Manufacture of Fork Holograms to Produce Laguerre-Gaussian Beams with Orbital Angular Momentum

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WHAT IS ORBITAL ANGULAR MOMENTUM?

- Orbital angular momentum (OAM) is defined as an optical vortex with the Poynting vector travelling about the beam path
- This vortex has an azimuthal phase dependence $e^{-il\varphi}$ where l is the number of helical phase fronts
- A Laguerre-Gaussian mode (LG) will be used for its explicit $e^{-il\varphi}$ component

WHAT IS ORBITAL ANGULAR MOMENTUM?

Intensity pattern of an OAM beam

Intensity pattern of an OAM interfered with
A plane wave at normal incidence

WHAT IS A HOLOGRAM?

By coherently adding a beam containing the phase and amplitude information of an object with a plane off-axis reference beam of the same wavelength, a hologram is produced. By putting this new hologram in the path of a reconstruction beam (once again, a plane wave), the object's phase and amplitude information is then reconstructed at the same angle as the original reference beam's angular separation.

WHAT IS A FORK HOLOGRAM?

A fork hologram is the combination of the phase information from an LG mode and an HG mode reference beam at an angle θ

HOW TO CREATE THE HOLOGRAM

Using Mathematica, the LG and HG modes can be represented by their respective wave equations. These wave equations can be combined to find the superposition of the two modes and Mathematica can make a model of the hologram created.

$$\Psi_1 = A_1 e^{il\theta} e^{ik_z z} \text{ (Laguerre – Gaussian Mode)}$$

$$\Psi_2 = A_2 e^{ik_x x} e^{ik_z z} \ (Plane\ Wave\ Mode)$$

$$I = |\Psi_1 + \Psi_2|^2 = (\Psi_1^* + \Psi_2^*)(\Psi_1 + \Psi_2)$$

$$I = \left(A_1 e^{-il\theta} e^{-k_z z} + A_2 e^{-ik_x x} e^{-ik_z z}\right)\left(A_1 e^{il\theta} e^{ik_z z} + A_2 e^{ik_x x} e^{ik_z z}\right)$$

$$I = \left(A_1 e^{-il\theta} + A_2 e^{-ik_x x}\right)\left(A_1 e^{il\theta} + A_2 e^{ik_x x}\right)$$

Assume $A_1 and\ A_2$ have the same value

$$I = A^2 e^{-il\theta} e^{il\theta} + A^2 e^{-il\theta} e^{ik_x x} + A^2 e^{-ik_x x} e^{il\theta} + A^2 e^{-ik_x x} e^{ik_x x}$$

