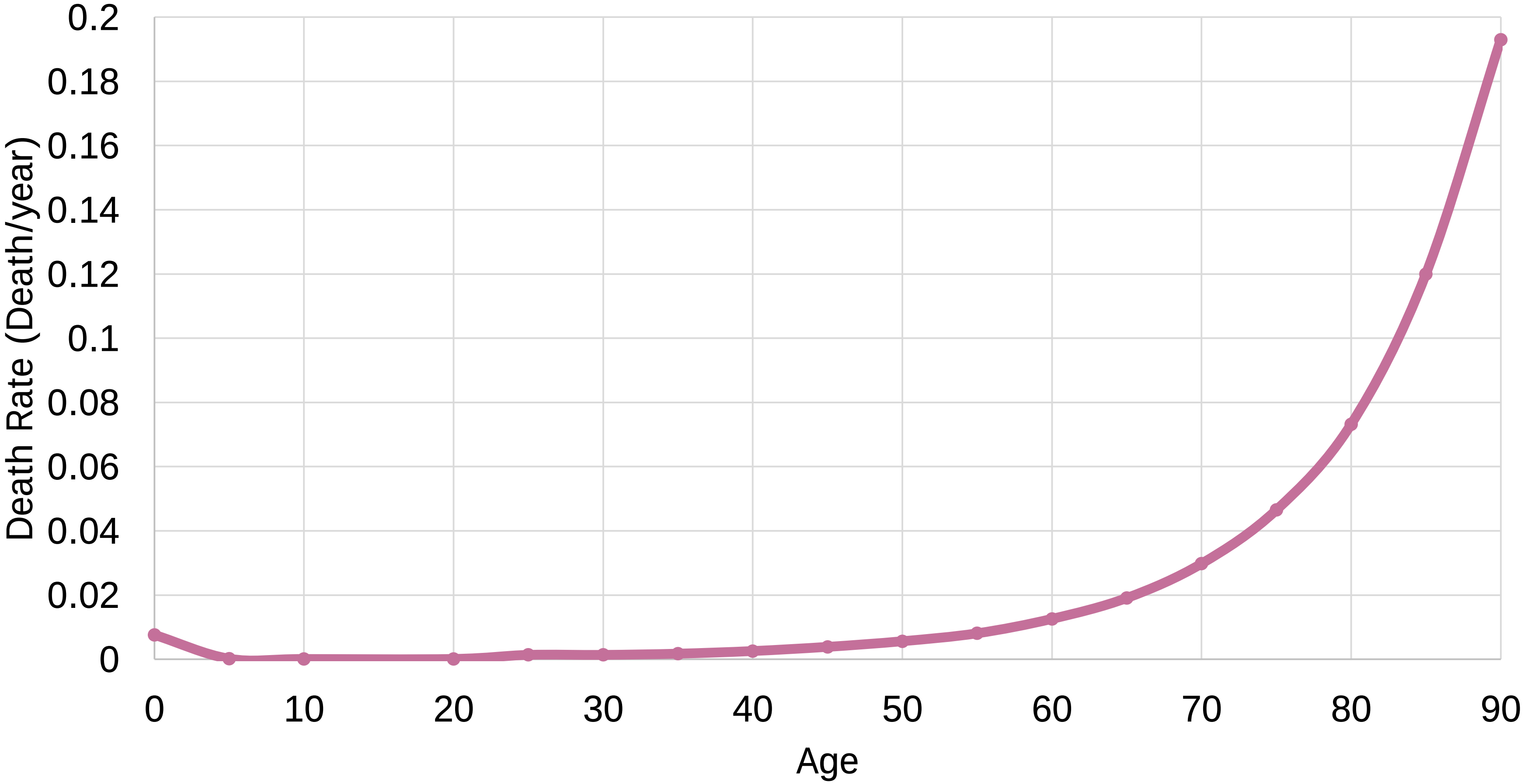
$$R(t) = \int_t^{\infty} f(t')\,dt' = exp\left[-\int_0^t \lambda(t')\,dt'\right]$$

- Failure rates can be characterized as:
  - Increasing Failure Rate (IFR) when λ(t) increasing
  - Decreasing Failure Rate (DFR) when λ(t) decreasing
  - Constant Failure Rate (CFR) when λ(t) constant

Bathtub Curve



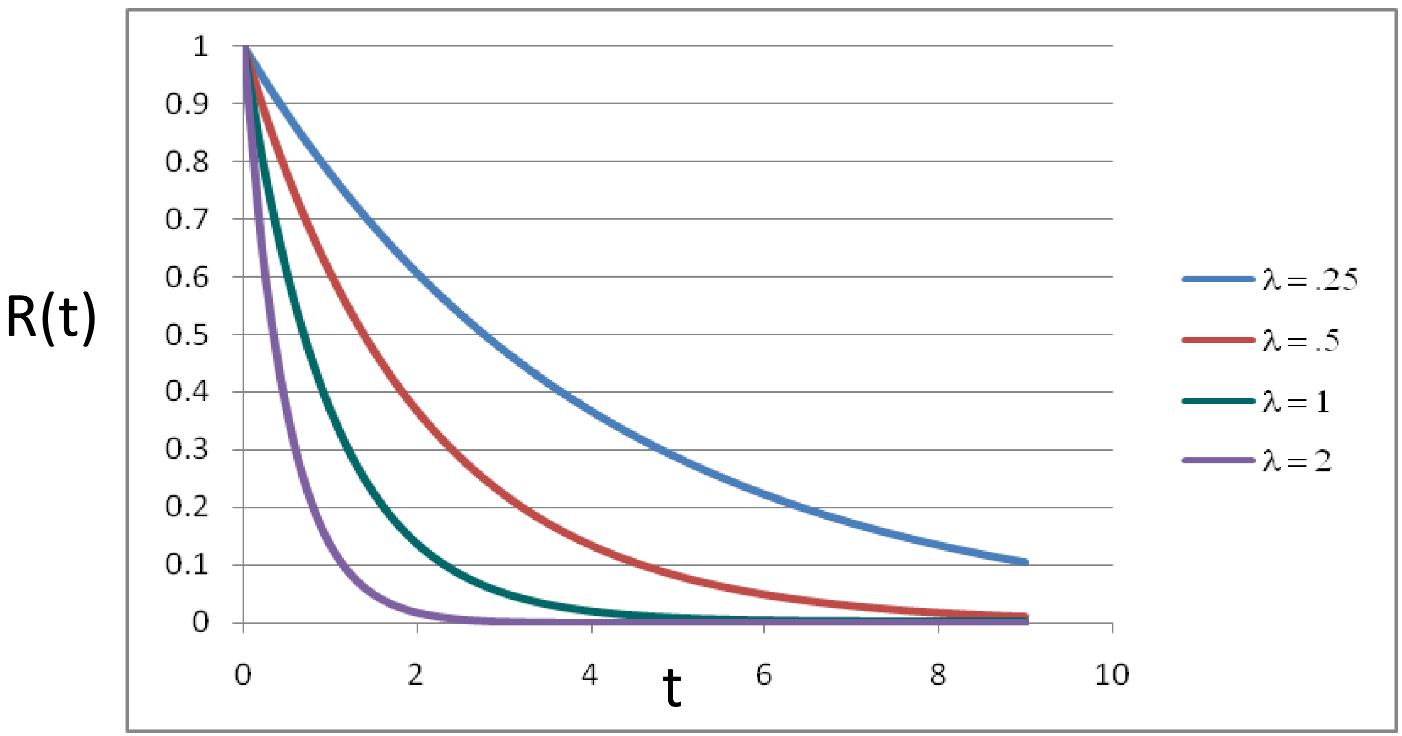
Human Mortality Curve



Exponential Distribution

- A failure distribution that has a constant failure rate is called an exponential probability distribution

$$\lambda(t) = \lambda \longrightarrow R(t) = \exp(-\lambda t)$$



Weibull Distribution

- The most useful probability distributions in reliability is the Weibull
- Used to model increasing, decreasing, or constant failure rates

