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A-P-T Research, Inc.

RAM XII Workshop Tutorial
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November 13, 2019

RELIABILITY ENGINEERING

MISSION SUCCESS STARTS WITH RELIABILITY

SAFETY ENGINEERING
SEAC
& ANALYSIS CENTER

Safety Engineering and Analysis Center
A Division of A-P-T Research, Inc.
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Introduction

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- This tutorial is a brief summary of a three-day reliability engineering course offered by A-P-T Research, Inc.
- The course is intended to provide a better understanding of reliability engineering as a discipline with focus on the reliability analysis tools and techniques and their application in technical assessments and special studies.
- The material in the course is based on over 30 years of extensive industry and Government experience in reliability engineering and risk assessment.
- For offerings, contact: Megan Stroud, 256-327-3373, training@apt-research.com.
- **Note:** Attendees of the full course will be credited with 2.0 Continuing Education Units (CEU).

Table of Contents

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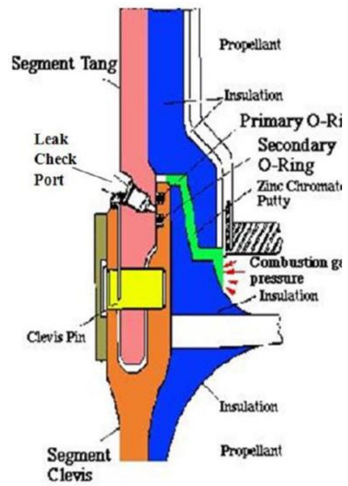
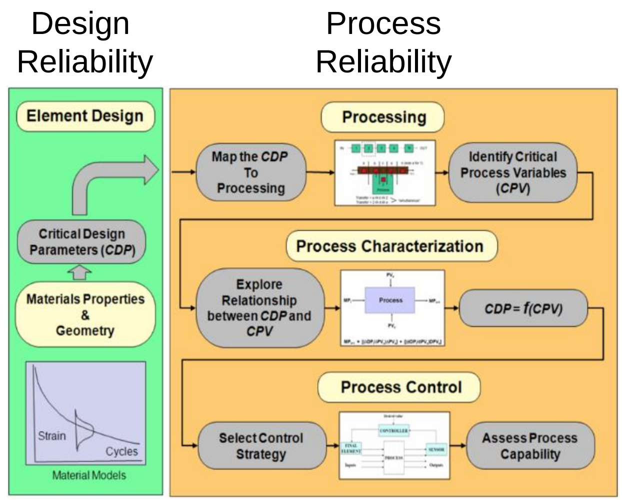
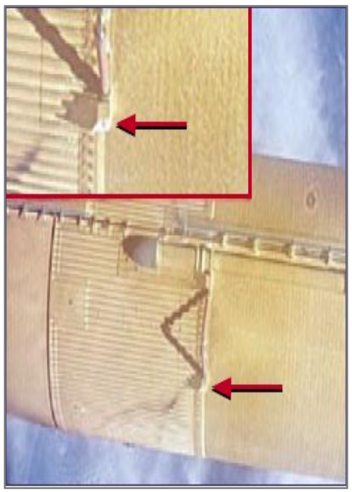
This tutorial covers only sections shown in blue.

SEAC Courses

- System Safety
- Software System Safety
- Explosives Safety
- Launch Safety (inactive)
- Risk Management
- Reliability Engineering
- Probabilistic Risk Assessment (PRA)







RELIABILITY ENGINEERING OVERVIEW

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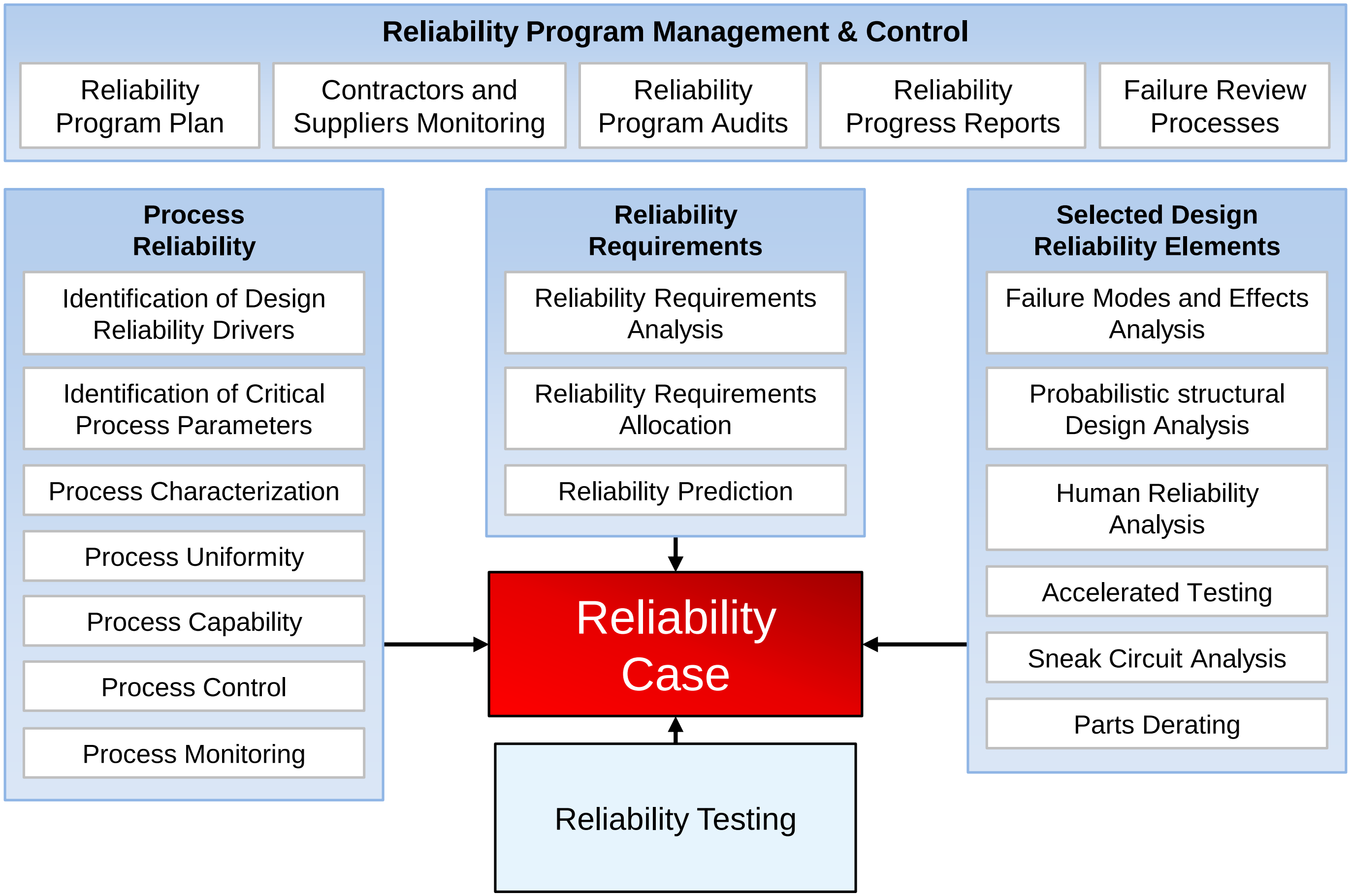
Definitions

- **Reliability Engineering** is the engineering discipline that deals with how to design, produce, ensure, and assure reliable products to meet pre-defined product functional requirements.
- **Reliability Metric** is the probability that a system or component performs its intended functions under specified operating conditions for a specified period of time. Other measures used: Mean Time Between Failures (MTBF), Mean Time to Failure (MTTF), Safety Factors, and Fault Tolerances, etc.
- **Operational Reliability Prediction** is the process of quantitatively estimating the mission reliability for a system, subsystem, or component using both objective and subjective data.
- **Design Reliability Prediction** is the process of predicting the reliability of a given design based on failure physics using statistical techniques and probabilistic engineering models.
- **Process Reliability** is the process of mapping the design drivers in the manufacturing process to identify the process parameters critical to generate the material properties that meet the specs. A high process reliability is achieved by maintaining a uniform, capable, and controlled processes.
- **Reliability Demonstration** is the process of quantitatively demonstrating certain reliability level (i.e., comfort level) using objective data at the level intended for demonstration.

Why Reliability Engineering

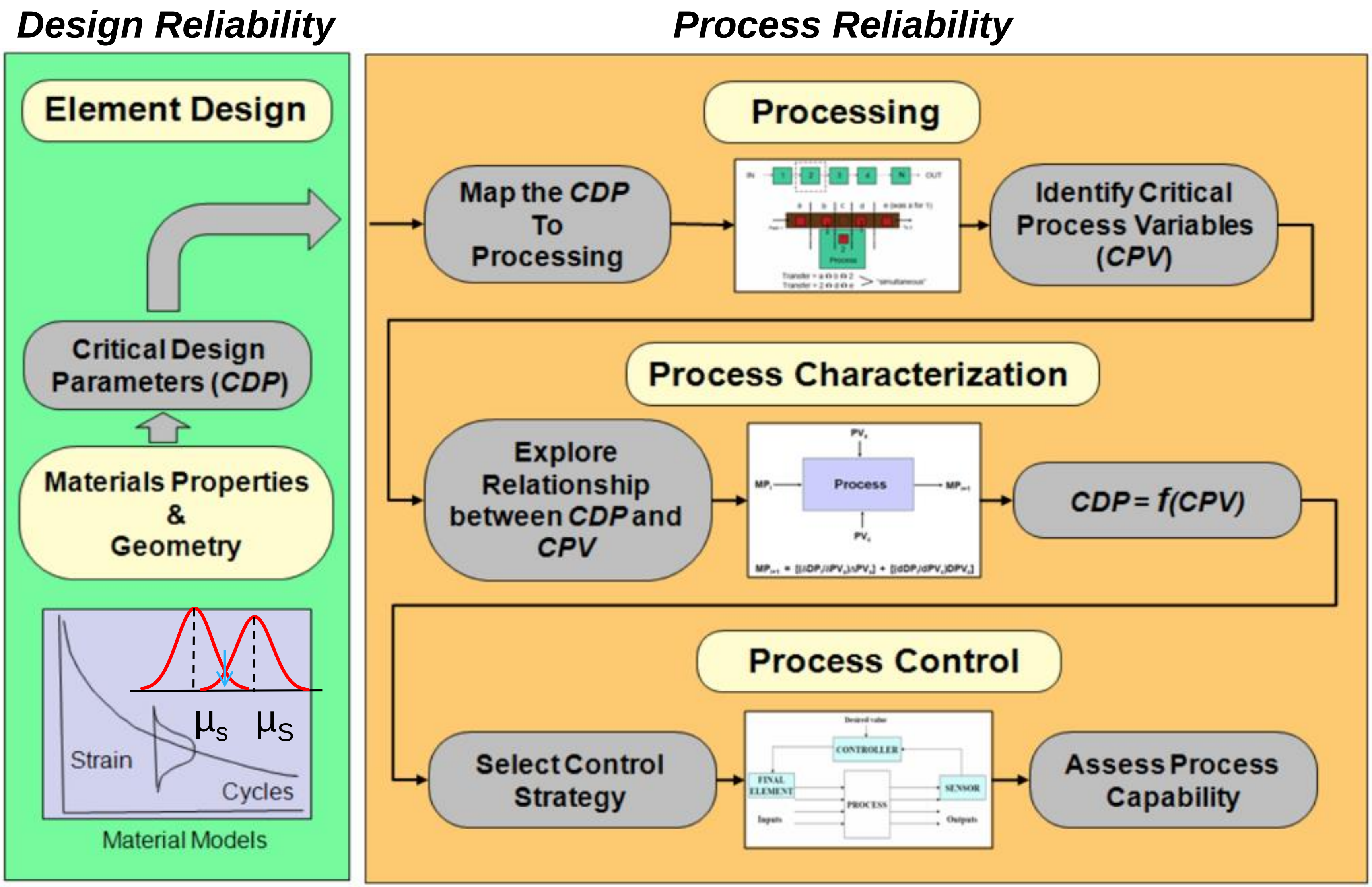
- Reliability engineering is a design-support discipline.
- Reliability engineering is critical for understanding component failure mechanisms and identifying critical design and process drivers.
- Reliability engineering has important interfaces with, and input to, design engineering, maintainability and supportability engineering, test and evaluation, risk assessment, risk management, system safety, sustainment cost, and quality engineering.