$$I = A^2 + A^2 + A^2[e^{i(l\theta - k_x x)} e^{-i(l\theta - k_x x)}]$$

$$I = A^2 + A^2 + 2A^2 \cos\,(l\theta - k_x x)$$

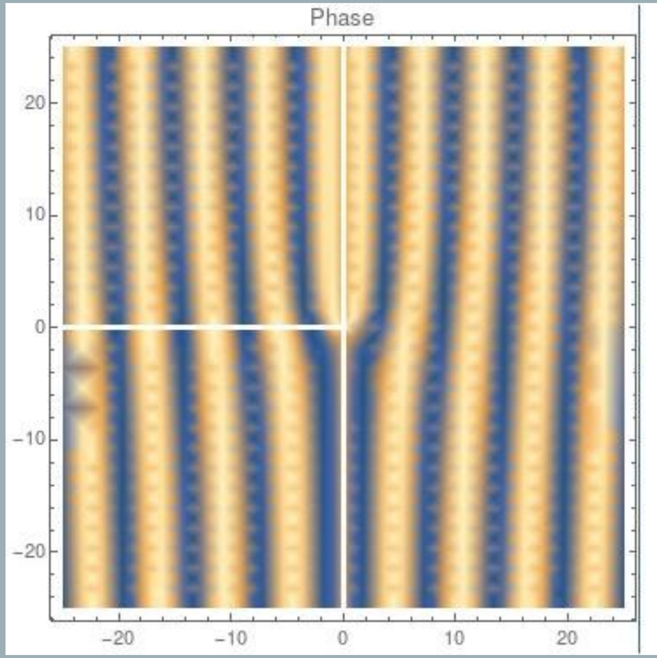
Then normalize as

$$I_n = \frac{[A^2 + A^2 + 2A^2 \cos(l\theta - k_x x)]}{A^2 + A^2}$$

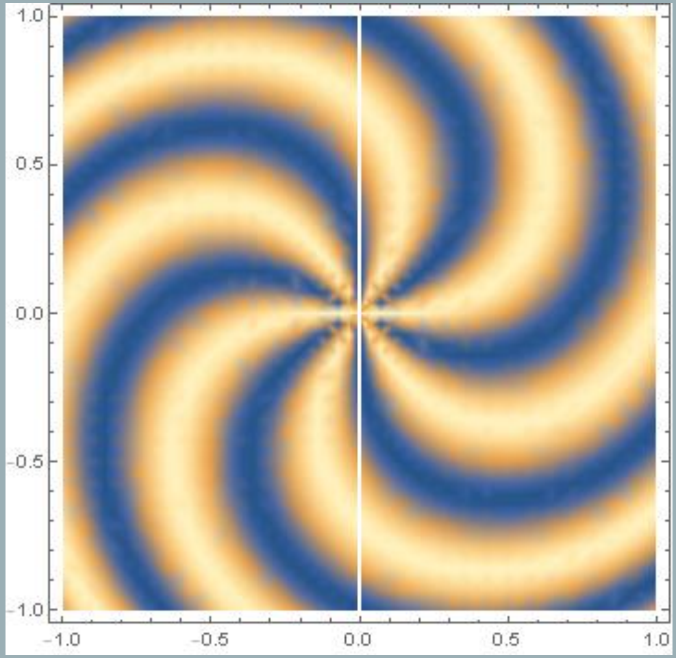
$$I_n = \frac{2A^2(1 + \cos(l\theta - k_x x))}{2A^2}$$

