Addressing Finitely Repeated Problems in Engineering Decision Making Under Uncertainty

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Engineering Design Background

- Decision-based design views engineering design as a series of decision problems.
- Subfield of value-based engineering uses value models instead of performance attributes as objectives.

Value-Based Engineering

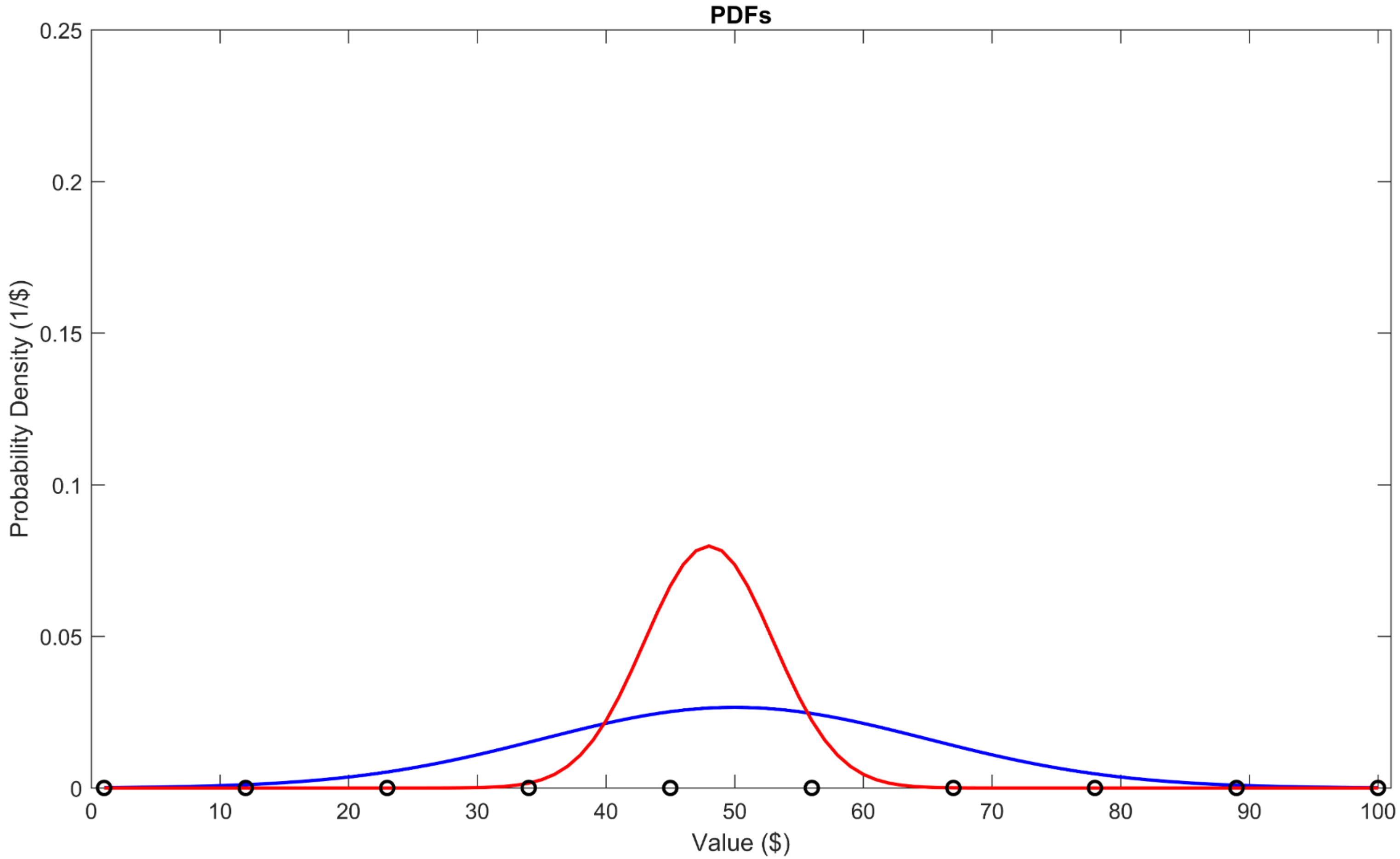
- Fundamentals of Decision Making for Engineering Design and Systems Engineering [2].
- Proposes value-modeling and utility theory as the basis for engineering decision making, including design
- Heavy emphasis on Von Neumann and Morgenstern [3]

Value-Based Engineering

- Hazelrigg simultaneously proposes ordinal preference functions and expected utility.

- These are not compatible.