- The Weibull failure rate function:

$$\lambda(t) = at^b$$

- λ(t) is increasing for b >0, decreasing for b < 0
  constant for b =0

Weibull Distribution

- For mathematical convenience it is better to express λ(t) in the following manner:

$$\lambda(t) = at^b \Rightarrow \lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}$$

$\boldsymbol{\beta}$ is the shape parameter

$\boldsymbol{\theta}$ is the scale parameter (characteristic life)

$$R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}} \qquad F(t) = 1 - e^{-\left(\frac{t}{\theta}\right)^{\beta}}$$

Component Reliability Estimation

1. Reliability testing: Collect time to failure data (t)
2. Fit the data to a statistical distribution (Weibull, use Weibull plot)
3. Estimate the parameter of the distribution (shape and scale for Weibull)
4. Develop the Reliability function (R(t))

Example

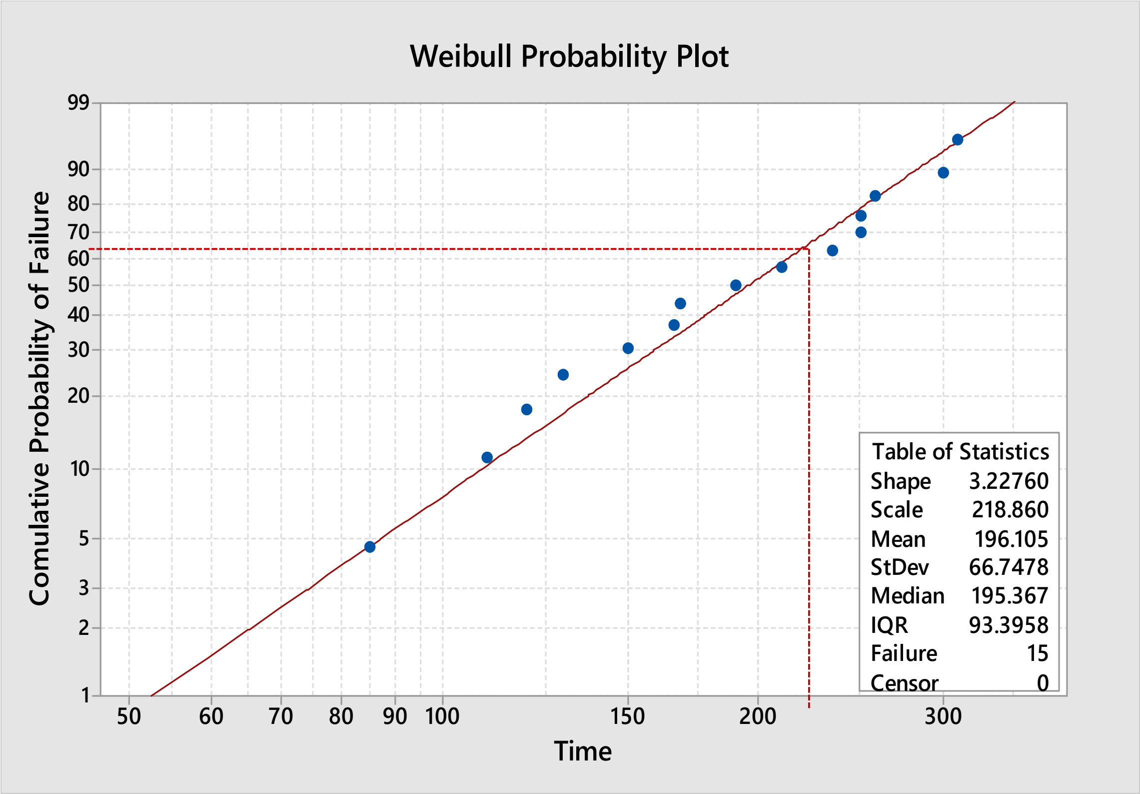
15 Electronic Components are test until failure. The time to failure data is below. Develop a Weibull reliability function?

Failure order	Time to Failure	Cumulative Probability
1	85	0.05
2	110	0.11
3	120	0.18
4	130	0.24
5	150	0.31
6	166	0.37
7	168	0.44
8	190	0.50
9	210	0.56
10	235	0.63
11	250	0.69
12	250	0.76
13	258	0.82
14	300	0.89
15	310	0.95

$$= \frac{Failure\ order\ - 0.3}{Total\ \#\ of\ components + 0.4}$$

Example – Weibull Plot

- Plot cumulative probability of failure vs. time to failure on a Weibull paper



Scale Parameter:

θ = time at 63.2% of failure

θ = 218.86

Shape parameter:

β = slope of the fitting line

β = 3.23

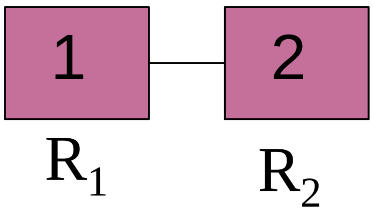
Estimate the reliability:

$$\boldsymbol{R(t) = e^{-\left(\frac{t}{\theta}\right)^{\beta}}}$$

$$\boldsymbol{R(t) = e^{-\left(\frac{t}{218.86}\right)^{3.23}}}$$

Reliability of Serial System

- Reliability Block Diagram



- How do we calculate the Reliability of this system?
- Go back to the basic probability:

E_1 = the event, component 1 does not fail

E_2 = the event, component 2 does not fail

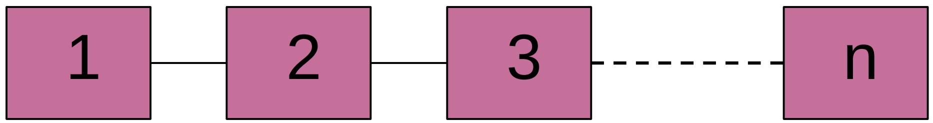
$P\{E_1\} = R_1$ *and* $P\{E_2\} = R_2$ where

Therefore assuming independence:

$$R_s = P\{E_1 \cap E_2\} = P\{E_1\}\,P\{E_2\} = R_1\,R_2$$

Reliability of Serial System

- Reliability Block Diagram



- Generalizing to n mutually independent components in series:

$$R_s(t) = R_1\, R_2 \;....\; R_n$$

- For Serial System:

$$R_s(t) \leq \min\{R_1, R_2, ..., R_n\}$$

Component Count vs. System Reliability

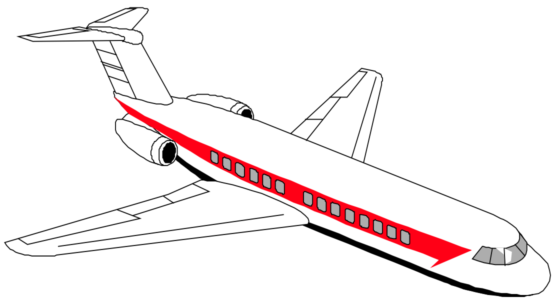
Number of Components

Comp. Rel.	10	100	1000
.900	.3487	$.266 \times 10^{-4}$	$.1748 \times 10^{-45}$
.950	.5987	.00592	$.5292 \times 10^{-22}$
.990	.9044	.3660	$.443 \times 10^{-4}$
.999	.9900	.9048	.3677

System Reliability

Exercise

- The failure distribution of the main landing gear of a commercial airliner is Weibull with a shape parameter of 1.6 and a characteristic life of 10,000 landings.
- The nose gear also has a Weibull distribution with a shape parameter of 0.90 and a characteristic life of 15000 landings.
- What is the reliability of the landing gear system if the system is to be overhauled after 1000 landings?