$$I_n = 1 + \cos(l\theta - k_x x)$$

MODELS MADE IN MATHEMATICA



Interference at 30^o angle of incidence

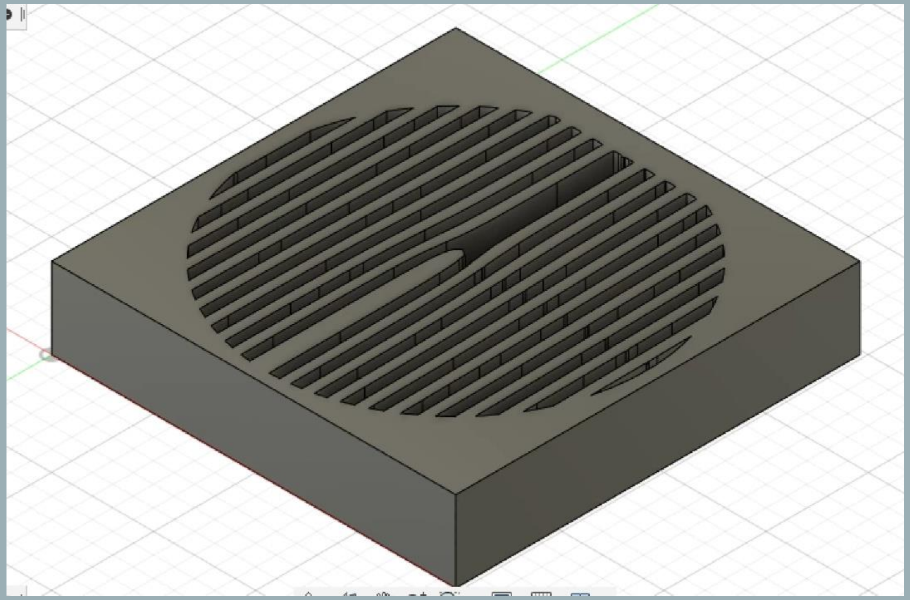


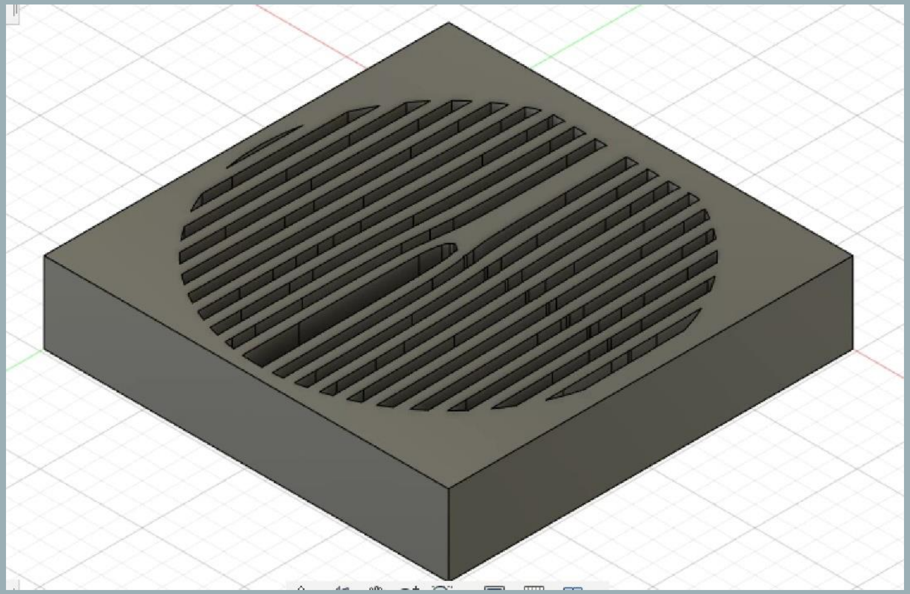
Interference at normal incidence

HOW TO PRODUCE A HOLOGRAM FROM THE MATHEMATICA MODEL

- Convert the PNG file from the model into an SVG file
  - SVG files contain a vector outline of the input image
- Input the SVG file containing the hologram into a 3D rendering software to be given depth
- Use the STL from the 3D software and print the model