An Example



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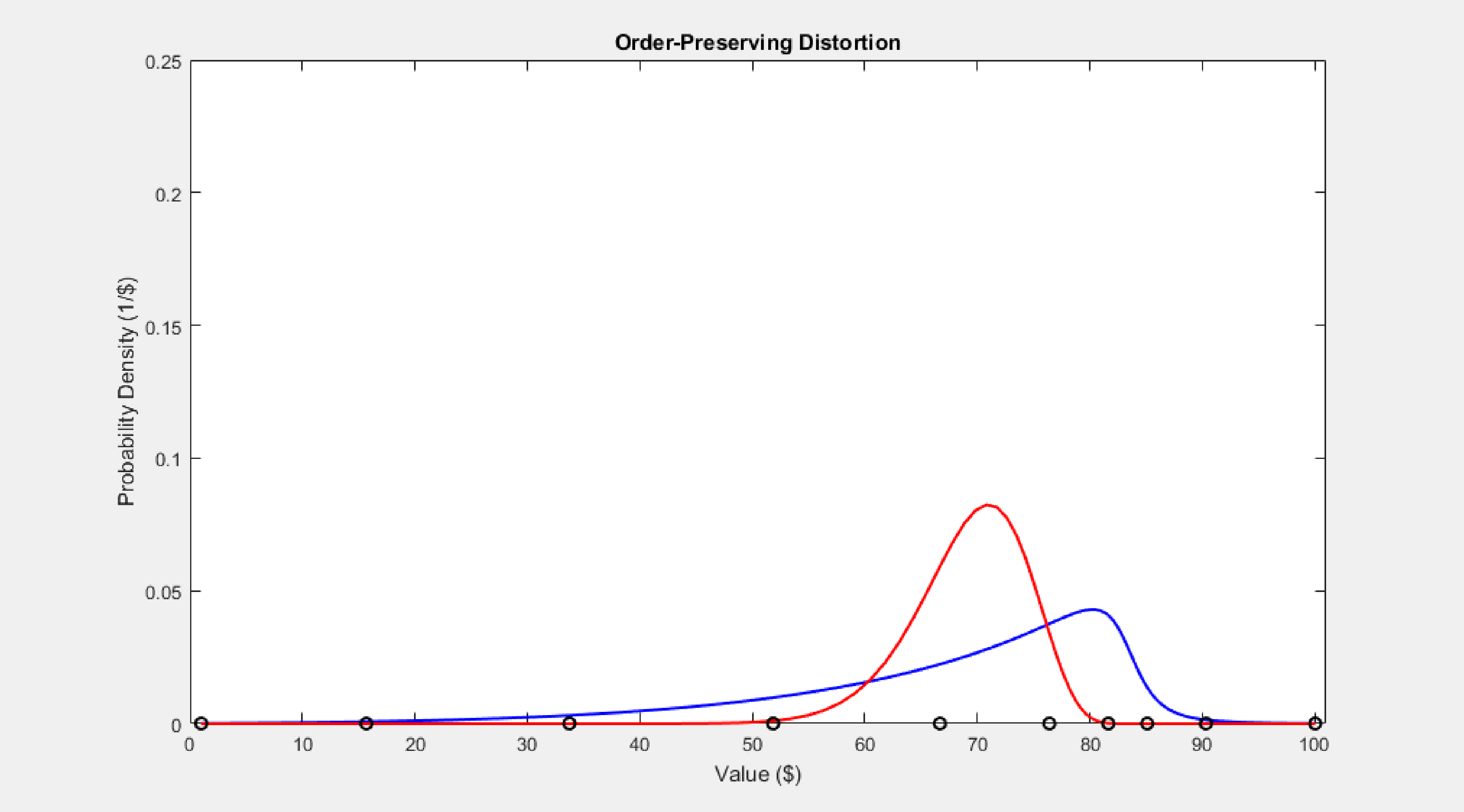
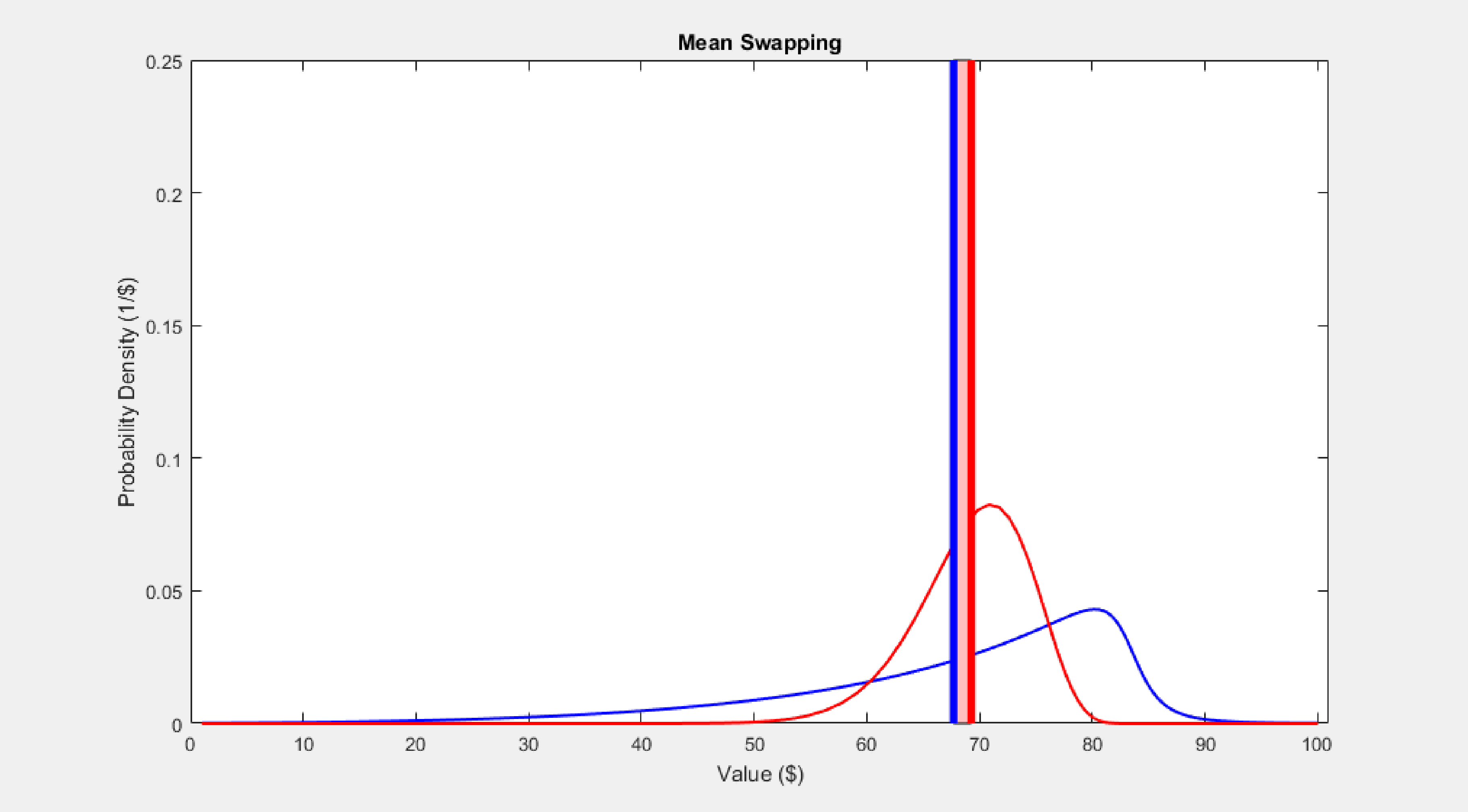
An Example