Exercise

R$_1$
θ =10,00
β = 1.6

R$_2$
θ =15,00
β = 0.9

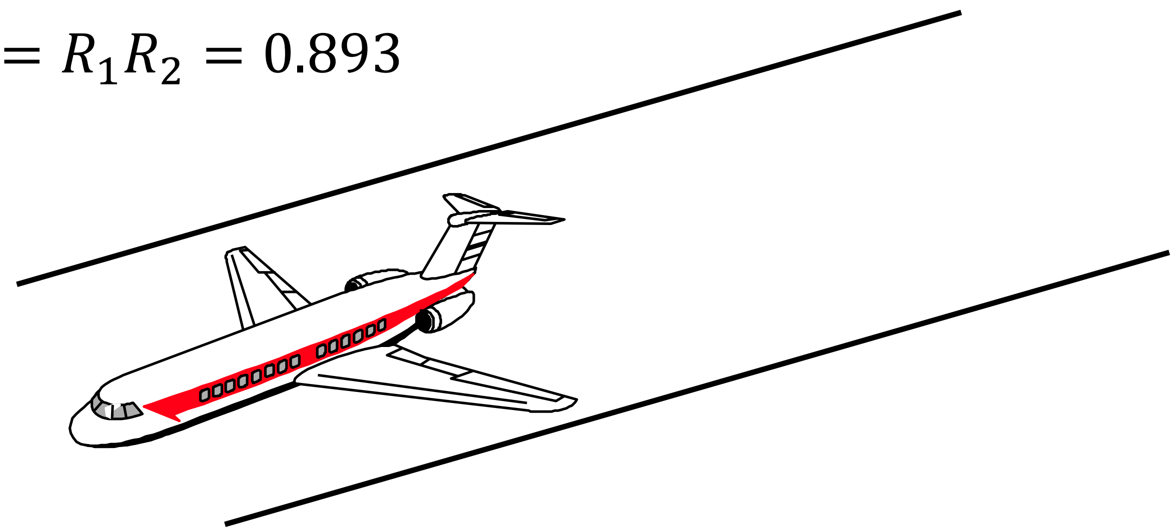
- For Weibull: $R(t) = exp - \left(\frac{t}{\theta}\right)^{\beta}$

- What is the system reliability after 1000 landing?

$$R_1 = exp - \left(\frac{1{,}000}{10{,}000}\right)^{1.6} = 0.975$$

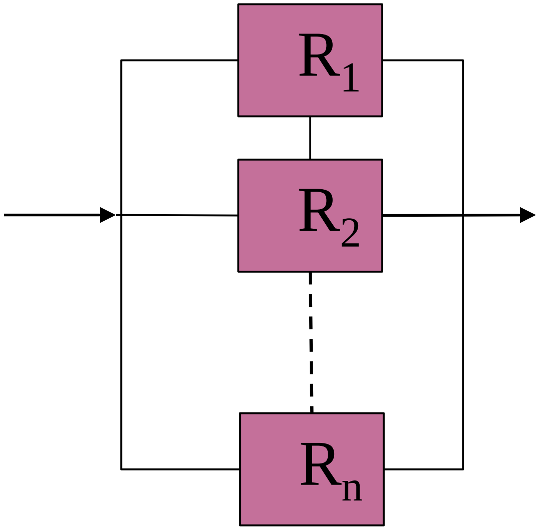
$$R_2 = exp - \left(\frac{1{,}000}{15{,}000}\right)^{0.9} = 0.916$$

$$R_s = R_1 R_2 = 0.893$$



Reliability of Parallel System

- Reliability Block Diagram



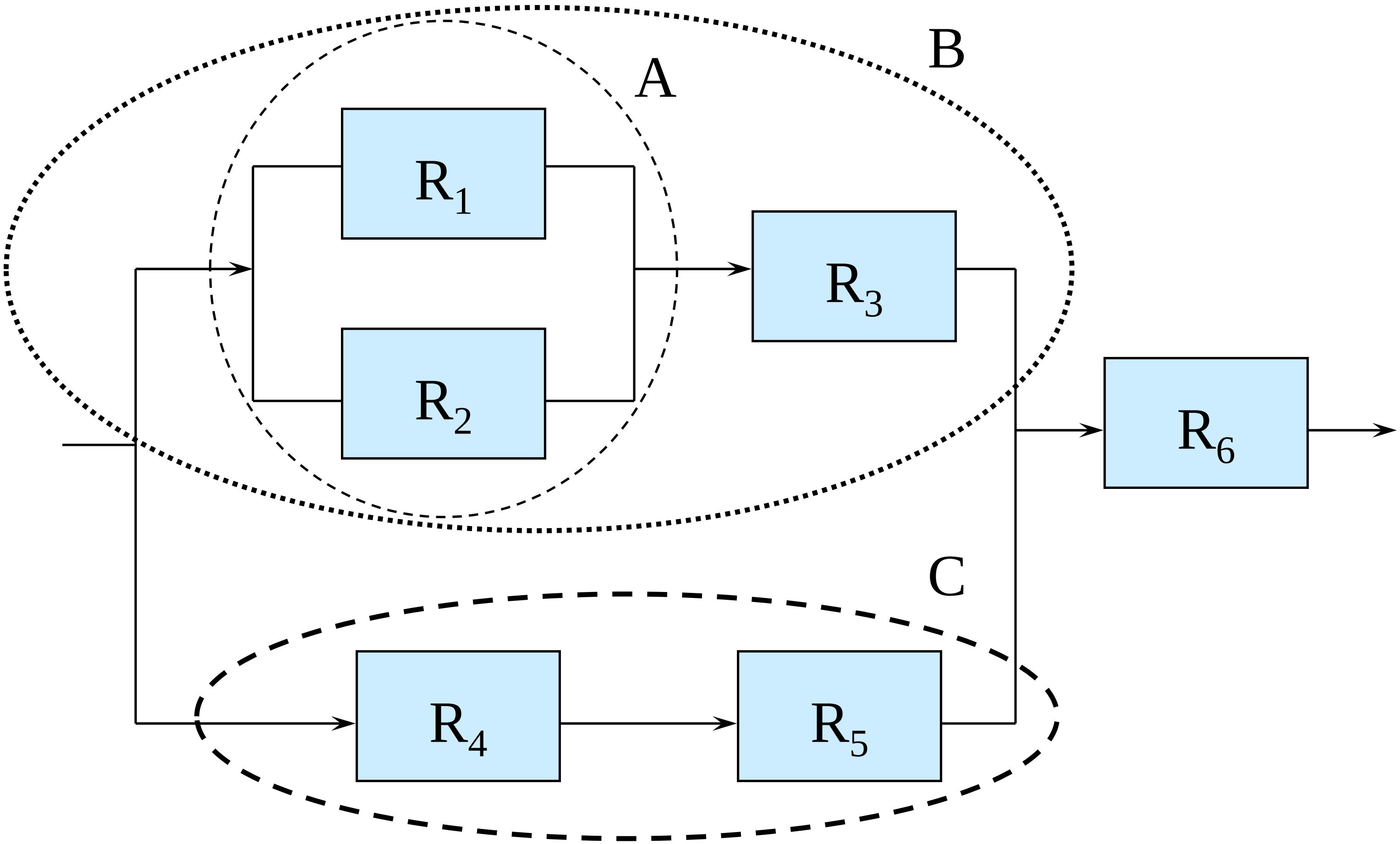
- Reliability of parallel system is the probability that at least one component does NOT fail!

$$R_s(t) = 1 - [(1 - R_1)(1 - R_2) \ldots (1 - R_n)]$$

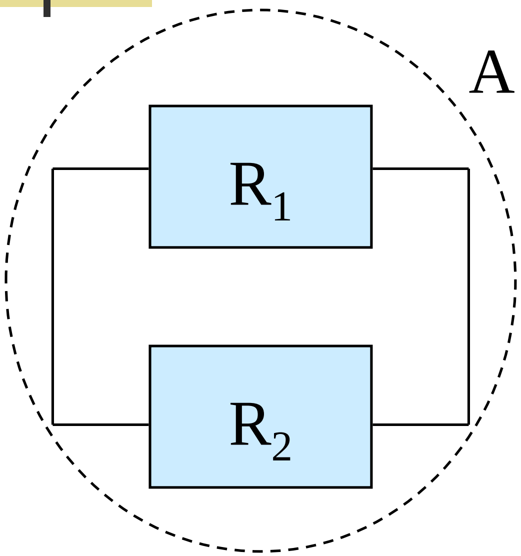
- For Serial System:

$$R_s\ (t) >= \max\ \{R_1, R_2, \ldots, R_n\}$$

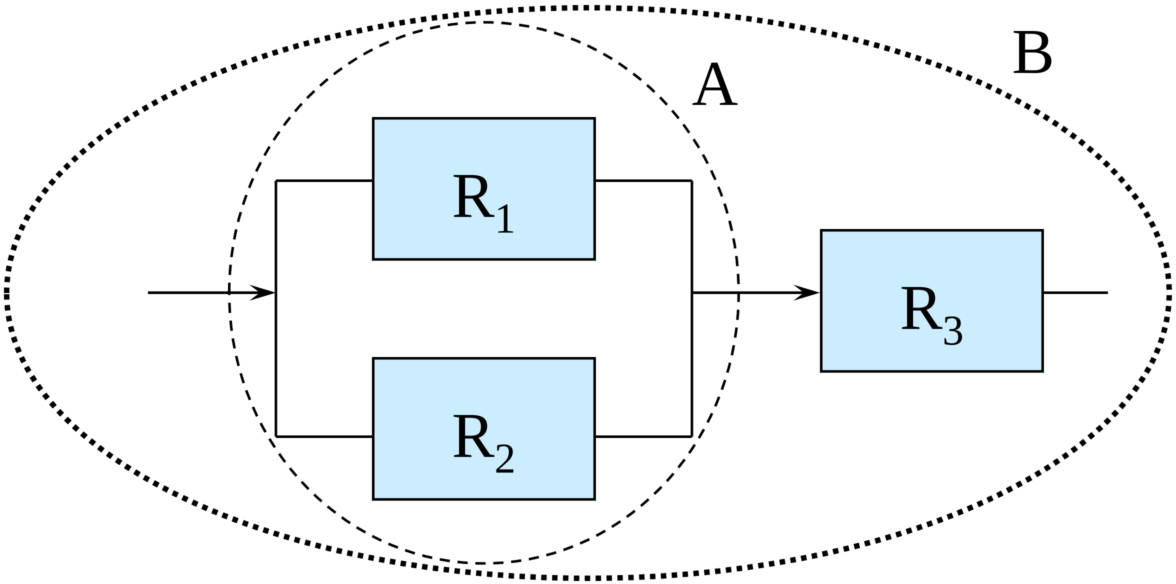
Combined Series - Parallel Systems



Combined Series - Parallel Systems

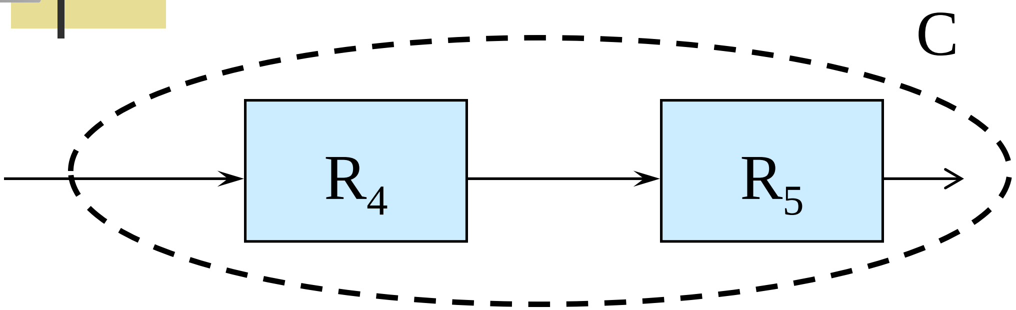


$$R_A = [\,1 - (1 - R_1)\,(1 - R_2\,)]$$

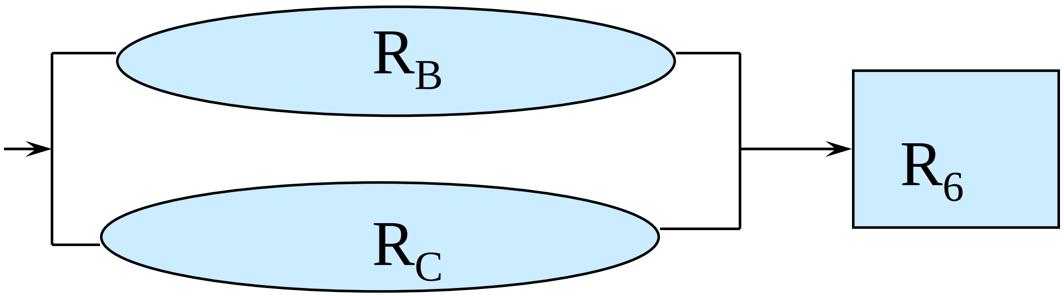


$$R_B = R_A\, R_3$$

Combined Series - Parallel Systems

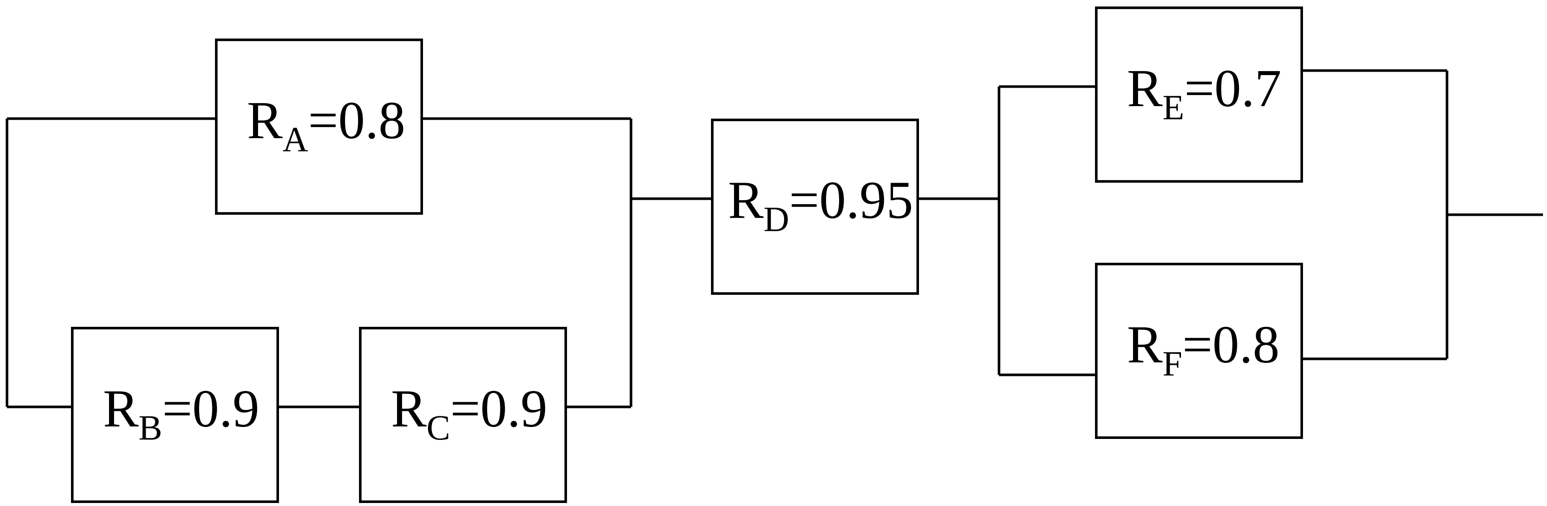


$$R_C = R_4 \, R_5$$

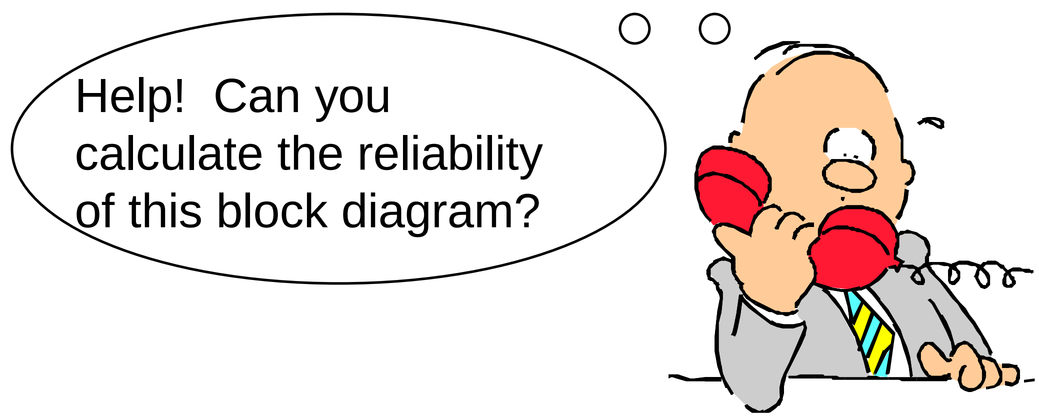


$$R_s = [1 - (1 - R_B)\,(1 - R_c)\,]\,R_6$$

Exercise

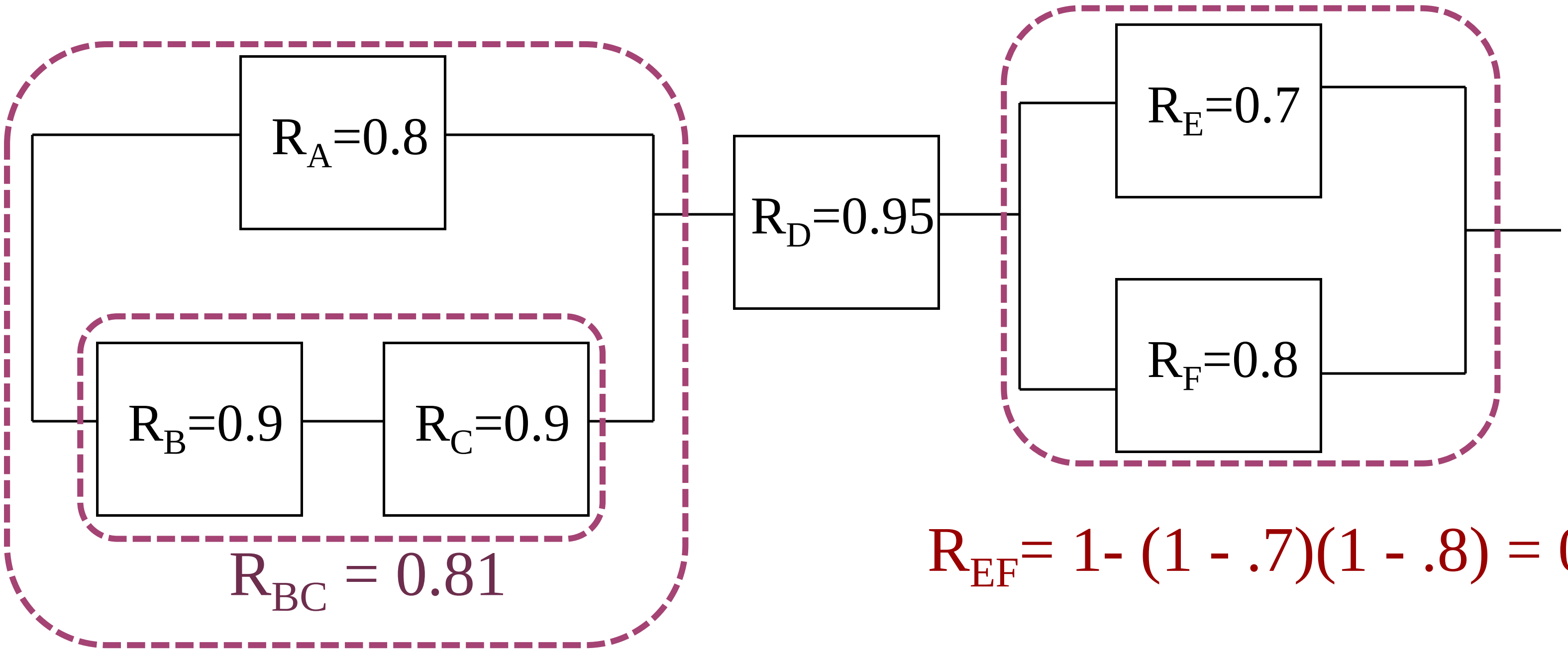


$R_A=0.8$
$R_B=0.9$
$R_C=0.9$
$R_D=0.95$
$R_E=0.7$
$R_F=0.8$



Help! Can you calculate the reliability of this block diagram?

Exercise



R_{EF}= 1- (1 - .7)(1 - .8) = 0.94

R_{ABC} = 1- (1 - .81)(1 - .8) = 0.962

R_s = (0.962) (0.95) (0.94) = 0.859

k-out-of-n Redundancy

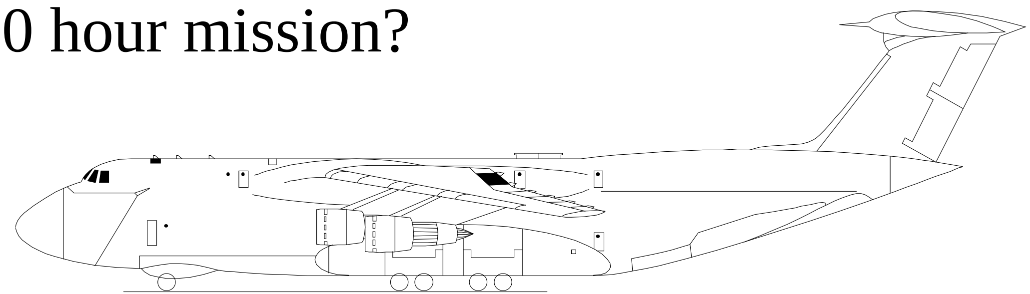
- Let n = the number of redundant, identical and independent components each having a reliability of R