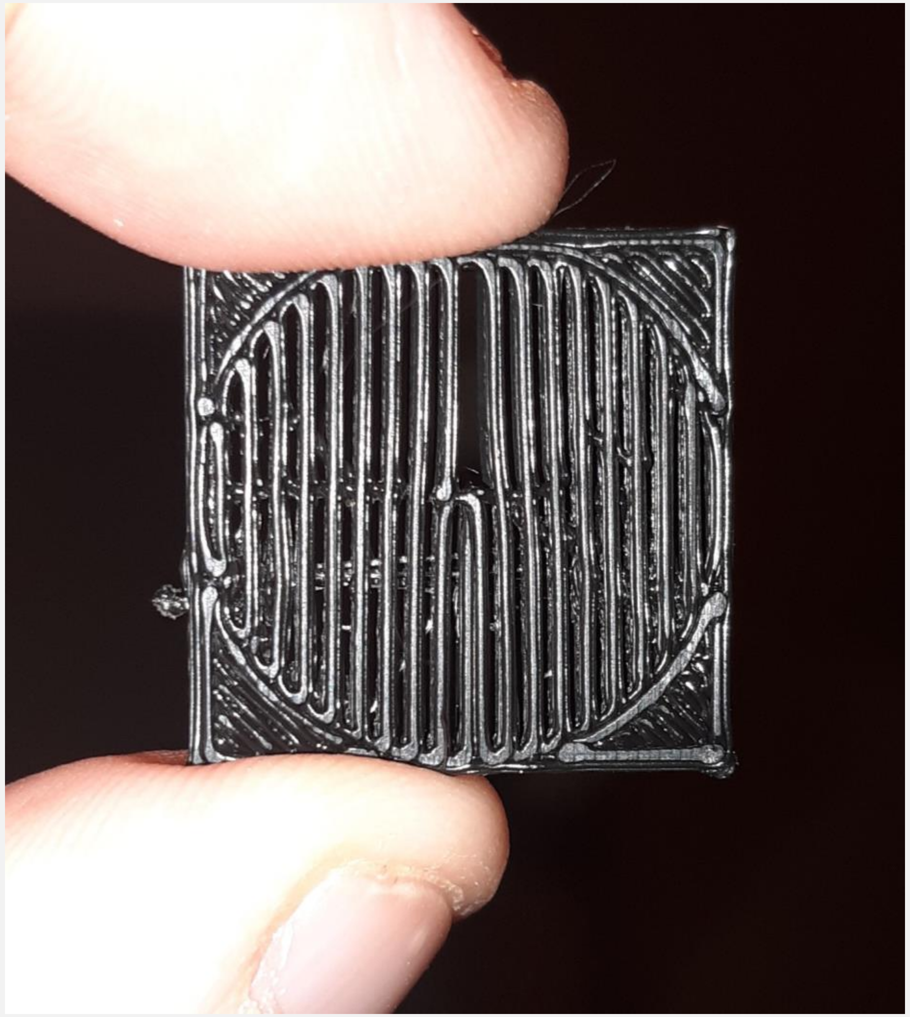
3D MODELS


Negative model to act as a mask in the NMDC


Positive model to be placed in the beam path

FIRST PROTOTYPE

The first prototype was printed with a 0.4mm nozzle so the best resolution that could be obtained was approximately 0.5mm. The model was initially much higher resolution than this so the grating had filament between the slits.



WHY A HIGHER RESOLUTION IS NEEDED

The distances between fringes is inversely proportional to the distance between slits. In the figure to the right, both Young's double slit and the grating equation both demonstrate this proportionality.

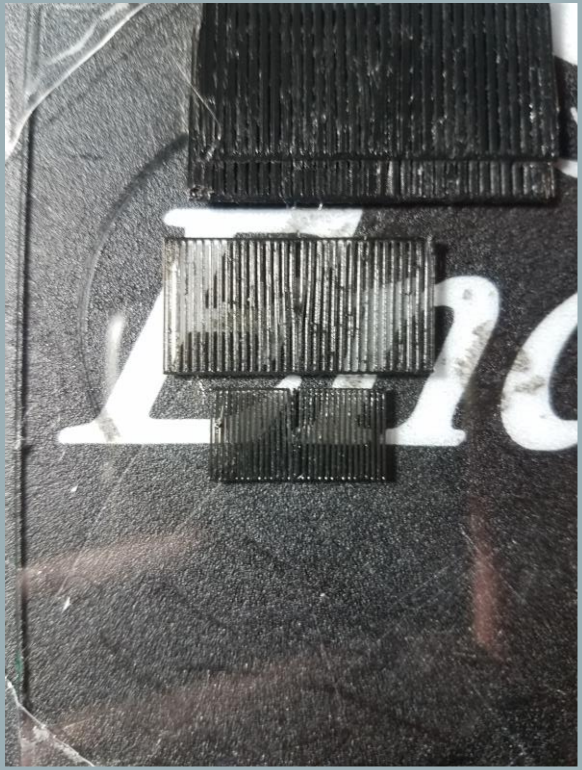
$$\Delta y = \frac{\lambda L}{a}$$

$$\frac{m\lambda}{a} = sin\Theta$$

FINAL PRODUCTS

After the prototype, a 0.2mm nozzle was used to achieve a higher resolution hologram. With this new resolution, not only were the holograms made smaller but in more detail allowing for higher l values to be used.