Order-Preserving Distortion
0.25
0.2
0.15
0.1
0.05
0
Probability Density (1/$)
0
10
20
30
40
50
60
70
80
90
100
Value ($)

An Example



Mean Swapping
Probability Density (1/$)
0.25
0.2
0.15
0.1
0.05
0
0
10
20
30
40
50
60
70
80
90
100
Value ($)

The St. Petersburg Paradox

- Described by Daniel Bernoulli [1].

- Flip a fair coin until it comes up heads.
- n = # of consecutive tails
- Win $\$2^n$

- Question: How much would you pay to play?

The St. Petersburg Paradox

- Expected Value:

$$E[V] = \sum_{n=0}^{\infty} \frac{1}{2^{n+1}} * \$2^{n} = \infty$$

- Infinite expected value
- No one will pay an infinite entrance fee.

The St. Petersburg Paradox

- Bernoulli's solution uses logarithmic utility.
- Based on players current wealth.
- Only has infinite expected utility if player is infinitely wealthy.

St. Petersburg Paradox: Relevance

- Value model validity standards can preclude the use of expected value.
- Engineering design problems present more issues, though.
- Highly unlikely, highly impactful outcome
  - This is the basis of the St. Petersburg Paradox

St. Petersburg Paradox: Simulation

- We will use simulation.
  - Analytical solutions for money quantities of interest is possible here.
  - Not possible/feasible for many real-world engineering problems.