Selected Elements of A Reliability Engineering Case



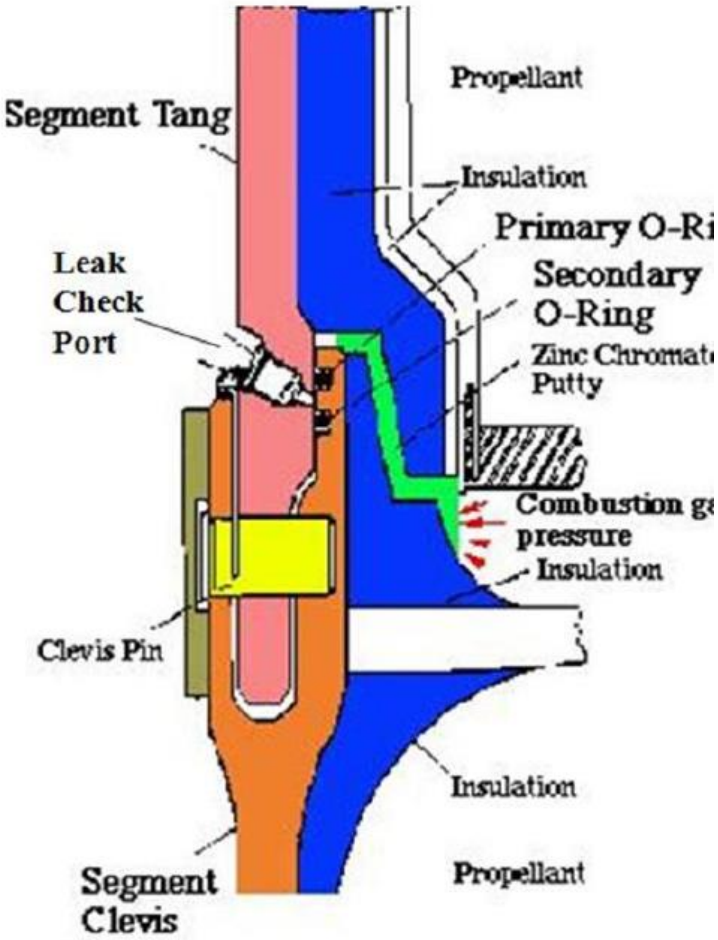
A comprehensive reliability program is essential to address the entire spectrum of engineering and programmatic concerns, from loss of function and loss of life to sustainment and system life cycle costs.

Design it Right and Build it Right



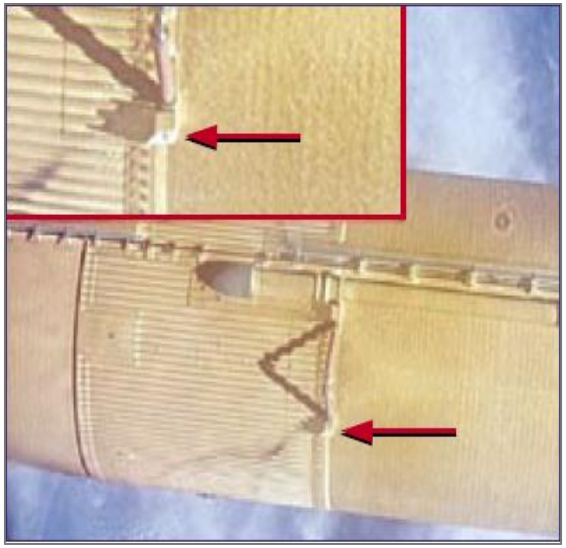
Causes and Contributing Factors

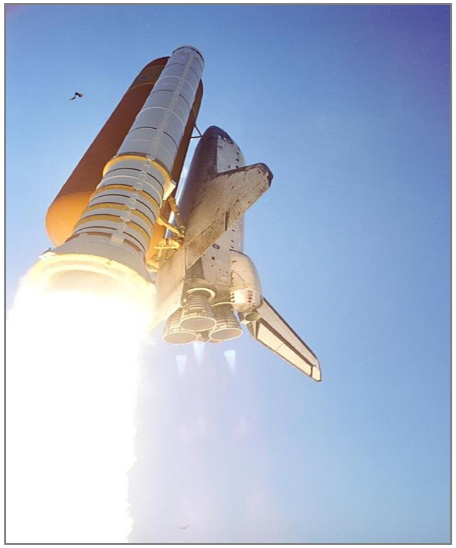
- The zinc chromate putty frequently failed and permitted the gas to erode the primary O-rings.
- The particular material used in the manufacture of the shuttle O-rings was the wrong material to use at low temperatures.
- Elastomers become brittle at low temperatures.



Causes and Contributing Factors

- Breach in the Thermal Protection System caused by the left bipod ramp insulation foam striking the left wing leading edge.
- There were <u>large gaps in NASA's knowledge</u> about the foam.
- Dissections of foam revealed subsurface flaws and defects as contributing to the loss of foam.





Reliability Check List

The following is a partial reliability check list:

- **Design Reliability**
  - Do we understand the design drivers?
  - Do we understand the design uncertainties?
  - Do we understand the physics of failure?
  - Do we understand the failure causes?
  - Do we have the right design margins?
- **Process Reliability**
  - Is the process capable of building the tolerances?
  - Do we have process uniformity?
  - Do we have process control?
- **Reliability Analysis and Testing**
  - Have we done a timely FMEA consistent with design timeline?
  - Do reliability predictions support the goals and requirements of the program?
  - Have we done enough reliability testing and demonstration to support the design?
- **Systems Engineering**
  - Do we understand the requirements?
  - Are we part of system integrated analysis environment?

Reliability Metrics

There are many ways to measure and evaluate reliability. The following are the most commonly used across government and industry:

- ***Mean Time Between Failures (MTBF)/ Mean Time to Failure (MTTF)***
  - MTBF is a basic measure of reliability for repairable items. MTBF is the expected value of time between two consecutive failures, for repairable systems
  - MTTF is a basic measure of reliability for non-repairable systems. It is the mean time expected until the first failure.
- ***Predicted Reliability Numbers***
  - Reliability prediction is the process of quantitatively estimating the reliability using both objective and subjective data (e.g. 0.99999).

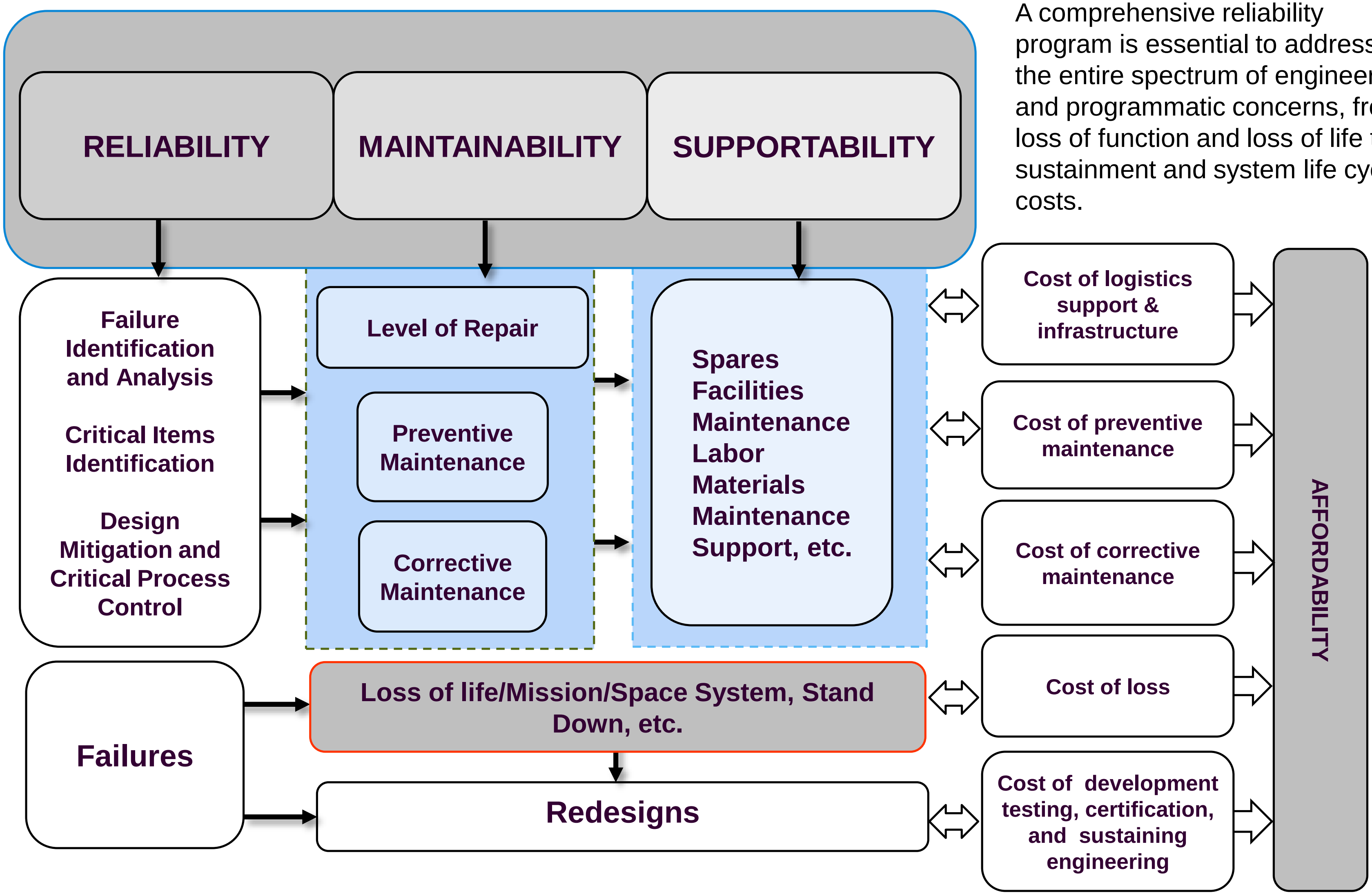
Reliability Metrics (Continued)