- Let k = the number of components that must operate for the system to operate
- The reliability of the system (from binomial distribution):

$$R_s = \sum_{x=k}^{n} \binom{n}{x} R^x (1 - R)^{n-x},$$

$$where \binom{n}{x} = \frac{n!}{x!(n-x)!}$$

A Very Good Example

Out of the 12 identical AC generators on the C-5 aircraft, at least 9 of them most be operating in order for the aircraft to complete its mission. Failures are known to follow an exponential distribution with a failure rate of 0.01 failure per hour. What is the reliability of the generator system over a 10 hour mission?



For exponential distribution:

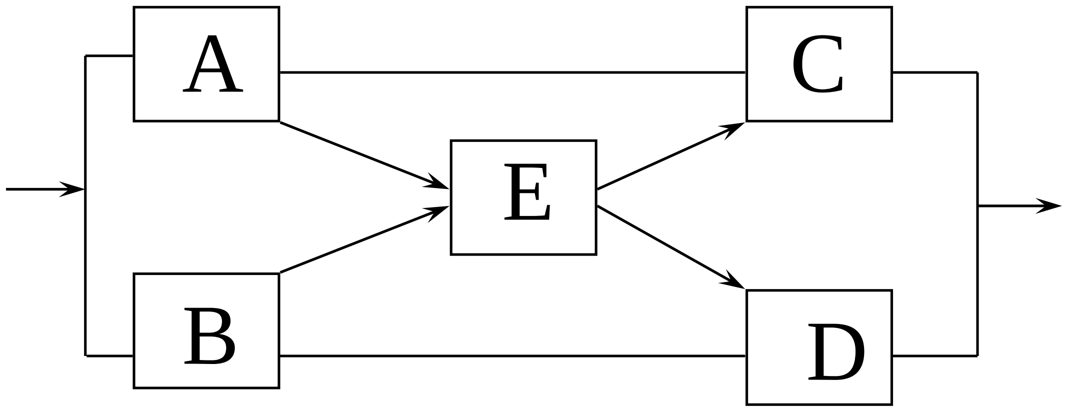
$$R(t) = \exp(-\lambda t)$$

$$R(10) = \exp(-0.01 * 10) = 0.9048$$

$$R_s = \sum_{x=k}^{n} \binom{n}{x} R^x (1 - R)^{n-x} = \sum_{x=9}^{12} \binom{12}{x} .905^x (1 - .905)^{12-x} = 0.978$$

Reliability of Complex Configurations

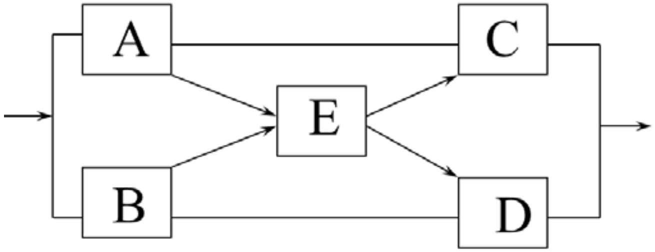
- Network:



- Two approaches:
  1. Decomposition Approach
  2. Enumeration Method

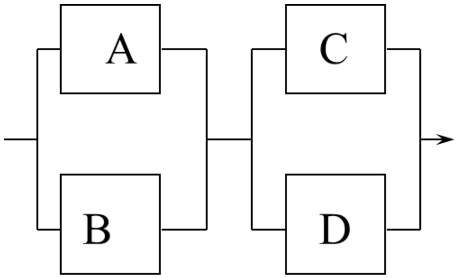
Decomposition Approach

- Decompose the network to combined (parallel and serial) system



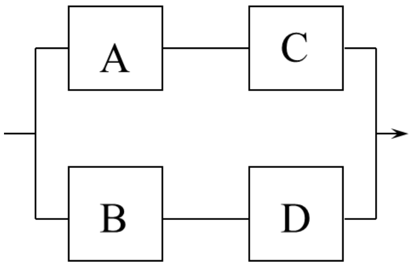
Case I: If E does not fail

Probability = R_E



Case II: If E fails

Probability = $1 - R_E$

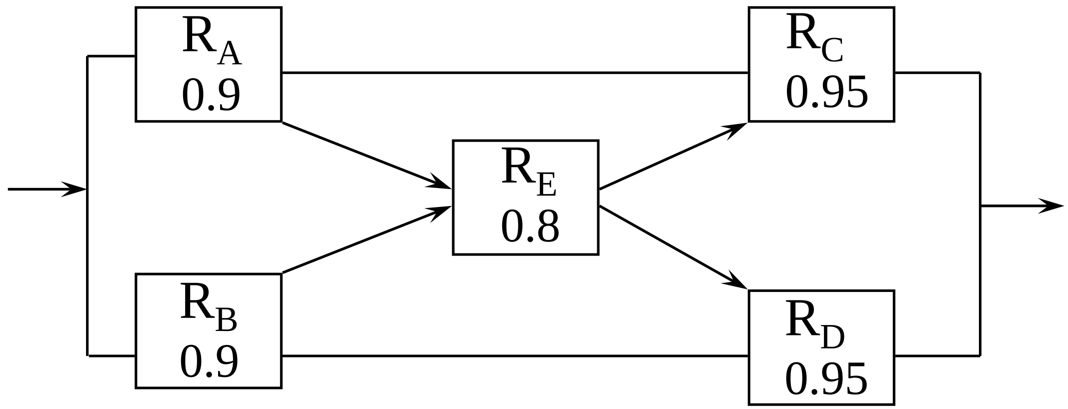


System Reliability = Σ (R of each Case x Case Probability)

$Rs = R_{CaseI} \times R_E + R_{case\ II} \times (1-R_E)$

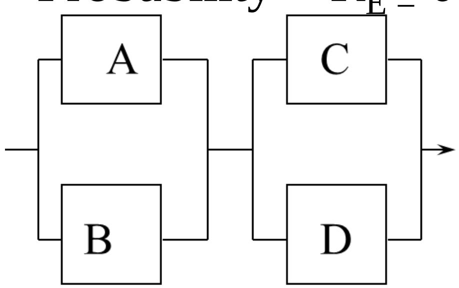
Decomposition Approach

- Example: Calculate the reliability of this system:



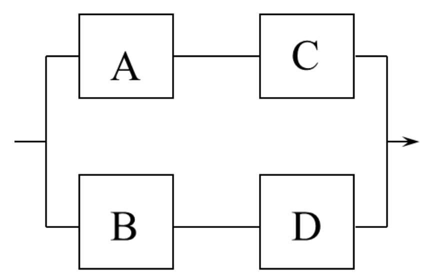
Case I: If E does not fail

Probability = R_E = 0.8



Case II: If E fails

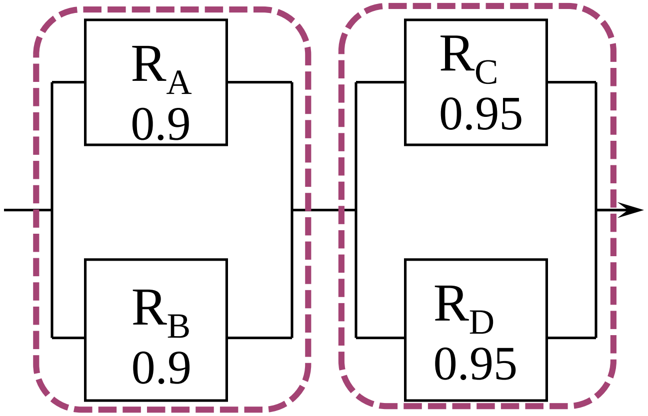
Probability = $1 - R_E = 0.2$



Decomposition Approach

- Case I:

Probability = 0.8



$$R_{CaseI} = [1- (1-R_A)(1-R_B)] \times [1- (1-R_C)(1-R_D)]$$

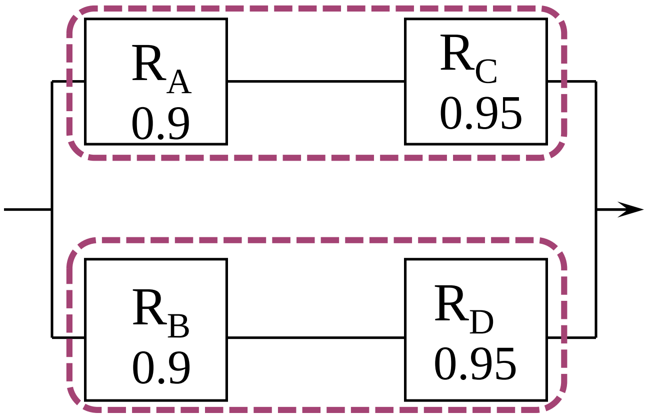
$$R_{CaseI} = [1- (1-0.9)(1-0.9)] \times [1- (1-0.95)(1-0.95)]$$

$$R_{CaseI} = 0.9875$$

Decomposition Approach

- Case II:

Probability = 0.2



$$R_{CaseII} = 1 - (1 - R_A R_C)(1 - R_B R_D)$$

$$R_{CaseII} = 1 - (1 - 0.9 \times 0.95)(1 - 0.9 \times 0.95)$$

$$R_{CaseII} = 0.979$$

- System Reliability:

System Reliability = Σ (R of each Case x Case Probability)

$$R_S = 0.8 \times 0.9875 + 0.2 \times 0.979$$

$$R_S = 0.9858$$

Enumeration Method

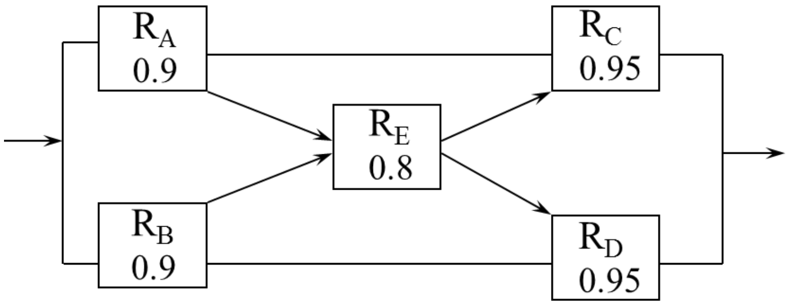
- Identify all possible combinations of success (S) or failure (F) of each component and the resulting success or failure of the system

- Calculate the probability of intersection of each possible combination of component successes or failures that lead to system success.