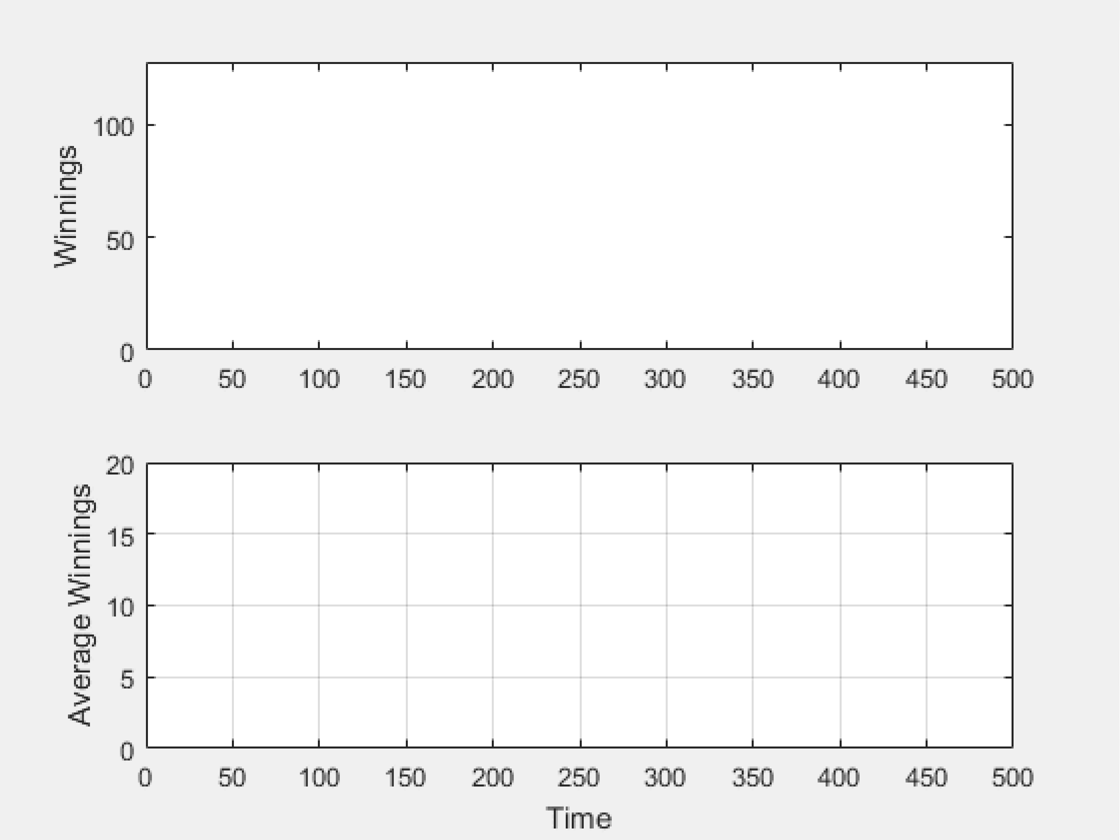
- St. Petersburg game:
  - Geometric outcome distribution
  - Exponential value function
  - Optional: exponential utility function

St. Petersburg Paradox: Simulation

- We will play many times.
  - Number of repetitions from 1 to 2,000
  - Winnings per play will be recorded.

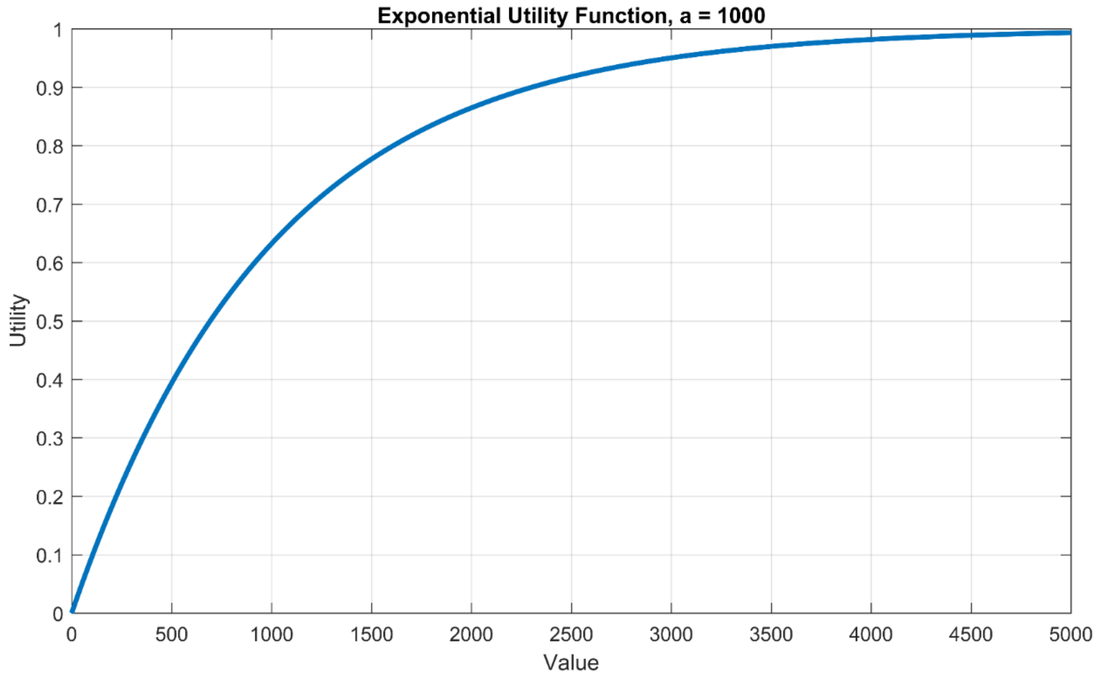
- Entrance fee based on exponential utility function will be noted.