- ***Demonstrated reliability numbers***
  - Unlike reliability prediction, reliability demonstration is the process of quantitatively estimating the reliability of a system using objective data at the level intended for demonstration. In general, demonstrated reliability requirement is set at a lower level than predicted reliability. It is intended to demonstrate a comfort level with a lower reliability than the predicted reliability because of the cost involved **(e.g., 0.99 with 90% confidence)**.
- ***Safety factors***
  - Safety factor (SF) is a term describing the capability of a system beyond the expected loads or actual loads (e.g., safety factor of 2).
- ***Fault tolerances***
  - Fault tolerance is the property that enables a system to continue operating properly in the event of the failure of some of its components (e.g., one fault tolerance means you can tolerate one failure and still operate successfully).

“How Reliable is Reliable Enough?”

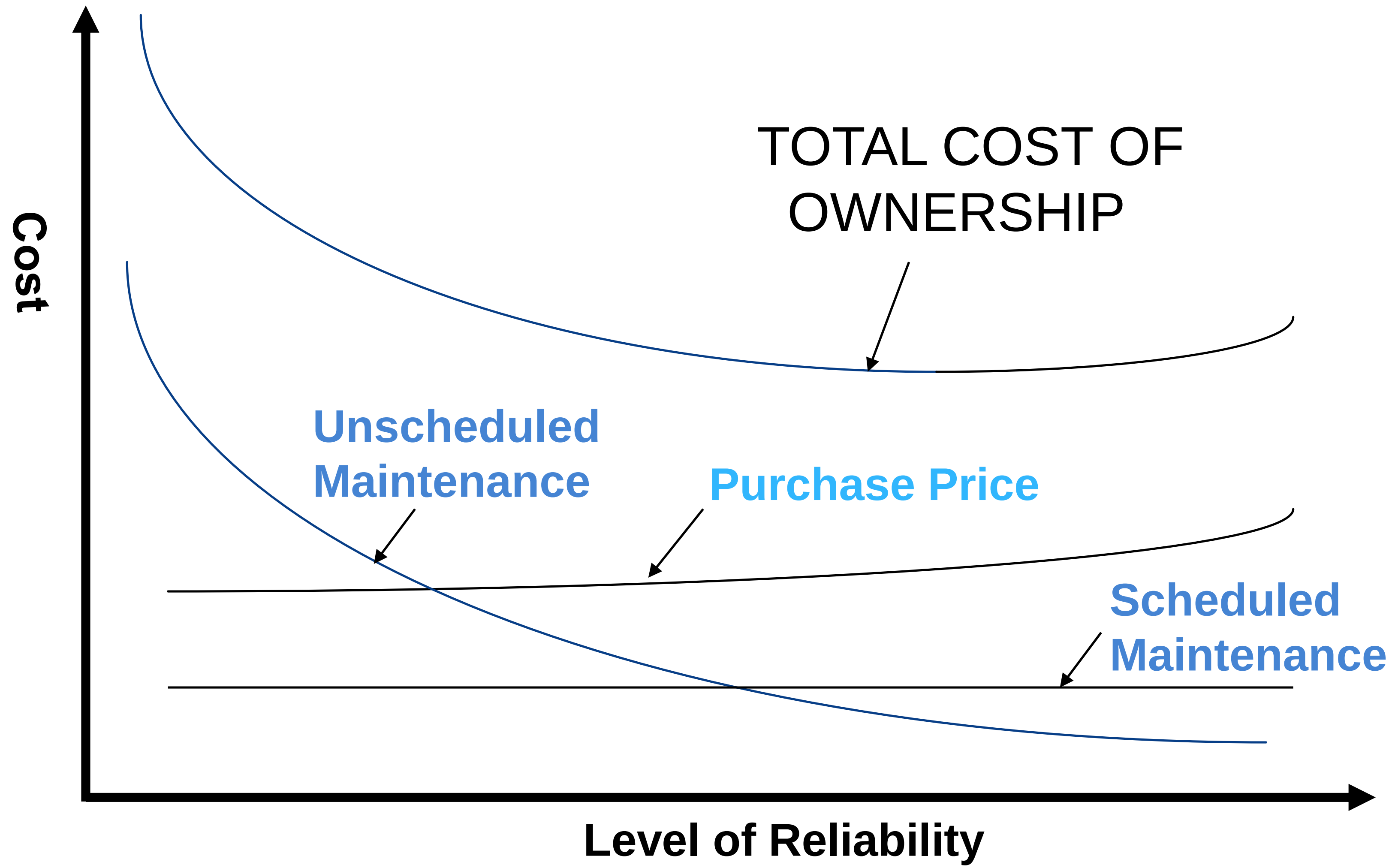
- In reliability engineering, no one likes things to fail. We don’t like bridges to collapse and we don’t like nuclear plants to leak radioactive material.
- Engineers still have to address the question “How reliable is reliable enough?” Is it one in a thousand? One in ten thousands? One in a million?
- The answer is: It depends. For example, “reliable enough” for a critical situation might mean a high safety factor (e.g., 2.0 or better), or high reliability (e.g., 0.999999 or better). For degraded performance, a lower safety factor or lower reliability might be acceptable.
- For these reasons, engineers must design things to certain reliability specifications depending on the safety and economics of the situation, technology availability, and design constraints.

Reliability Relationship To Maintainability, Supportability, and Affordability



A comprehensive reliability program is essential to address the entire spectrum of engineering and programmatic concerns, from loss of function and loss of life to sustainment and system life cycle costs.

Reliability Relationship to Life Cycle Cost



Reliability Relationship to Safety

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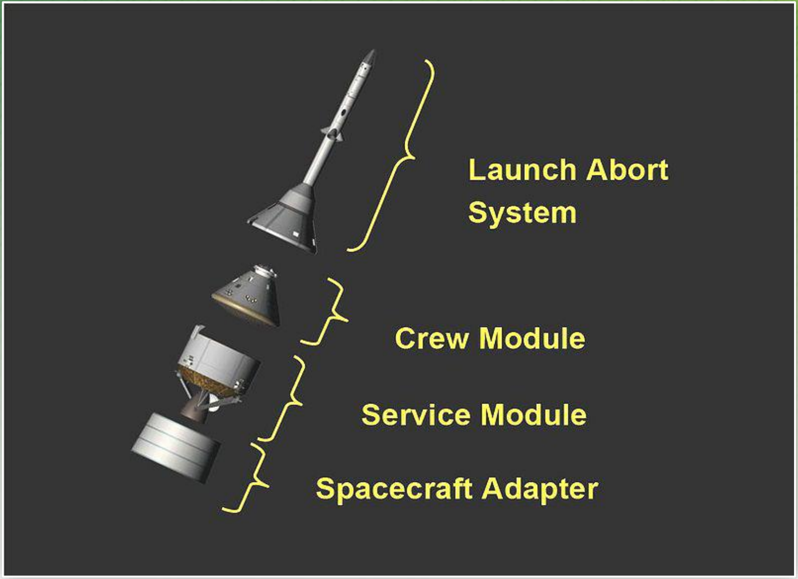
	Reliability	Safety
Roles	To ensure the product functions successfully.	To ensure the product and environment are safe and hazard free.
Requirements	Design function specific within the function boundary. Internally imposed.	Non-function specific such as “no fire,” “no harm to human beings.” Externally imposed.
Approaches	Bottom-up and start from the component or system designs at hand.	Top-down and trace the top-level hazards to basic events, then link to the designs.
Analysis Boundaries	Focus on the component or sub-system being analyzed (assumes others are at as-designed and as-built conditions). Component interactions and external vulnerability and uncertainty are usually not addressed.	System view of hazards with multiple and interacting causes. External vulnerability and uncertainty may be required to be addressed.

Safety and Reliability are unique but closely related —
they complement each other and need to be integrated.



RELIABILITY ALLOCATION

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Reliability Allocation Definitions

- **Reliability allocation** is the process of allocating the system reliability requirement or goal down to the subsystems level through apportionment.
- In general, reliability allocation is intended to drive a process to improve the product reliability during the design development process through prediction down to the subsystem or component levels.

- **Note:** Quantitative reliability requirements can be predicted, demonstrated, or both, depending on the objectives and the economics of the project or the program.
  - Predicted reliability requirement calls for estimating the reliability using both objective and subjective data, where reliability prediction is performed to the lowest identified level of design for which data is available.
  - Demonstrated reliability requirement calls for estimating the reliability of a system using objective data at the level intended for demonstration. Demonstrated reliability requirement is intended to provide empirical evidence of design reliability and can’t be allocated.

Reliability Allocation Process

- Reliability allocation involves solving the following inequality:

$$f(R_1, R_2, \ldots, R_n) \geq R_s$$

where:
R_i is the reliability allocated to the i^{th} subsystem/component.
f is the functional relationship between the subsystem/component and the system.
R_s is the required system reliability.

Reliability Allocation Methods/Techniques

- Several techniques have been used over the years for reliability allocation. Commonly used techniques are:
  - The simplest technique is **Equal Apportionment**, which distributes system reliability equally among all the subsystems.
  - The **ARINC** apportionment method designed by ARINC Research Corporation, a subsidiary of Aeronautical Radio, Inc (ARINC).
  - The **AGREE** apportionment method, designed by the Advisory Group on Reliability of Electronic Equipment (AGREE)
- Both the AGREE and ARINC techniques take additional weighting factors into consideration during allocation.
- To obtain good results, it is important to choose an appropriate apportionment method based on the system reliability requirement and the system properties.