FINAL PRODUCTS


Original size compared to the 0.2mm nozzle size

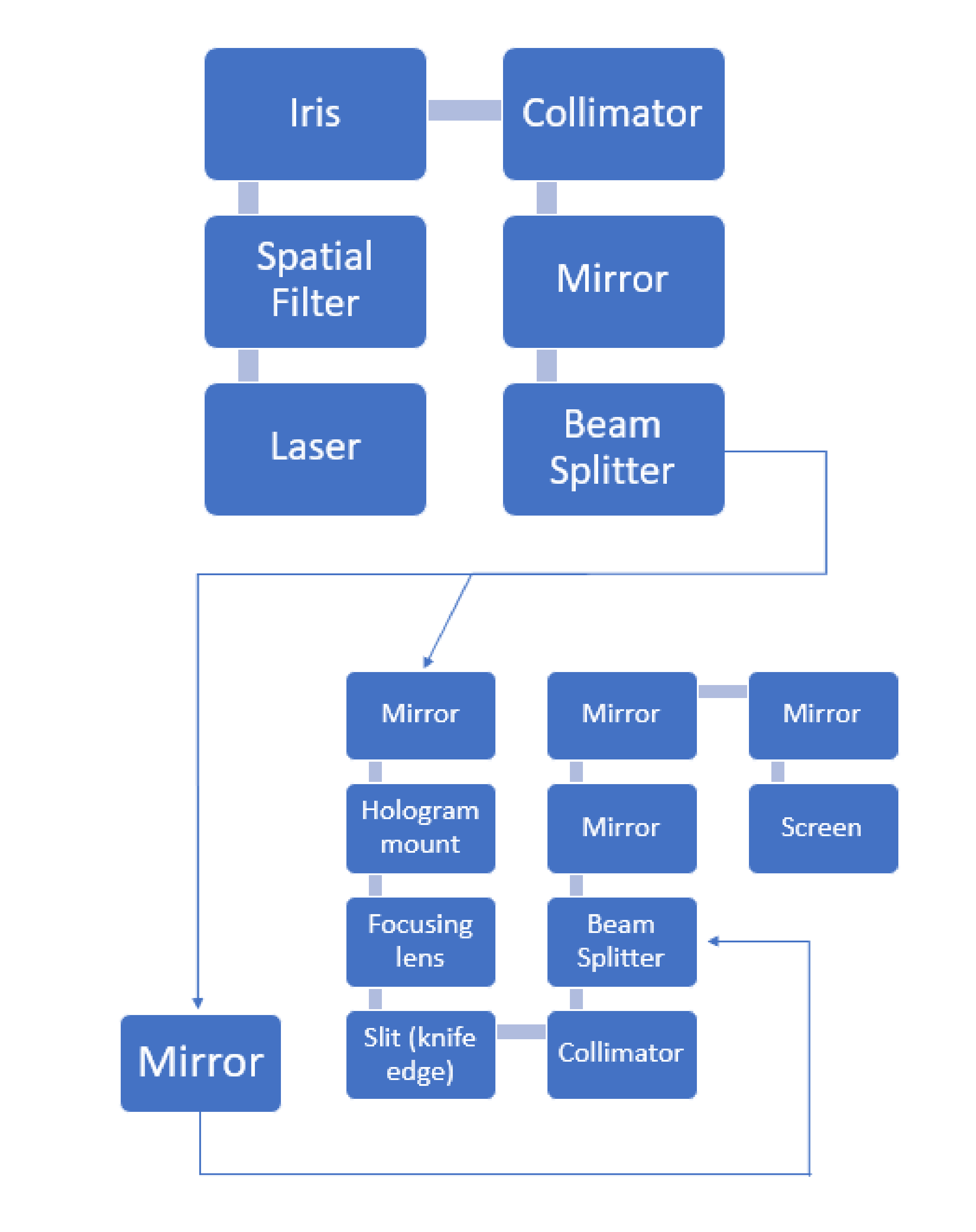

Higher l value ($l = 6$) printed on a mirror

TESTING THE HOLOGRAMS

To test how effective the hologram was in producing LG modes, the beam exiting the hologram was interfered with a plane wave at normal incidence.