St. Petersburg Paradox: Simulation



Exponential Utility

- $utility = 1 - e^{-\frac{value}{a}}$
- a is a wealth parameter
- We'll use a = 100, 1000, and 10,000



	a = 100	a = 1,000	a = 10,000
Expected Utility	$3.879 * 10^{-2}$	$5.538 * 10^{-3}$	$7.199 * 10^{-4}$
Certainty Equivalent	\$3.96	\$5.55	\$7.20

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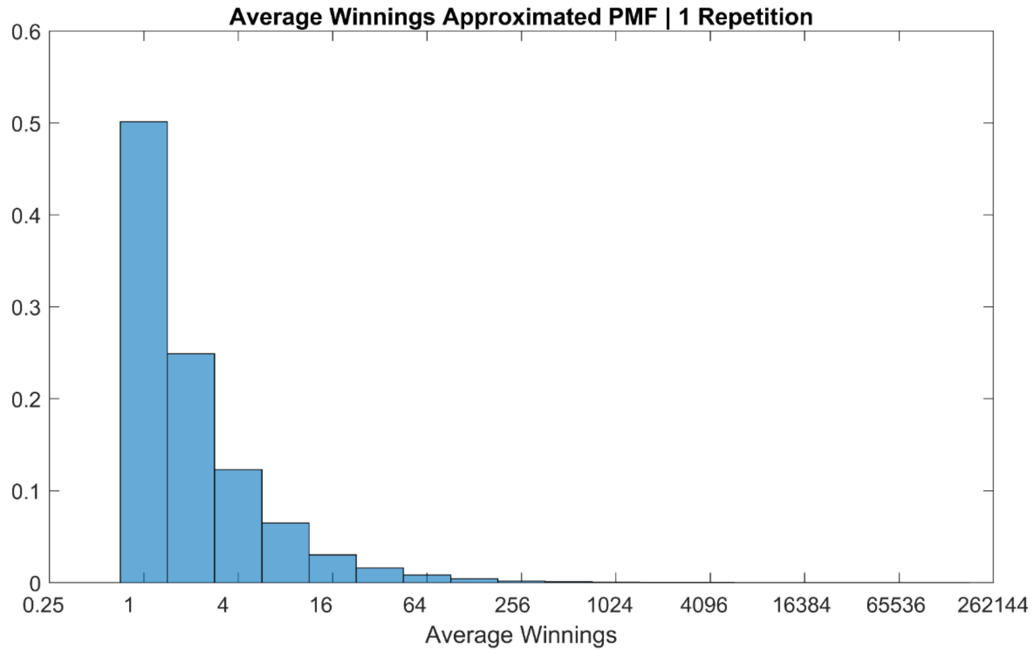
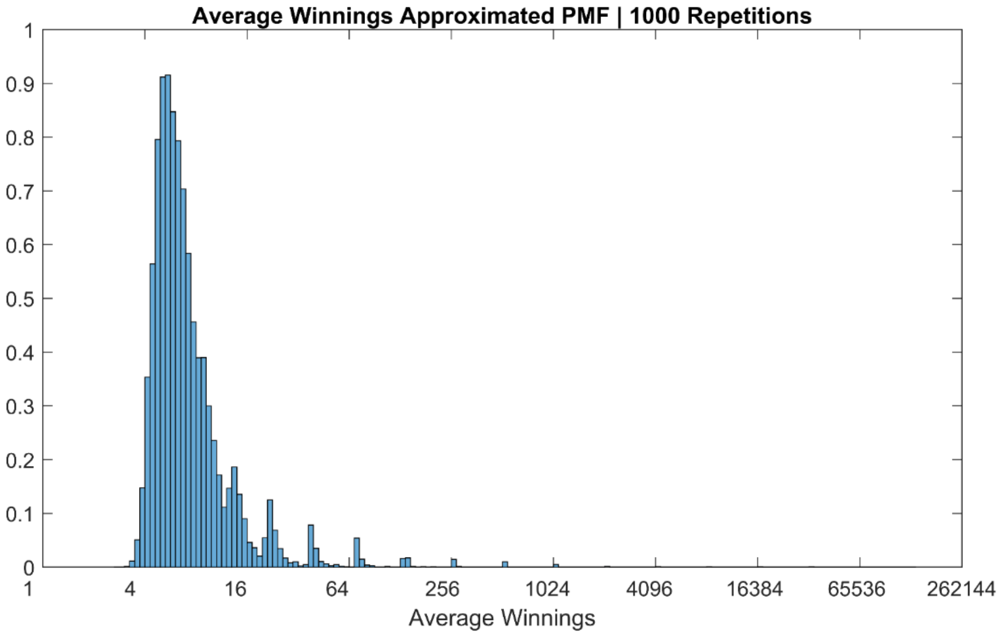
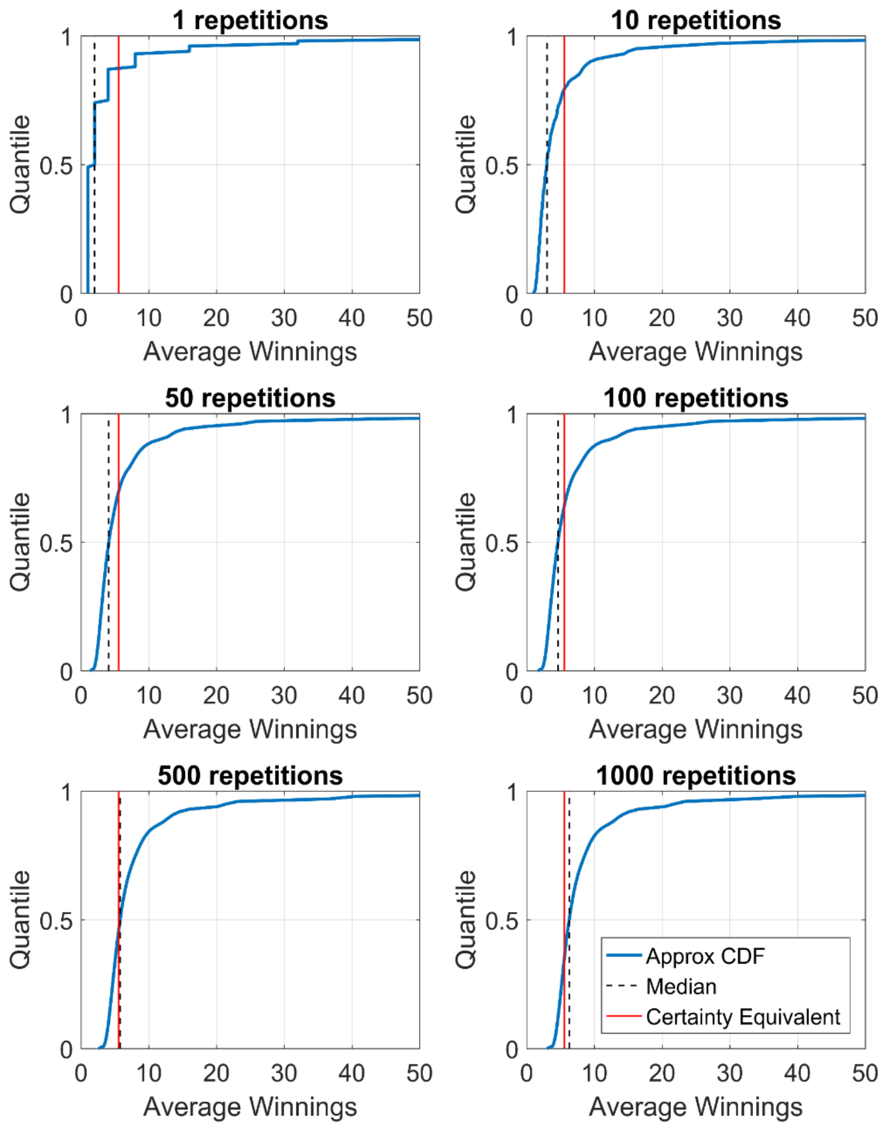
Average Winnings