The following charts cover the Equal Apportionment and the ARINC Methods. The AGREE method is included in the backup section.

Reliability Allocation Methods/Techniques

- **Equal Apportionment**
  - The simplest apportionment technique is to distribute the reliability uniformly among all components. This method is called equal apportionment.
  - Equal apportionment assumes a series of n subsystems, all in series and having an exponential failure distribution. Each subsystem is assigned the same reliability. The mathematical model can be expressed as:

$$R^* = \prod_{i=1}^{n} R_i^* \quad \Longrightarrow \quad R_i^* = (R^*)^{\frac{1}{n}} \text{ for } i = 1, 2, \ldots, n$$

Where:

R* is the required system reliability

R^*_i is the reliability requirement apportioned to subsystem i

n is the total number of subsystems.

Equal Apportionment Example

- Consider a proposed communication system which consists of three subsystems (transmitter, receiver, and coder), each of which must function if the system is to function. Each of these subsystems is to be developed independently. Historical data from previous programs showed that the three subsystems have very similar failure rates. What reliability requirement should be assigned to each subsystem in order to meet a system requirement R of 0.729?
- The apportioned subsystem requirements are found as:

$$R_T = R_R = R_C = (R)^{1/n} = (0.729)^{1/3} = 0.90$$

Where R_T, R_R, and R_C are the transmitter, receiver, and coder reliabilities, respectively.

- A reliability requirement of 0.90 should be assigned to each subsystem in order to meet a system reliability requirement of 0.729.

The ARINC Apportionment Method

- **The ARINC Apportionment Method** assumes that all subsystems are in series and have an exponential failure rate. Allocations are derived based on weighting factors. The mathematical expression is:

$$w_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i}$$

$$\lambda_i' = w_i \lambda_S$$

Where, n is the total number of subsystems, λ_i is the present failure rate of the ith subsystem, λ_S is the required system failure rate, and λ_i' is the failure rate allocated to the ith subsystem.

ReliaSoft Corporation, Lambda Predict, Tucson, AZ: ReliaSoft Publishing, 2007.

ARINC Apportionment Example

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Allocation

	Name	Part Number	Include	Present Failure Rate (FITS)	Allocated Failure Rate (FITS)	Allocated MTBF (hrs)	Current Failure Rate (FITS)
▸	Power Supply	1.1	☑	30.9971	6.3508	1.5746E+05	30.9971
	Transformer	1.2	☑	19.5772	4.0110	2.4931E+05	19.5772
	Switch	1.3	☑	3.8967	0.7984	1.2526E+06	3.8967
	Load	1.4	☑	4.2330	0.8673	1.1530E+06	4.2330

$$w_i = \frac{\lambda_i}{\sum_{i=1}^{n} \lambda_i}$$

$$\lambda_i' = w_i \lambda_S$$

Allocation | Plot

Calculations

Allocation Type

ARINC

Product: Power Circuit

Reliability Goal: 0.9

Elements: 4

Operating Time 8760

ARINC

☑ Use Failure Rate From Current Project

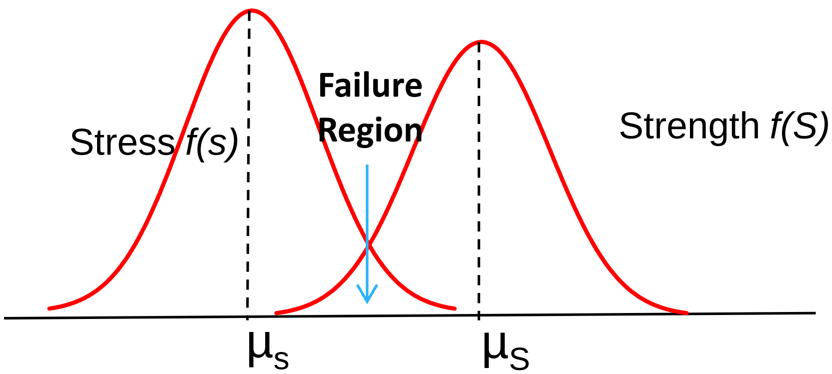
ReliaSoft Corporation, Lambda Predict, Tucson, AZ: ReliaSoft Publishing, 2007.

Advantages

- Main Advantages
  - Reliability allocation helps optimize the best combination of component reliability improvements that meet the intended reliability goals and at sufficient allocated costs.
  - It provides a realistic view of subsystem performance required to meet system objectives.
  - It shows the most cost-effective areas for design improvements; and avoids putting design efforts into subsystems that may not gain any additional reliability by improvements.

Limitations

- Main Limitations
  - Most allocation methods apply only to series configurations.
  - The apportionment process of reliability values between the various subsystems in many cases has high level of subjectivity. It is usually made on the basis of achievable reliability, or any other factors considered appropriate by the analyst making the allocation.
  - Most allocation methods require the availability of equipment historical data in order to reduce subjectivity and produce credible and reasonable allocation estimates.



RELIABILITY PREDICTION

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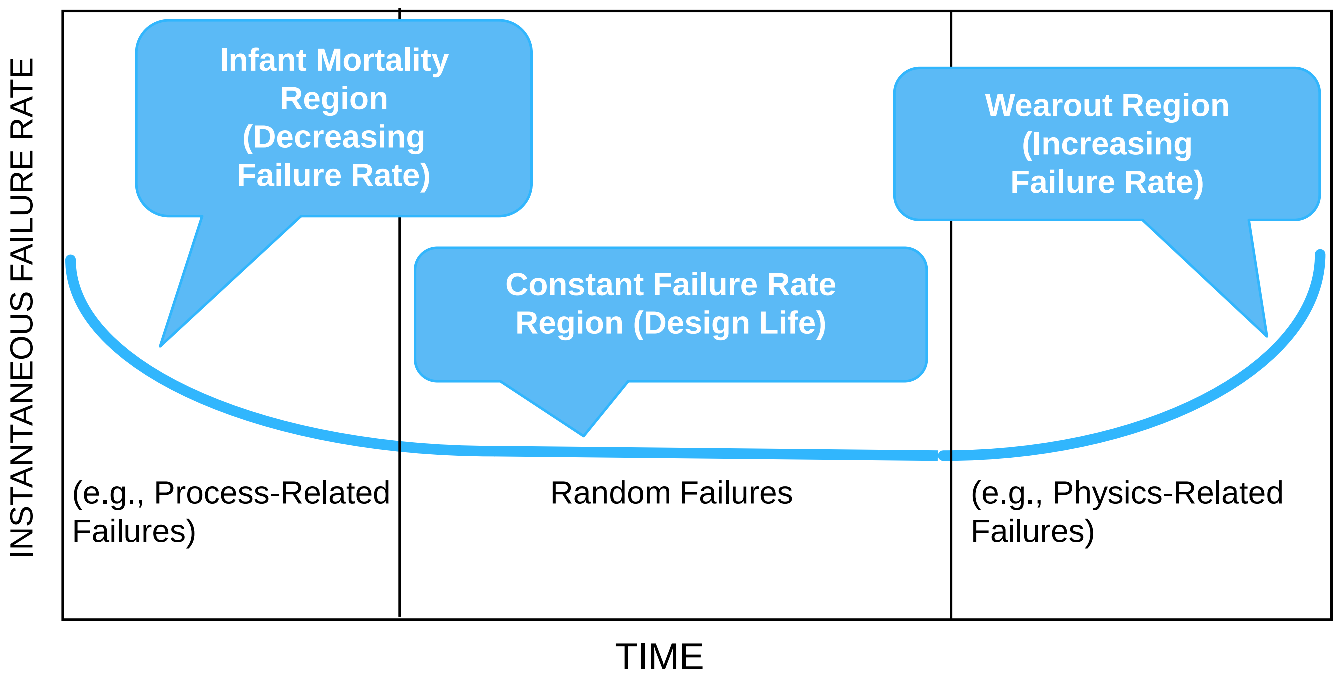
Reliability Prediction - Definition

- Reliability prediction is the process of quantitatively estimating the reliability using both objective and subjective data. It is one of the most common forms of reliability analysis.
- Reliability prediction is performed to the lowest identified level of design for which data is available.
- Reliability prediction techniques are dependent on the degree of the design definition and the availability of the relevant data.

The Bathtub Curve - Hardware Reliability



RELIABILITY PREDICTION USING RELIABILITY BLOCK DIAGRAMS (RBDS)

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Reliability Block Diagrams

- A Reliability Block Diagram (RBD) is a static form of reliability analysis using inter-connected boxes (blocks) to show and analyze the effects of failure of any component on the system reliability.
- The diagram represents the functioning state (i.e., success or failure) of the system in terms of the functioning states of its components. For example, a simple series configuration indicates that all of the components must operate for the system to operate, a simple parallel configuration indicates that at least one of the components must operate, and so on.

Reliability Block Diagrams

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- RBDs provide a success-oriented view of the system.
- RBDs provide a framework for understanding redundancy.
- RBDs facilitate the computation of system reliability from component reliabilities.
- RBDs and fault trees provide essentially the same information. However, RBDs are easier to use and communicate.

Reliability Block Diagrams Classifications

- The most commonly used types of RBDs are:
  - Simple series (all items have to function successfully)
  - Simple active parallel (all items operating simultaneously in parallel and only one is needed)
  - Standby parallel redundancy (alternate items are activated upon failure of the first item; only one item is operating at a time to accomplish the function)
  - Shared parallel (failure rate of remaining items change after failure of a companion item)
  - **r-out-of-n Systems** – Redundant system consisting of n items in which r of the n items must function for the system to function (voting decision).
  - Combination of series and parallel systems

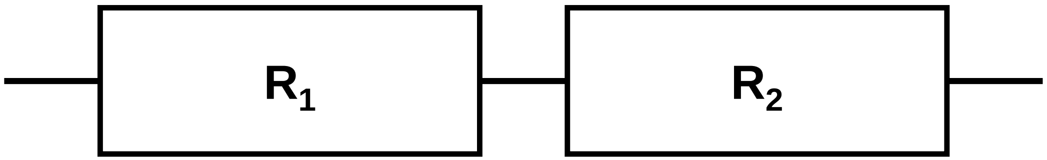
Note: We will not cover shared and r-out-of-n Systems redundancy

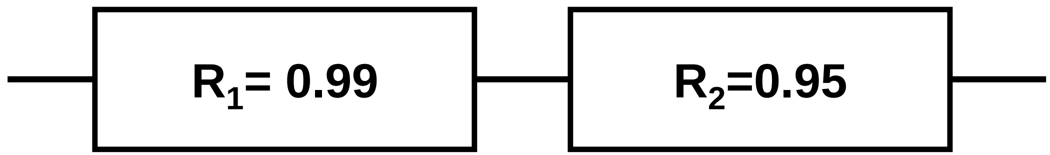
Simple Series Reliability Block Diagrams

2-Components Case

The general expression for a series system with two components is:

$R_{\text{System}} = R_1 \times R_2$





Example

$R_{\text{System}} = R_1 \times R_2$

$R_{\text{System}} = 0.99 \times 0.95$

General n Series Components Case

$R_{\text{System}} = R_1\, R_2\, R_3 \ldots Rn$ where R_s = probability that system will work.