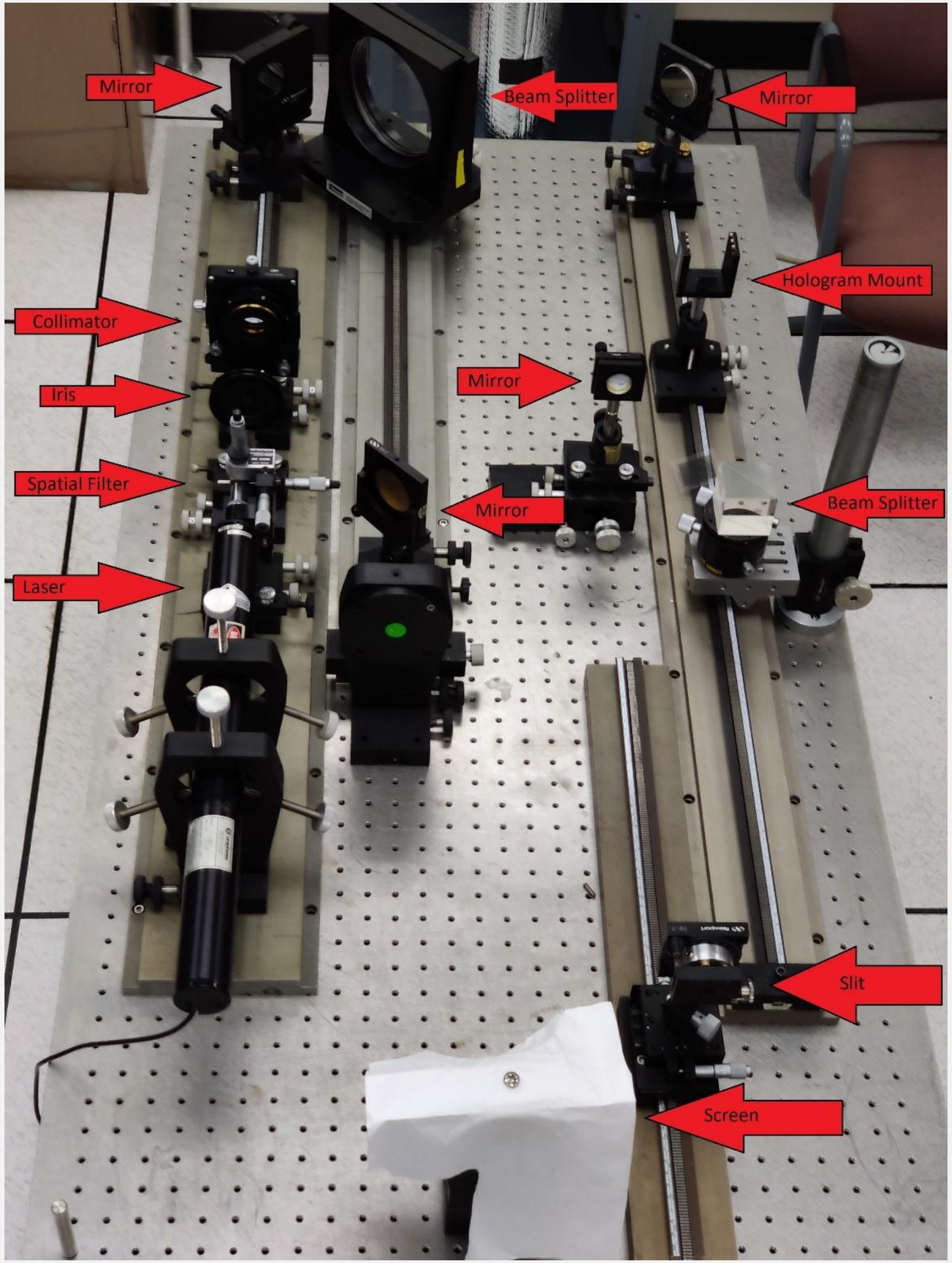
EXPERIMENT

Figure on the right shows the beam path of the experiment.