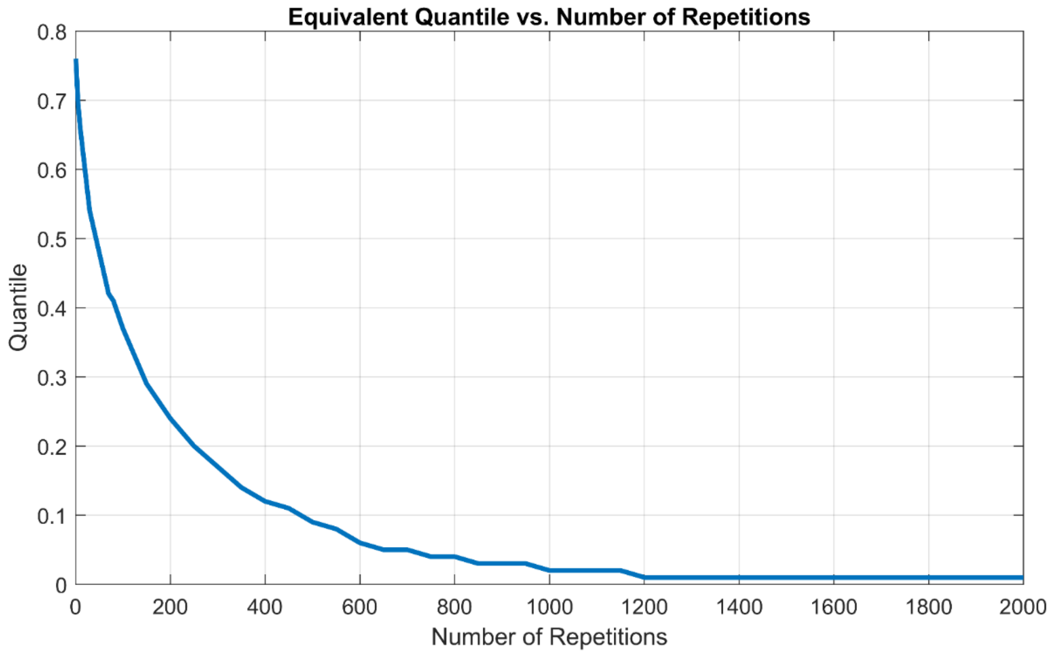
Average Winnings Quantiles

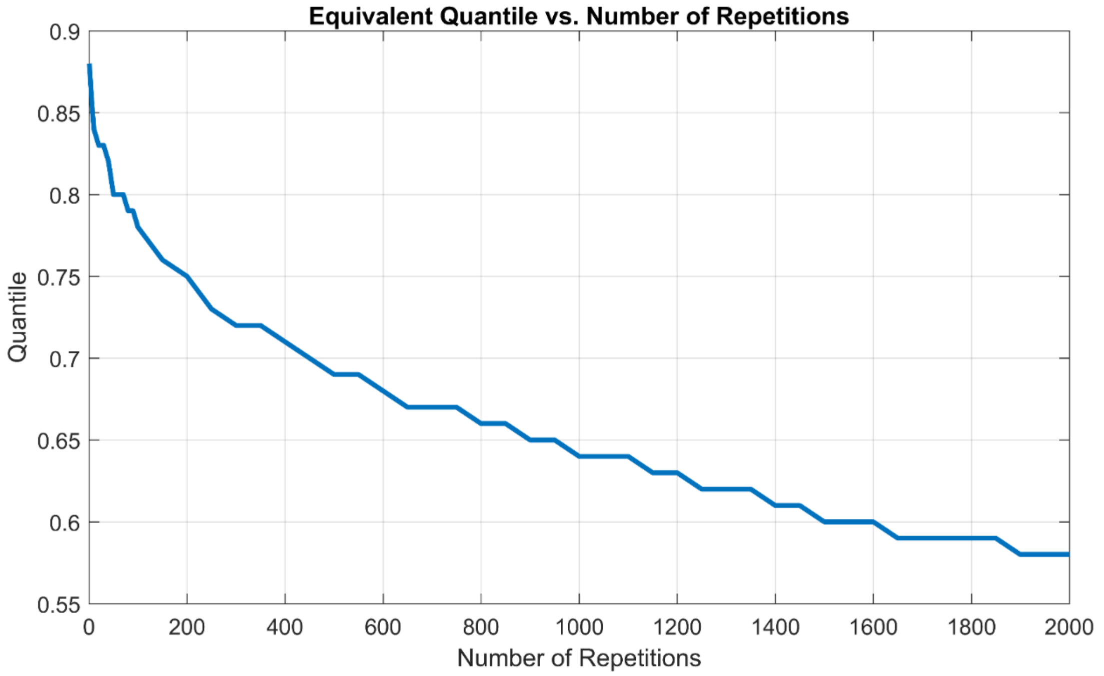


Equivalent Quantiles

- Probability of profit is *very* different for different numbers of repetitions.