Simple Active Parallel Reliability Block Diagrams

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- **2-Components Case**

  The general expression for a parallel system with two components is:

  $R_{System} = 1- (1-R_1)(1- R_2)$

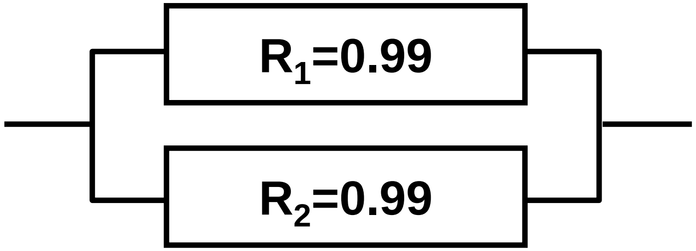
  If $Q_1=1-R_1$ and $Q_2=1- R_2$

  Then, $R_{System} = 1-Q_1 \times Q_2$

- **Example**

  The reliability of the redundant system

  $R = 1-Q_1 Q_2 = 1- (0.01)(.0.01) = 0.9999$



- General n redundant components Case

  $R_{system} = 1-(Q_1 Q_2 Q_3 \ldots Q_n)$

Standby Redundancy

The general reliability formula for n exponentially, identically distributed, and independent units in a standby redundant configuration (with perfect switching, $R_s = 1$) is:

$$R = \sum_{i=0}^{n-1} \{(\lambda t)^i / i!\} e^{(-\lambda t)}$$

Two Component Case

Assume, one shot switching reliability = 1, $\lambda_{switch} = 0$, failure rates are constant $\lambda_1 = \lambda_2 = 0.0001$ and

Mission duration $t_1 = t_2 = 1000$ hrs.

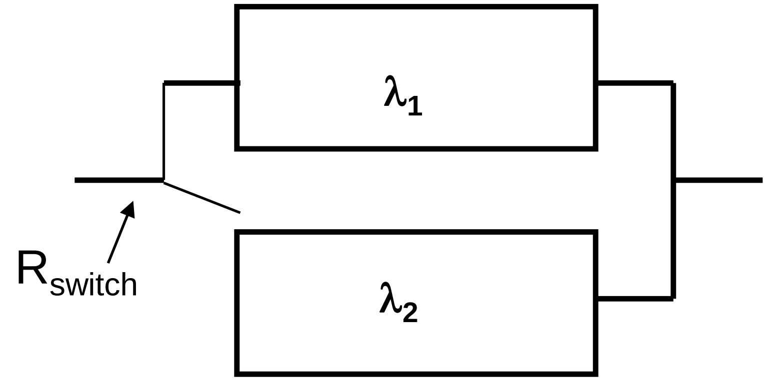
Substituting λ =.0001 and t = 1000 into the above equation we have:

$$R = ((\lambda t)^0/0!)e^{-\lambda t} + ((\lambda t)^1/1!) \times e^{-\lambda t}$$

$$R = ((1/1) \quad e^{-\lambda t} + ((\lambda t)^1/1) \quad \times \; e^{-\lambda t}$$

$$R = e^{-0.0001 \times 1000} + (0.0001 \times 1000) \times e^{-0.0001 \times 100}$$

$$R = 0.90484 + (0.1) \times 0.90484 = 0.9953$$



Complex System Reliability Block Diagrams

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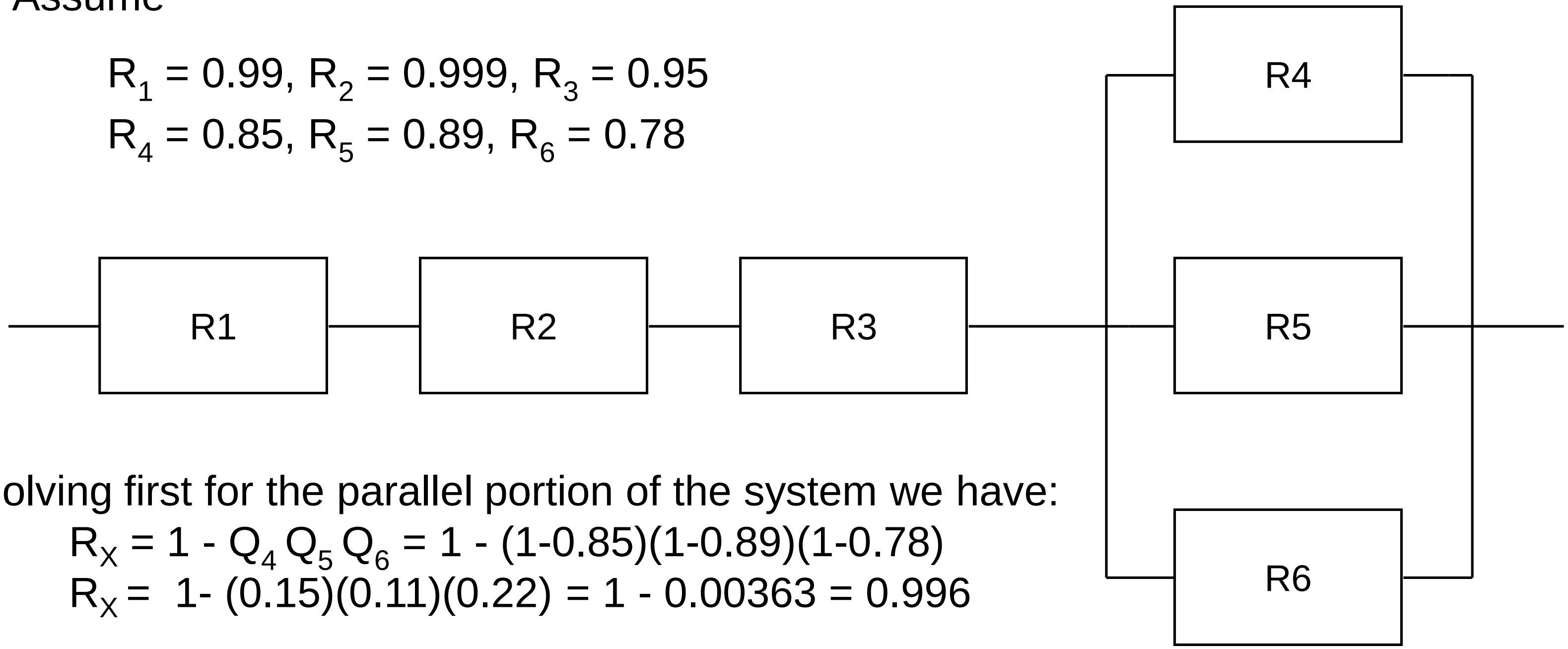
Assume

$R_1 = 0.99$, $R_2 = 0.999$, $R_3 = 0.95$
$R_4 = 0.85$, $R_5 = 0.89$, $R_6 = 0.78$



Solving first for the parallel portion of the system we have:

$R_X = 1 - Q_4 Q_5 Q_6 = 1 - (1-0.85)(1-0.89)(1-0.78)$
$R_X = 1 - (0.15)(0.11)(0.22) = 1 - 0.00363 = 0.996$

Now solving the series and then combine with parallel portion of the diagram, we have:

$R_s = R_1 R_2 R_3 R_X$
$R_s = (0.99)(0.999)(0.95)(0.996) = 0.936$

PHYSICS BASED RELIABILITY PREDICTION

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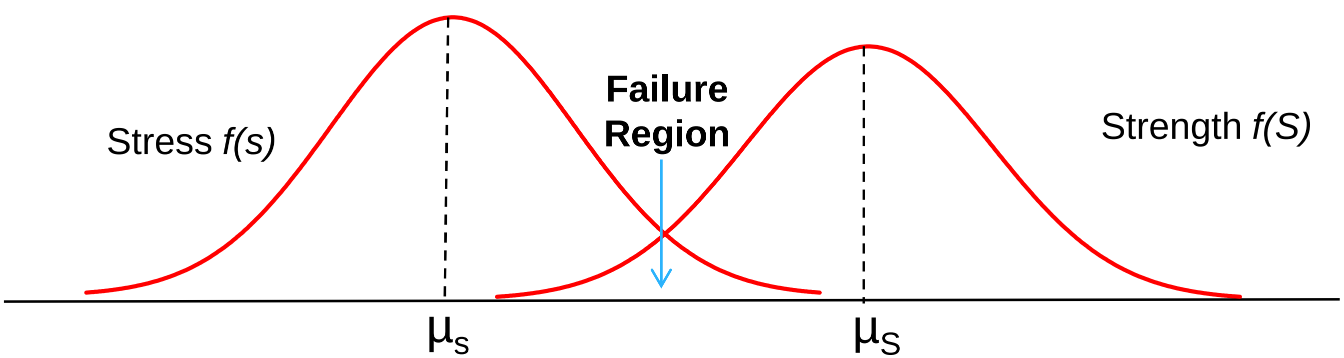
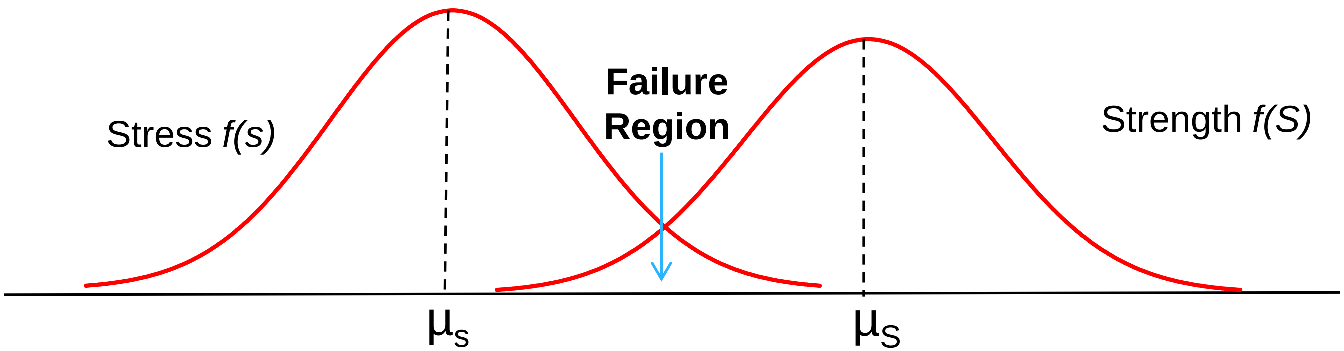
Physics-Based Reliability Prediction

- Physics-based reliability prediction is a methodology to assess component reliability for given failure modes.
- The component is characterized by a pair of transfer functions that represent the load (stress, or burden) that the component is placed under by a given failure mode, and capability (strength) the component has to withstand failure in that mode.
- The variables of these transfer functions are represented by probability density functions.
- The interference area of these two probability distributions is indicative of failure.