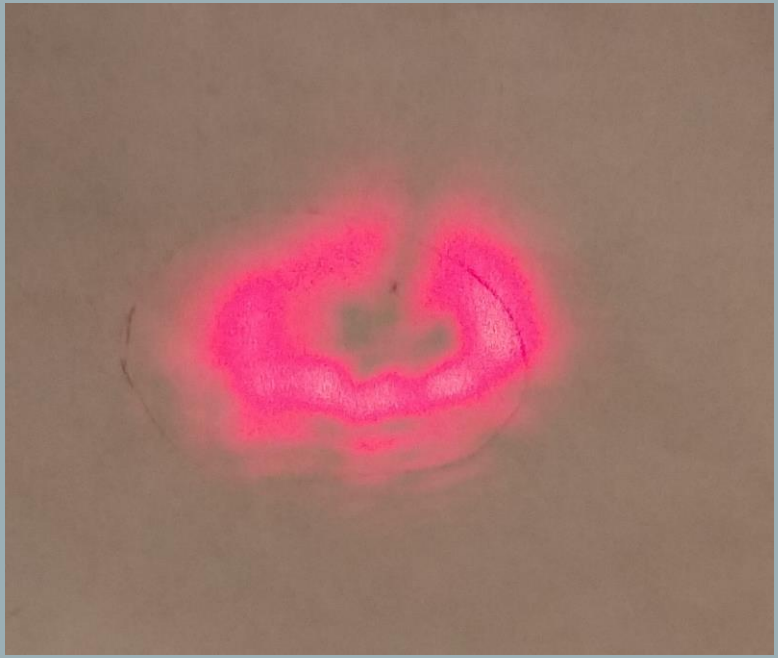
EXPERIMENT

Figure on the right shows the lab setup.



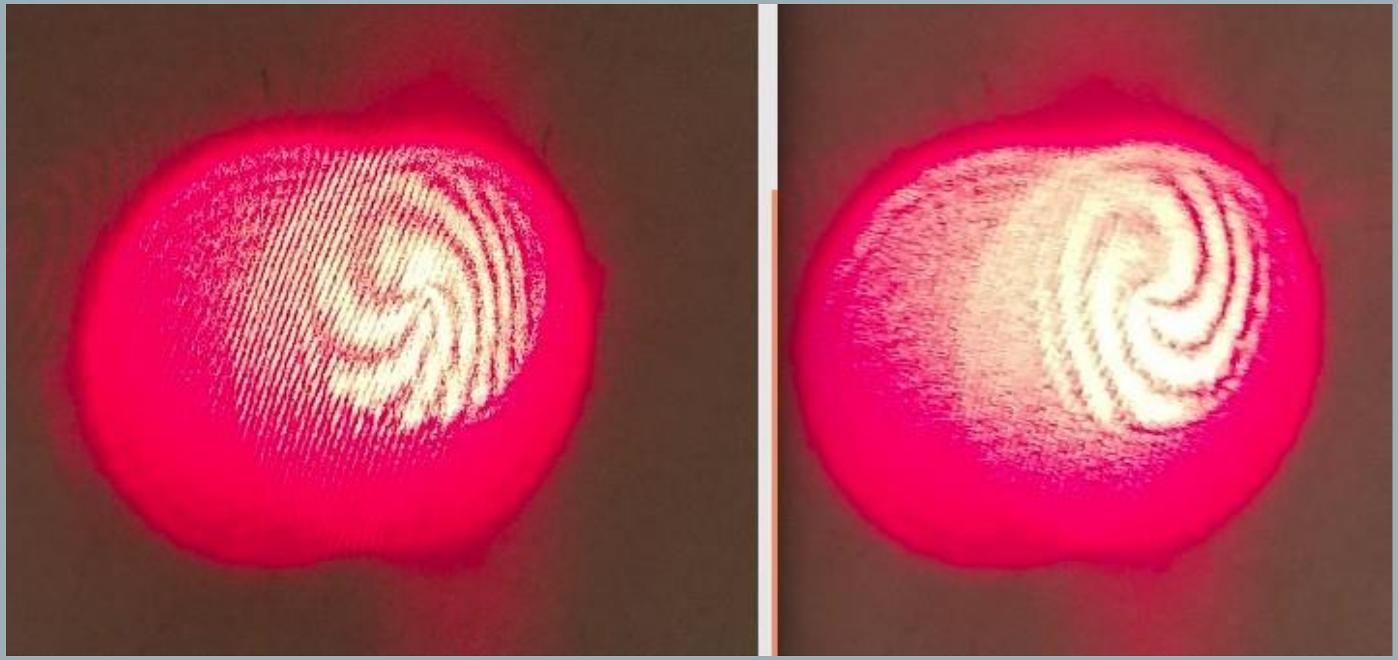
RESULTS

OAM without $m = 2,3, \ldots$ being blocked by slit



Intensity pattern from an OAM beam

RESULTS



Spiral interference from the OAM beam and plane wave

CONCLUSIONS