- Figures are a = 100 and 10000





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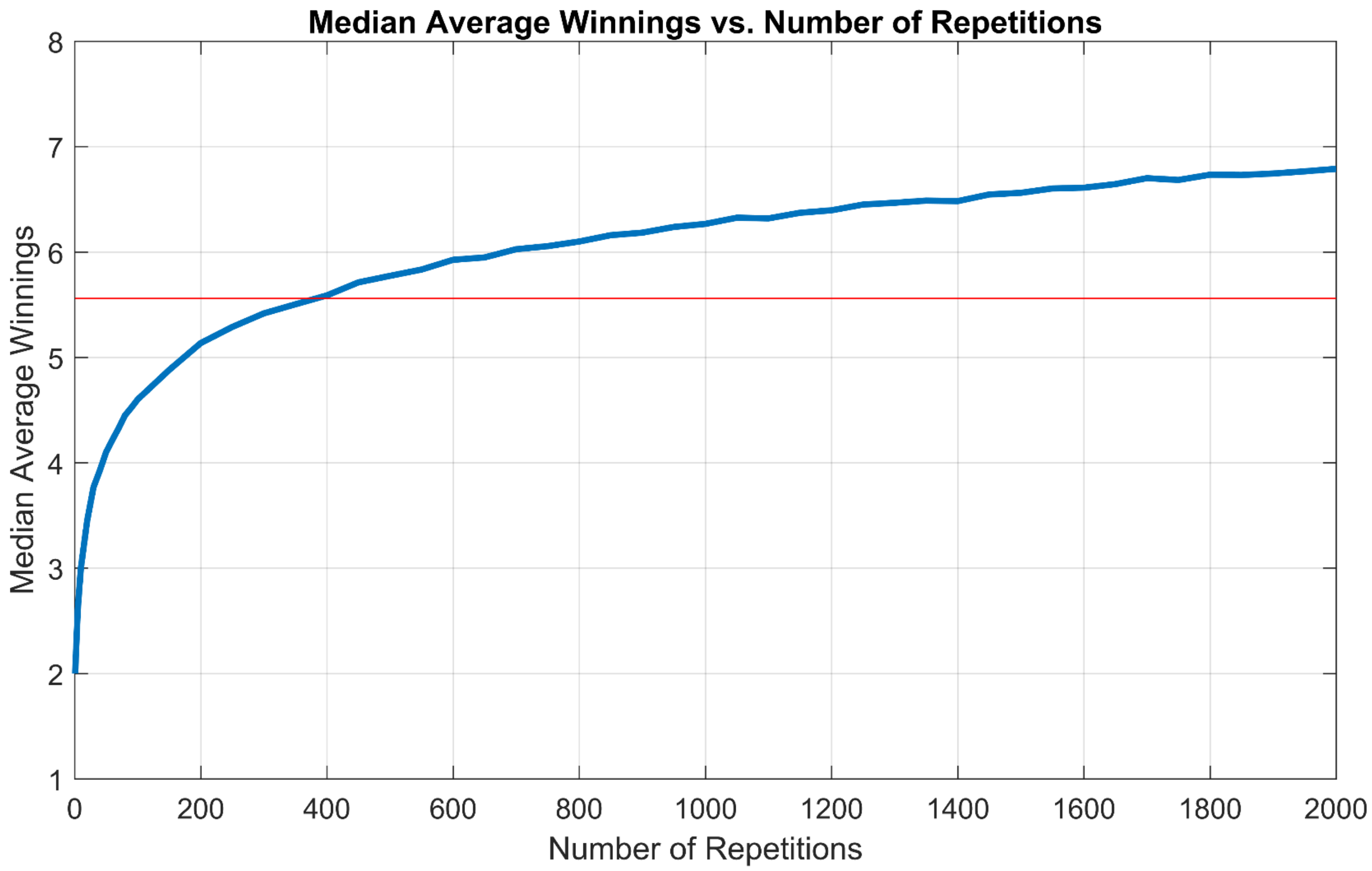
Median Winnings



Discussion

- St. Petersburg games look wildly different depending on how long they are played.
- Number of repetitions is important.
- Real world engineering problems often have few repetitions.

- St. Petersbug games may serve as a test-bed for developments in engineering design theory.

References

- [1] D. Bernoulli, “Exposition of a new theory on the measurement of risk,” *Econom. J. Econom. Soc.*, pp. 23–36, 1954.
- [2] G. A. Hazelrigg, *Fundamentals of decision making for engineering design and systems engineering*. 2012.
- [3] J. Von Neumann and O. Morgenstern, *Theory of games and economic behavior*. Princeton university press, 1953.
- [4] S. S. Stevens, “On the Theory of Scales of Measurement,” Sci. New Ser., vol. 103, no. 2684, pp. 677–680, 1946.

Questions?