Physics Based Reliability Prediction
The Normal Case



Assuming both the stress and strength are normally distributed, the following expression defines the reliability for a structural component. If

$$R = \Phi\left[\frac{(\mu_S - \mu_s)}{\sqrt{\sigma_S^2 + \sigma_s^2}}\right]$$

Where

μ_s = mean value of the stress

σ_s = standard deviation of the stress

μ_S = mean value of the strength

σ_S = standard deviation of the strength

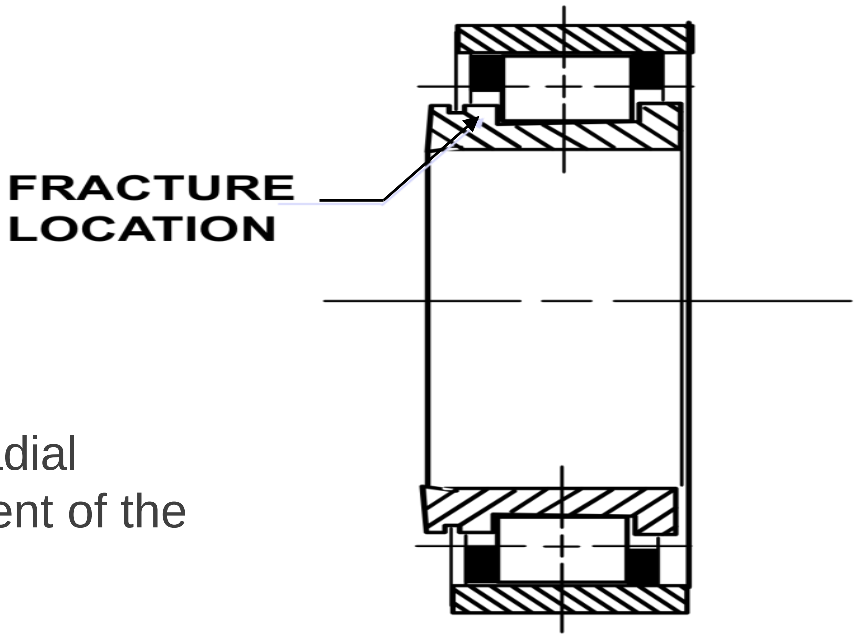
Note 1: In general, reliability is defined as the probability that the strength exceeds the stress for all values of the stress.

Note 2: Normality assumption does not apply to all engineering phenomena; and, under these special circumstances when the Normal does not apply, different methodology is used to determine reliability. As long as the engineering phenomena can be modeled, by whatever distribution, reliability could be obtained by methods such as the Monte Carlo method. Since the overwhelming majority of engineering phenomena do follow the normal distribution, the normality assumption is certainly the place to start.

Physics-Based Reliability Prediction
A Rocket Engine Roller Bearing Example

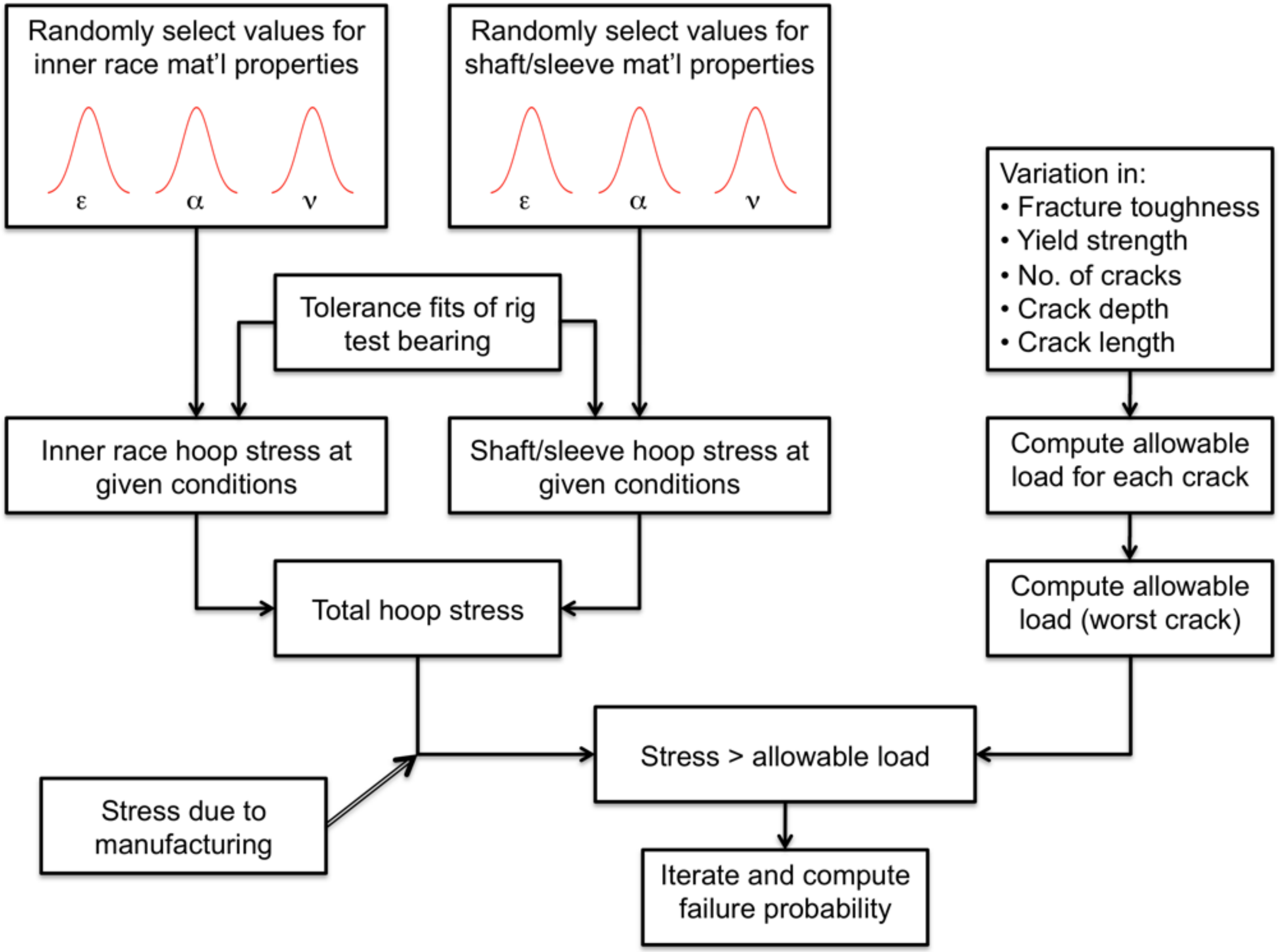
- During rig testing, the High Pressure Fuel Turbo-pump (HPFTP) Bearing of the Space Shuttle Main Engine (SSME) experienced several cracked races. Three out of four tests failed (440C bearing races fractured). As a result, a study was formulated to:
  - Determine the probability of failure due to the hoop stress exceeding the material's capability strength causing a fracture.
  - Study the effect of manufacturing stresses on the fracture probability for two different materials, the 440C (current material) and the 9310 (alternative material).

The **hoop stress** is the force exerted circumferentially (perpendicular both to the axis and to the radius of the object) in both directions on every particle in the cylinder wall. Along with axial **stress** and radial **stress**, circumferential **stress** is a component of the **stress** tensor in cylindrical coordinates.



Physics-Based Reliability Prediction
A Rocket Engine Roller Bearing Example

- The Simulation Model



The Simulation Model

- Since this failure model is a simple overstress model, only two distributions need to be simulated: the hoop stress distribution and the materials capability distribution.
- In order to calculate the hoop stress distribution it was necessary to determine the materials properties variability.
- Of those materials, properties that affected the total inner race hoop stress, a series of equations was derived which mapped these life drivers (such as modulus of elasticity, coefficient of thermal expansion, etc.) into the total inner race hoop stress.
- In order to derive these equations, several sources of information were used which included design programs, equations from engineering theory, manufacturing stress data, and engineering judgment. This resulted in a distribution of the total hoop stress.

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The Simulation Model

- In a similar fashion, a distribution on the materials capability strength was derived.
- In this case, life drivers such as fracture toughness, crack depth/length, yield strength, etc., were important. The resulting materials capability strength distribution was then obtained through a similar series of equations.
- The Monte Carlo simulation in this case would calculate a random hoop stress and a random materials capability strength. If the former is greater than the latter, a failure due to overstress occurs in the simulation. Otherwise, a success is recorded.
- The simulation was run for two different materials: 440C (current material) and 9310.
- After several thousand simulations are conducted, the percent which failed are recorded.

Analysis Results

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Test Failures	Race Configuration	Failures in 100,000 firings**
3 of 4	440C w/ actual* mfg. stresses	68,000
N/A	440C w /no mfg. stresses	1,500
N/A	440 C w/ ideal mfg. stresses	27,000
0 of 15	9310 w/ ideal mfg. stresses	10

* ideal + abusive grinding
** Probabilistic Structural Analysis

- The results of this analysis clearly showed that the 9310 material was preferred over the 440C in terms of the inner race fracture failure mode.
- Manufacturing stresses effect for the 440C material was very significant.
- Material selection has a major impact on reliability.
- Probabilistic engineering analysis is critical to perform sensitivity analysis and trade studies for material selection and testing.