- System reliability is the sum of the success probabilities

Enumeration Method

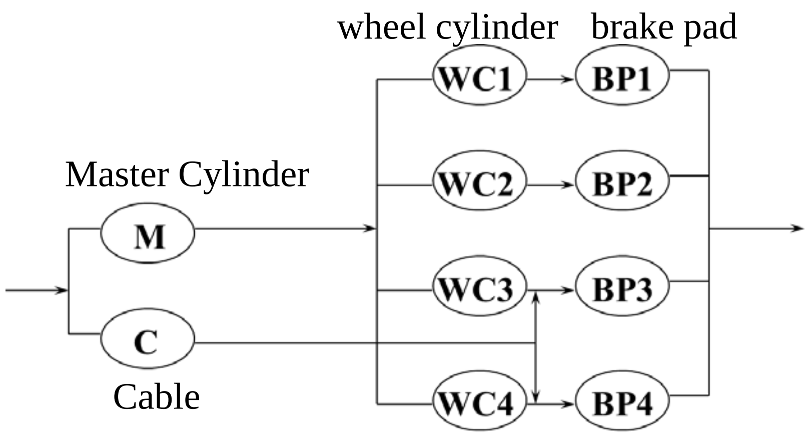
S = success, F = failure



A	B	C	D	E	System	Probability
S	S	S	S	S	S	0.584820
F	S	S	S	S	S	0.064980
S	F	S	S	S	S	0.064980
S	S	F	S	S	S	0.030780
S	S	S	F	S	S	0.030780
S	S	S	S	F	S	0.146205
F	F	S	S	S	F	
S	F	F	S	S	S	0.003420
S	S	F	F	S	F	
⋮	⋮	⋮	⋮	⋮	⋮	
F	F	F	S	F	F	
F	F	F	F	F	F	
				Total		0.985800

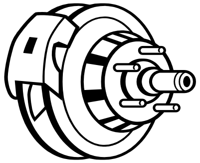
# of combinations: $5^{\wedge 2} = 32$

Case Study: Automotive Braking System



- An automobile braking system consists of a fluid braking subsystem and a mechanical braking subsystem (parking brake)
- Both subsystems must fail in order for the system to fail

- The fluid braking subsystem will fail if the Master cylinder fails (M) (which includes the hydraulic lines) or all four wheel braking units fail
- A wheel braking unit will fail if either the wheel cylinder fails (WC1, WC2, WC3, WC4) or the brake pad assembly fails (BP1, BP2, BP3 , BP4)
- The mechanical braking system will fail if the cable system fails (event C) or both rear brake pad assemblies fail (events BP3, BP4)

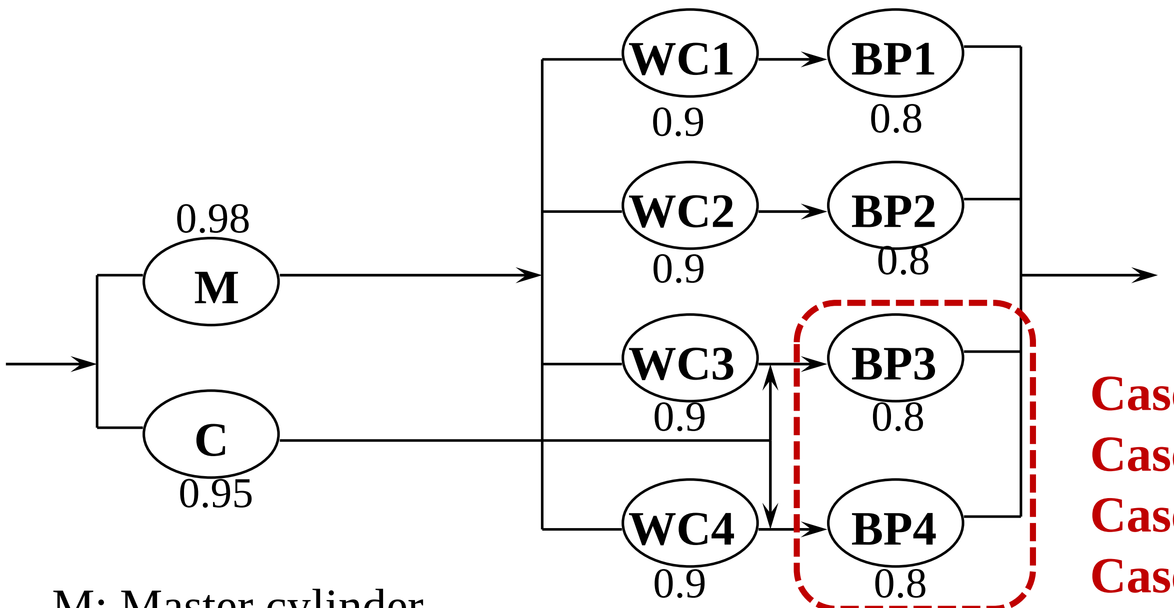


Case Study: Automotive Braking System

The reliability of driving 5k miles without brake maintenance:

$R_M = 0.98$, $R_C = 0.95$, $R_{WC} = 0.9$, $R_{BP} = 0.8$

What is the reliability of the brake system?



Case I = BP3 fails & BP4 works
Case II = BP3 works & BP4 fails
Case III = both BP3 & BP4 fail
Case IV = both BP3 & BP4 work

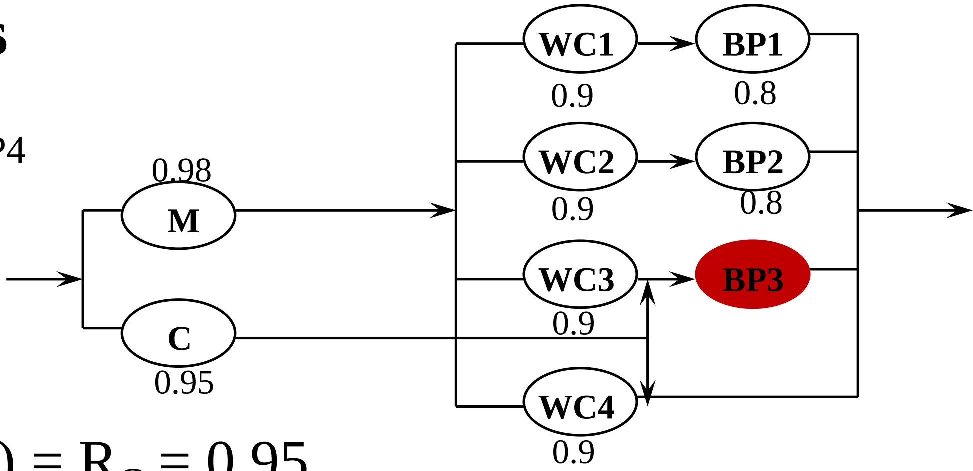
M: Master cylinder
C: Cable
WC: Wheel Cylinder
BP: Brake Pad

Case Study: Automotive Braking System

Case I = BP3 fails & BP4 works

Probability (P_I) = $(1 – R_{BP3})\, R_{BP4}$

$P_I = (1 – 0.8)\ 0.8 = 0.16$



Reliability of parking brake (R_{park}) = R_C = 0.95

Reliability of hydraulic brake (R_H):

$$R_H = R_M\,[\,1- (1 - R_{WC}\,R_{BP})^2\,(1 - R_{WC})]$$
$$R_H = 0.98\,[\,1- (1 – 0.9 \times 0.8)^2\,(1 – 0.9)]$$
$$R_H = 0.972$$

These two subsystems operate in parallel, therefore:

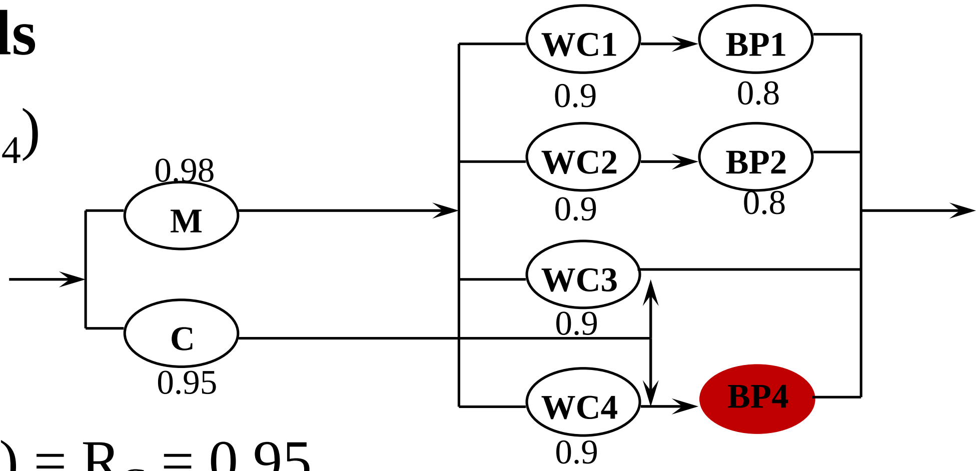
$$R_I = 1 – (1 - R_H)\,(1 – R_{park})$$
$$R_I = 1 – (1 – 0.972)\,(1 – 0.95) = 0.9986$$