RELIABILITY PREDICTION BASED ON OPERATIONAL DATA

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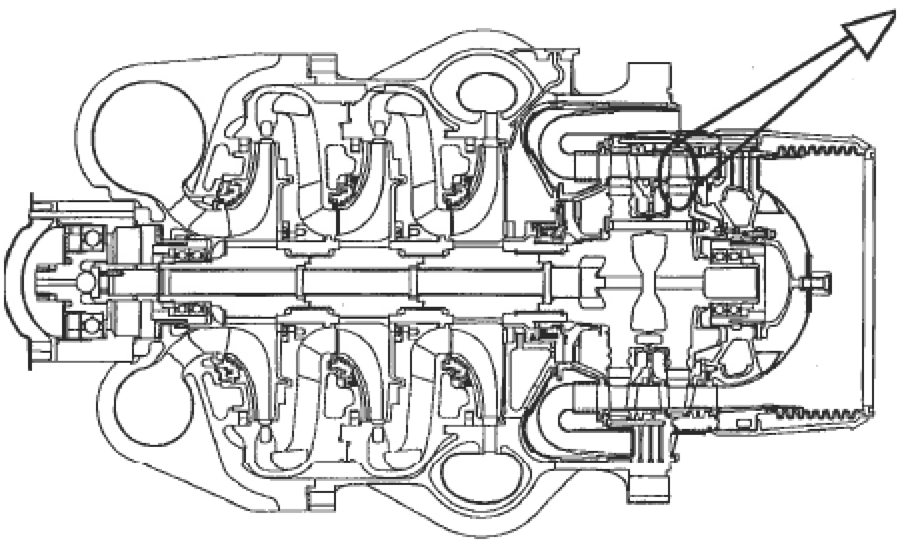
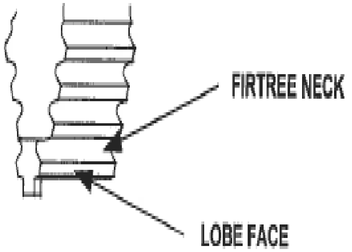
High Pressure Fuel Turbo-Pump (HPFTP) First Stage Turbine Blade Example

- During the inspection of the High Pressure Fuel Turbo-pump (HPFTP) Turbine blades of the Space Shuttle Main Engine (SSME) cracks were found in the blade firtree area. As a result, a study was formulated to determine the Space Shuttle flight risk due to a HPFTP first stage turbine blade failure.

HPFTP





HPFTP Turbine Blade Example

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Background

- A crack was found in a first stage turbine blade in HPFTP development unit 2423 during dye penetrant inspection 1/19/96.
- The subject blade had accumulated 20 starts and 9,826 seconds of operation.
- A total of 34 blade sets of the current configuration have been dye penetrant inspected, with no other crack being found.
- Metallurgical evaluation of the blade showed:
  - Fracture is hydrogen-assisted cracking.
  - Fracture origin approximately in middle of bottom firtree lobe – starting on pressure side.
  - No clear evidence of crack progression.

HPFTP Turbine Blade Example

Assumptions

- A crack in a blade is a failure.
- Only last dye penetrant inspection times are used (34 sets).
- One failure (crack) at 20 starts and 9,826 seconds.

Assumptions and Database

Starts	Seconds	Starts	Seconds
46	22,241	15	7,604
21	10,394	27	7,344
28	11,314	5	2,337
17	13,997	15	7,302
38	13,269	8	3,759
32	13,028	10	6,308
20	9,826	11	4,792
25	12,362	8	4,178
21	10,219	11	4,076
30	10,139	5	2,402
22	9,822	5	2,337
19	9,314	4	2,110
17	9,011	5	1,871
28	8,577	4	1,851
21	8,285	5	1,612
19	8,250	4	1,598
36	7,839	2	600

HPFTP Turbine Blade Example

Analysis Results

STS-75 Risk Summary for HPFTP First Stage Turbine Blade

STS-75 Engine	HPFTP Unit #	Based on Starts Reliability	Based on Starts Risk	Based on Time Reliability	Based on Time Risk
ME-1	4112R1	0.999486	1 / 1,944	0.999436	1 / 1,773
ME-2	2128	0.999882	1 / 8,475	0.999943	1 / 17,507
ME-3	4016R1	0.999221	1 / 1,283	0.999517	1 / 2,070
3 Engine Cluster		0.998589	1 / 708	0.998896	1 / 906

The starts and run time for the three pumps:

2 STARTS / 817 SEC

2 STARTS / 780 SEC

4 STARTS / 1856 SEC

Weibull model was used for reliability predictions.

Concluding Remarks

- Manufacturing records review for the flight set showed no discrepancies.
- Fleet leader blade set with 22,241 seconds and 46 tests.
- 53 blade sets were tested greater than the flight units.
- Flight reliability was assessed and risk was accepted by Shuttle program.

Reliability Prediction Advantages

Main Advantages

- Allows the analyst to quantitatively and statistically analyze the relative reliability during the design or operational phase.
- Can aid in determining the resource allocation during the test and evaluation phase.
- Provides a means to quantify the uncertainty of design variables and their impact on reliability and risk.
- Identifies regions of high risk in a design.
- Provides a means to compare competing designs.
- Can reduce unnecessary conservatism.
- Estimates of the failure rates of components generated by reliability predictions are critical input to safety, maintainability, supportability, and cost.
- Reliability predictions are also the main source of data for Probabilistic Risk Assessments (PRAs).

Reliability Prediction Limitations

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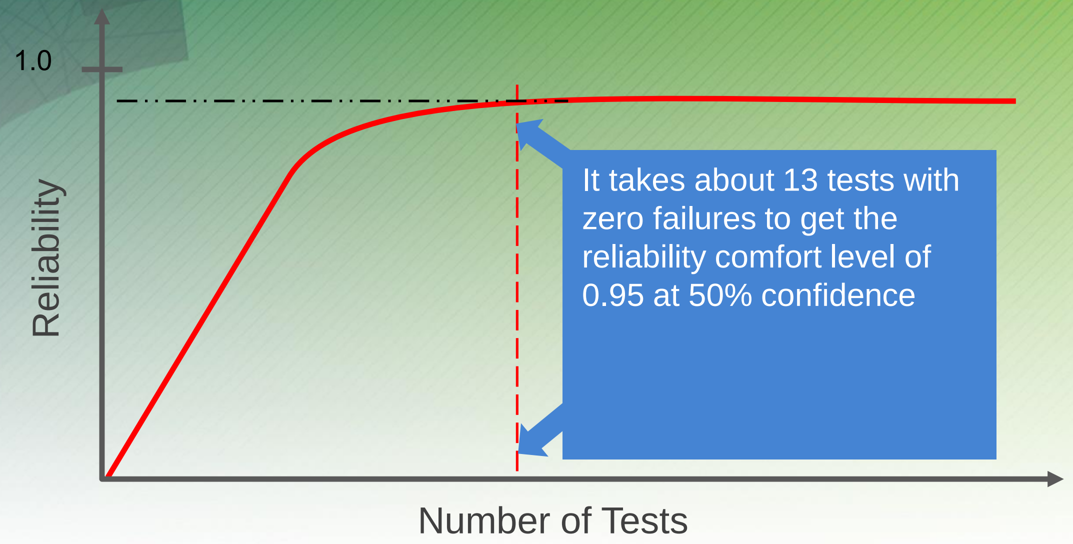
Main Limitations

- Reliability prediction can be resource intensive.
- The analyst must have knowledge of engineering disciplines and experience in probability and statistics.
- For reliability predictions using historical population, data used must be very close to the as-planned design population to be viable. Extrapolation between populations can render the technique nonviable.
- For physics-based reliability predictions, it may be difficult to get an accurate and detailed description of failure modes, failure mechanisms, and acting loads and environments (i.e., determining the density functions of the random variables in the load and capability transfer functions).



RELIABILITY DEMONSTRATION

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Reliability Demonstration Definition

- Reliability Demonstration is the process of quantitatively estimating the reliability of a system using objective data at the level intended for demonstration.
- It is used to provide empirical evidence of design reliability.
- It is the process of demonstrating the reliability of a design through testing and operation.
- It applies from test and evaluation through operation.
- Models and techniques used in reliability demonstration include Binomial, Exponential, Weibull models, etc.

Commonly Used Distributions

- There are a variety of probability distribution functions used for calculating reliability demonstration.
- They cover both discrete and continuous data cases.
- The most commonly used distributions are: The Exponential distribution for continuous data and the Binomial distribution for discrete data.

In the following charts we will cover the Binomial distribution for discrete data. The Exponential distribution for continuous data is included in the backup section.

http://reliabilityanalyticstoolkit.appspot.com/

The Binomial Distribution Case - Exact Method

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Two-sided confidence, exact method

- For a sample size of (N), a number of defects/failures of (N_d), and a confidence level of (1 – α)X100:
  - The equation to calculate the Binomial lower limit of the two-sided confidence interval, p_L

$$\sum_{k=0}^{N_d-1} \binom{N}{k} p_L^k (1-p_L)^{(N-k)} = 1 - \frac{\propto}{2}$$

  - The equation to calculate binominal upper limit of the two-sided confidence interval, p_U

$$\sum_{k=0}^{N_d} \binom{N}{k} p_U^k (1-p_U)^{(N-k)} = \frac{\propto}{2}$$

The following equations are solved iteratively to determine the two-sided upper confidence limit (p_U) or two-sided lower confidence limit (p_L):

https://reliabilityanalyticstoolkit.appspot.com/binomial_confidence_details

The Binomial Distribution Case - Exact Method

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One-sided confidence, exact method

- The calculation method for single sided limits are nearly identical to the two-sided case, except all the α is in either the upper or lower tail of the distribution
  - The equation to calculate binominal lower single-sided confidence limit

$$\sum_{k=0}^{N_d-1} \binom{N}{k} p_L^k (1-p_L)^{(N-k)} = 1-\propto$$

  - The equation to calculate binominal upper single-sided confidence limit

$$\sum_{k=0}^{N_d} \binom{N}{k} p_U^k (1-p_U)^{(N-k)} = \propto$$

The following equations are solved iteratively to determine the single-sided upper confidence limit (p_U) or single-sided lower confidence limit (p_L):

Note 1: For the zero failure case**,** the Binomial upper limit on the probability of failure is: $P_U = 1- \alpha^{1/n}$, and the reliability Lower confidence Limit:
$R_L = 1 - P_U = \alpha^{1/n}$ Where α = 1- Confidence Level

https://reliabilityanalyticstoolkit.appspot.com/binomial_confidence_details

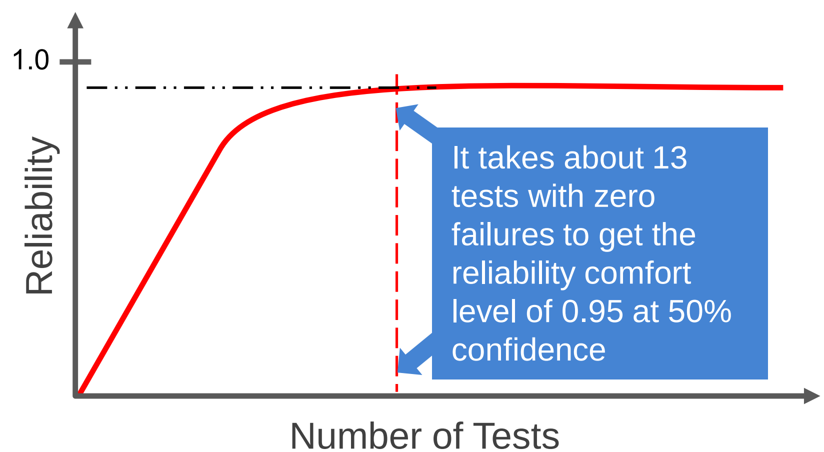
The Binomial Distribution Case
One-sided Exact Method Example

Demonstrated Reliability* at 50% confidence
Using the Binomial Model With Zero Failure Case

Number of tests	Reliability*	1-Reliability
1	0.500 (50.0%)	0.500
2	0.707 (70.7%)**	0.293
3	0.794 (79.4%)	0.206
4	0.841 (84.1%)	0.159
5	0.871 (87.1%	0.129
6	0.891 (89.1%)	0.109
7	0.906 (90.6%)	0.094
8	0.917 (91.7%)	0.083
9	0.926 (92.6%)	0.074
10	0.933 (93.3%)	0.067
11	0.939 (93.9%)	0.061
12	0.944 (94.4%)	0.056
13	0.948 (94.8%)	0.052

***Reliability** as a metric is the probability that an item will perform its intended function for a specified mission profile.