Case Study: Automotive Braking System

Case II = BP3 works & BP4 fails

Probability (P_{II}) = $R_{BP3}(1 - R_{BP4})$

$P_{II} = 0.8(1 – 0.8) = 0.16$



Reliability of parking brake (R_{park}) = $R_C = 0.95$

Reliability of hydraulic brake (R_H):

$$R_H = R_M [1 - (1 - R_{WC} R_{BP})^2 (1 - R_{WC})]$$
$$R_H = 0.98 [1 - (1 – 0.9 \times 0.8)^2 (1 – 0.9)]$$
$$R_H = 0.972$$

These two subsystems operate in parallel, therefore:

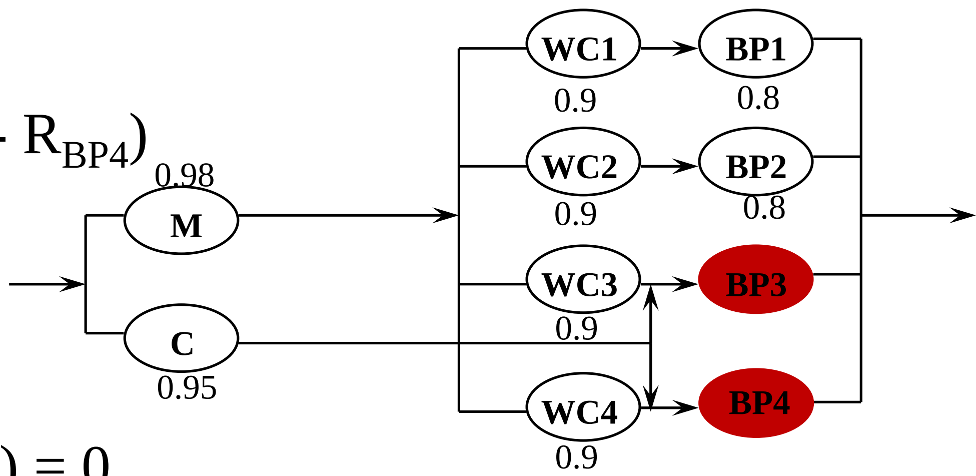
$$R_{II} = 1 – (1 - R_H)(1 – R_{park})$$
$$R_{II} = 1 – (1 – 0.972)(1 – 0.95) = 0.9986$$

Case Study: Automotive Braking System

Case III = both BP3 & BP4 fail

Probability (P_{III}) = $(1 - R_{BP3})(1 - R_{BP4})$

$P_{III} = (1 - 0.8)(1 - 0.8) = 0.04$



Reliability of parking brake (R_{park}) = 0

Reliability of hydraulic brake (R_H):

$$R_H = R_M [\, 1 - (1 - R_{WC} R_{BP})^2 \,]$$

$$R_H = 0.98 [\, 1 - (1 - 0.9 \times 0.8)^2]$$

$$R_H = 0.903$$

The parking brake is not operating, therefore:

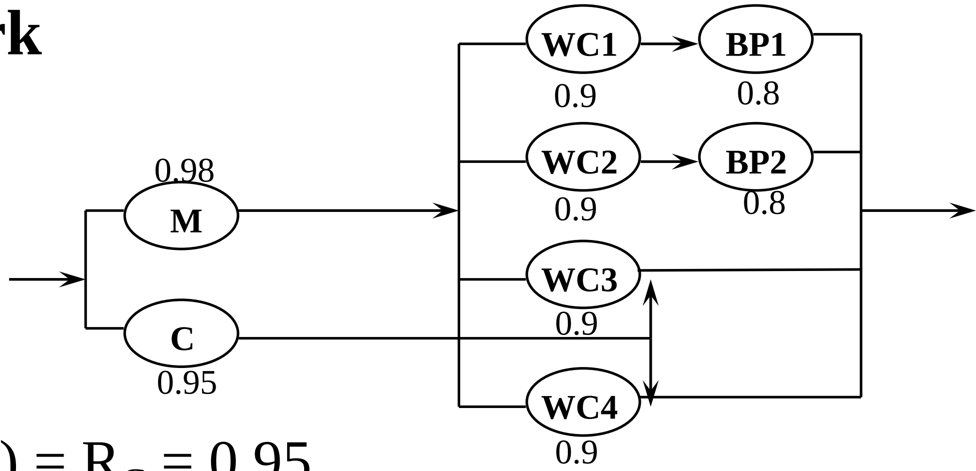
$$R_{III} = R_H = 0.903$$

Case Study: Automotive Braking System

Case IV = both BP3 & BP4 work

Probability (P_{IV}) = R_{BP3} x R_{BP4}

P_{IV} = 0.8 x 0.8 = 0.64



Reliability of parking brake (R_{park}) = R_C = 0.95

Reliability of hydraulic brake (R_H):

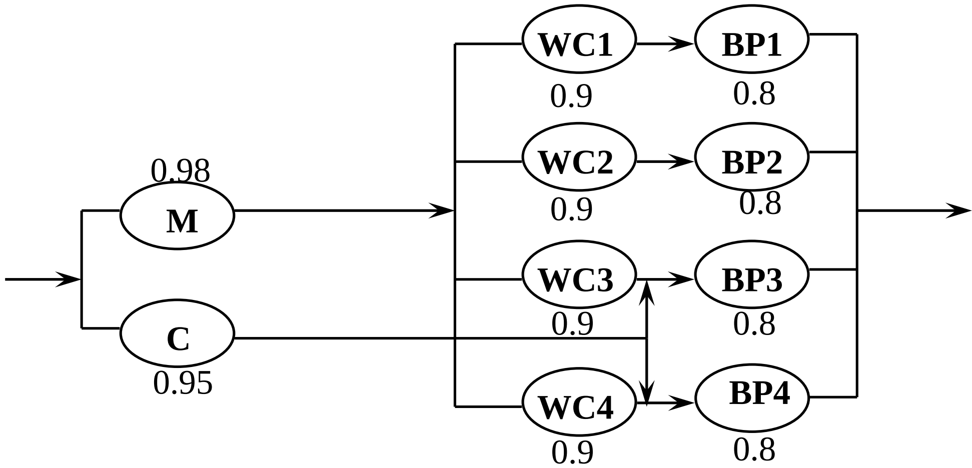
$$R_H = R_M [1- (1 - R_{WC} R_{BP})^2 (1 - R_{WC})^2]$$
$$R_H = 0.98 [1- (1 – 0.9 \text{ x } 0.8)^2 (1 – 0.9)^2]$$
$$R_H = 0.979$$

These two subsystems operate in parallel, therefore:

$$R_{II} = 1 – (1 - R_H) (1 – R_{park})$$
$$R_{II} = 1 – (1 – 0.979) (1 – 0.95) = 0.999$$

Case Study: Automotive Braking System

Reliability of the System:



System Reliability = Σ (R of each Case x Case Probability)

$$R_S = R_I P_I + R_{II} P_{II} + R_{III} P_{III} + R_{IV} P_{IV}$$

$$R_S = (0.16 \times 0.9986 \times 2 + 0.04 \times 0.903 + 0.64 \times 0.999$$

$$R_S = 0.995$$

Summary

- Failure Distributions
  - Exponential Distribution
  - Weibull Distribution
- Series Configuration
- Parallel Configuration
- Combined Series-Parallel Configuration
- K out-of-n Redundancy
- Complex Configurations – linked networks

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Assessment of System Reliability

Sa'd Hamasha, Ph.D.

Assistant Professor
Industrial and Systems Engineering

Phone: 607-372-7431
Email: hamasha@auburn.edu

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