**A reliability, R, at 50% confidence level of 0.707, for example, means, 50% of the time the probability of success will be as good as or exceeds 0.707. Mathematically: P(R≥0.0.707)=0.5



Reliability Demonstration

- Advantages
  - Provides empirical information on reliability.
  - Reduces the uncertainty of analytically based reliability estimates.
  - Supports the determining of the resource allocation during the test and evaluation phase.
  - Used to support the reliability prediction of a design through testing and operation.

Reliability Demonstration

- Limitations
  - Dedicated pre-operational demonstration testing cannot be performed for high levels of design indenture (e.g., launch vehicle) due to cost and schedule constraints.
  - Reliability testing at lower-levels of design indenture is highly limited due to the same constraints (i.e., cost and schedule).
  - Data from piggyback demonstration through other engineering testing can lack the resolution desired for good reliability modeling.

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BACKUP

RELIABILITY ALLOCATION
THE AGREE METHOD

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The AGREE Apportionment Method

- The AGREE apportionment method determines a minimum acceptable mean life for each subsystem in order to fulfill a minimum acceptable system mean life.
- The AGREE method assumes that all subsystems are in series and have an exponential failure distribution. This method takes into account both the complexity and the importance of each subsystem.

The AGREE Apportionment Method

The mathematical model:

Let:

i = a counter representing each module, i = 1, 2, 3 …, n

t = system operating time

R(t) = system reliability requirement at time t

For n total modules in the system, the contribution of each module containing m components to the overall system reliability is:

$$R(t_i) = \left[R^*(t)\right]^{\frac{m_i}{n}}$$

Where,

$$n = \sum_{i=1}^{m} m_i = total\ number\ of\ components\ in\ the\ system$$

m_i is the number of components in module i.

t_i = operating time of module i

The AGREE Apportionment Method Example 1

Allocating the System Reliability

Module (i)	Number of components (mi)		$[R^*(t)]^{\frac{m_i}{n}}$
1	20		0.98
2	40		0.96
3	20		0.98
4	20		0.98
n =	100	Rsystem =	0.90

The AGREE Apportionment Method

Determining the module failure rate

Each module's unreliability is: $1 - [R(t)]^{\frac{m_i}{n}}$

If an exponential failure is assumed, then the unreliability of a module is also given by: $1 - e^{-\lambda_i t_i}$

The probability that the module is *critical and fails* is:

$$w_i(1 - e^{-\lambda_i t_i})$$

Where,

λ_i = failure rate of module i

w_i = probability that the system fails given that module i is critical and fails

Equating the above two quantities and solving for λ_i:

$$w_i\left(1 - e^{-\lambda_i t_i}\right) = 1 - [R(t)]^{\frac{m_i}{n}} \longrightarrow \lambda_i^* = -\frac{1}{t_i} ln\left[1 - \frac{1 - [R^*(t)]^{\frac{m_i}{n}}}{w_i}\right]$$

*R(t) is the required system reliability

Source: http://www.reliabilityanalytics.com/blog/2011/10/09/reliability-allocation/

The AGREE Method
Example 2

- A system has four subsystems, each with 20 modules. The required system reliability is 0.9 for a four hour mission. Assume all subsystems are critical (i.e. the probability that the system fails when a subsystem fails is 1.0). What should the allocated module reliability and failure rate be if:
  - All subsystem are equally important?
  - Module 3 becomes twice as complex as the other modules?
  - Module 3 is only 10% as important as the other modules?

Question 1

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Answer 1: For the stated inputs, each subsystem must have an MTBF of 152 hours. The reliability of each subsystem must be 0.974, which when multiplied together results in an overall system reliability of 0.90.

	A	B	C	D	E	F	G
1							
2							
3	Required System Reliability, R*(t)	0.9					
4	Mission Time (ti, Hours)	4					
5							
6	AGREE Allocation Input Data				Output Data		
7	Module (i)	Number of components (mi)	Importance Factor (wi, probability that the system fails given that subsystem i is critical and fails)		$\lambda_i^* = -\frac{1}{t_i} \ln\left[1 - \frac{1-[R^*(t)]^{\frac{m_i}{n}}}{w_i}\right]$ Allocated module failure rate, failures/hours	Allocated module MTBF, hours	$e^{-\lambda_i t_i}$ Allocated module reliability
8	1	20	1		0.00659	152	0.974
9	2	20	1		0.00659	152	0.974
10	3	20	1		0.00659	152	0.974
11	4	20	1		0.00659	152	0.974
12	n =	80				Rsystem=	0.900

Source: http://www.reliabilityanalytics.com/blog/2011/10/09/reliability-allocation/

Question 2

Answer 2: If module 3 has 40 components instead of 20, this module now has an allocated MTBF of 95 hours and the remaining three modules must have an MTBF of 190 hours to achieve the overall system reliability goal of 0.9 for a 4 hour mission.

	A	B	C	D	E	F	G
1							
2							
3	Required System Reliability, R*(t)	0.9					
4	Mission Time (ti, Hours)	4					
5							
6	AGREE Allocation Input Data				Output Data		
7	Module (i)	Number of components (mi)	Importance Factor (wi, probability that the system fails given that subsystem i is critical and fails)		$\lambda_i^* = -\frac{1}{t_i} ln\left[1 - \frac{1 - [R^*(t)]^{\frac{m_i}{n}}}{w_i}\right]$ Allocated module failure rate, failures/hours	Allocated module MTBF, hours	$e^{-\lambda_i t_i}$ Allocated module reliability
8	1	20	1		0.00527	190	0.979
9	2	20	1		0.00527	190	0.979
10	3	40	1		0.01054	95	0.959
11	4	20	1		0.00527	190	0.979
12	n =	100				Rsystem=	0.900

Source: http://www.reliabilityanalytics.com/blog/2011/10/09/reliability-allocation/

Question 3

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Answer – 3: If the quantity of components is put back to 20, but module 3 now has an importance of only 0.1, meaning that 90% of the failures will not cause the system to fail, the allocated MTBF for this module is only 13 hours instead of 152 hours. Note, the product of the module reliability values, 0.684, does not equal the requirement of 0.9 because not all failures of module 3 will cause a system failure.

	A	B	C	D	E	F	G
1							
2							
3	Required System Reliability, R*(t)	0.9					
4	Mission Time (ti, Hours)	4					
5							
6	AGREE Allocation Input Data				Output Data		
7	Module (i)	Number of components (mi)	Importance Factor (wi, probability that the system fails given that subsystem i is critical and fails)		$\lambda_i^* = -\frac{1}{t_i} ln\left[1 - \frac{1 - [R^*(t)]^{\frac{m_i}{n}}}{w_i}\right]$ Allocated module failure rate, failures/hours	Allocated module MTBF, hours	$e^{-\lambda_i t_i}$ Allocated module reliability
8	1	20	1		0.00659	152	0.974
9	2	20	1		0.00659	152	0.974
10	3	20	0.1		0.07526	13	0.740
11	4	20	1		0.00659	152	0.974
12	n =	80				Rsystem=	0.684

Source: http://www.reliabilityanalytics.com/blog/2011/10/09/reliability-allocation/

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DEMONSTRATED RELIABILITY
THE EXPONENTIAL CASE

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Confidence Intervals

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- For estimating confidence intervals for the MTBF, two cases have to be considered:
  - Failure terminated case: A test that is run until a pre-assigned number of failures have occurred.
  - Time terminated case: A test that is stopped after a pre-assigned number of test hours have accumulated.
- The formula for the confidence interval employs the χ^2 (chi-square) distribution.
- For tests with no failures occurring, only the one-sided lower confidence limit can be calculated.

Exponential Example - Confidence Intervals

- One-sided Confidence interval
  - Lower limit, Failure Terminated.

$$\left(\frac{2T}{\chi^2(\alpha, 2r)}, \infty\right)$$

  - Lower limit, Time Terminated.

$$\left(\frac{2T}{\chi^2(\alpha, 2r+2)}, \infty\right)$$

Where:

T = total accumulated unit-hours
r = total number of failures
(1 – α)×100 = confidence level (%)

- Two-sided Confidence interval
  - Lower and Upper Limits, Failure Terminated

$$\left(\frac{2T}{\chi^2(\frac{\alpha}{2}, 2r)}, \frac{2T}{\chi^2(1-\frac{\alpha}{2}, 2r)}\right)$$

  - Lower and Upper Limits, Time Terminated

$$\left(\frac{2T}{\chi^2(\frac{\alpha}{2}, 2r+2)}, \frac{2T}{\chi^2(1-\frac{\alpha}{2}, 2r)}\right)$$

http://reliabilityanalyticstoolkit.appspot.com/confidence_limits_exponential_distribution

Exponential Example - Confidence Intervals

- Transport vehicle example: One failure in 100 hours of operation

Confidence bounds – Time Terminated	MTBF at 50%	
One-sided lower 50% limit	60	$\left(\frac{2T}{\chi^2(\alpha, 2r+2)}, \infty\right)$
Two-sided 50% limits	37 – 348	$\left(\frac{2T}{\chi^2(\frac{\alpha}{2}, 2r+2)}, \frac{2T}{\chi^2(1-\frac{\alpha}{2}, 2r)}\right)$

For the operating time = *t,* the Reliability is:

$$R(t) = e^{-(t/\ MTBF)}$$

For the t = *MTBF,* the Reliability is:

$$R(MTBF) = e^{-(MTBF/\ MTBF)} = e^{-1} = 0.368 = 36.8\%$$

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