

Application of Bayesian Inference for Increasing Rocket Engine Reliability and its Uncertainty Quantification

Presenter: Dani McDowell | Co-Authors: Shreyas Lakshmipuram Raghu

Advisor: Dr. L. Dale Thomas

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Presentation Agenda

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Introduction

Project Overview & Research Context

Research Team



Dani McDowell
Industrial & Systems
Engineering
Graduate Researcher



**Shreyas
Lakshmpuram
Raghu**
Aerospace Systems
Engineering
Ph.D. Candidate



**L. Dale Thomas,
Ph.D.**
CSIL Director,
Professor & Eminent
Scholar of Systems
Engineering



**Gang Wang,
Ph.D.**
Mechanical &
Aerospace
Engineering
Associate
Professor

RS-25 Affordability Project Overview

Context



The RS-25 Engine



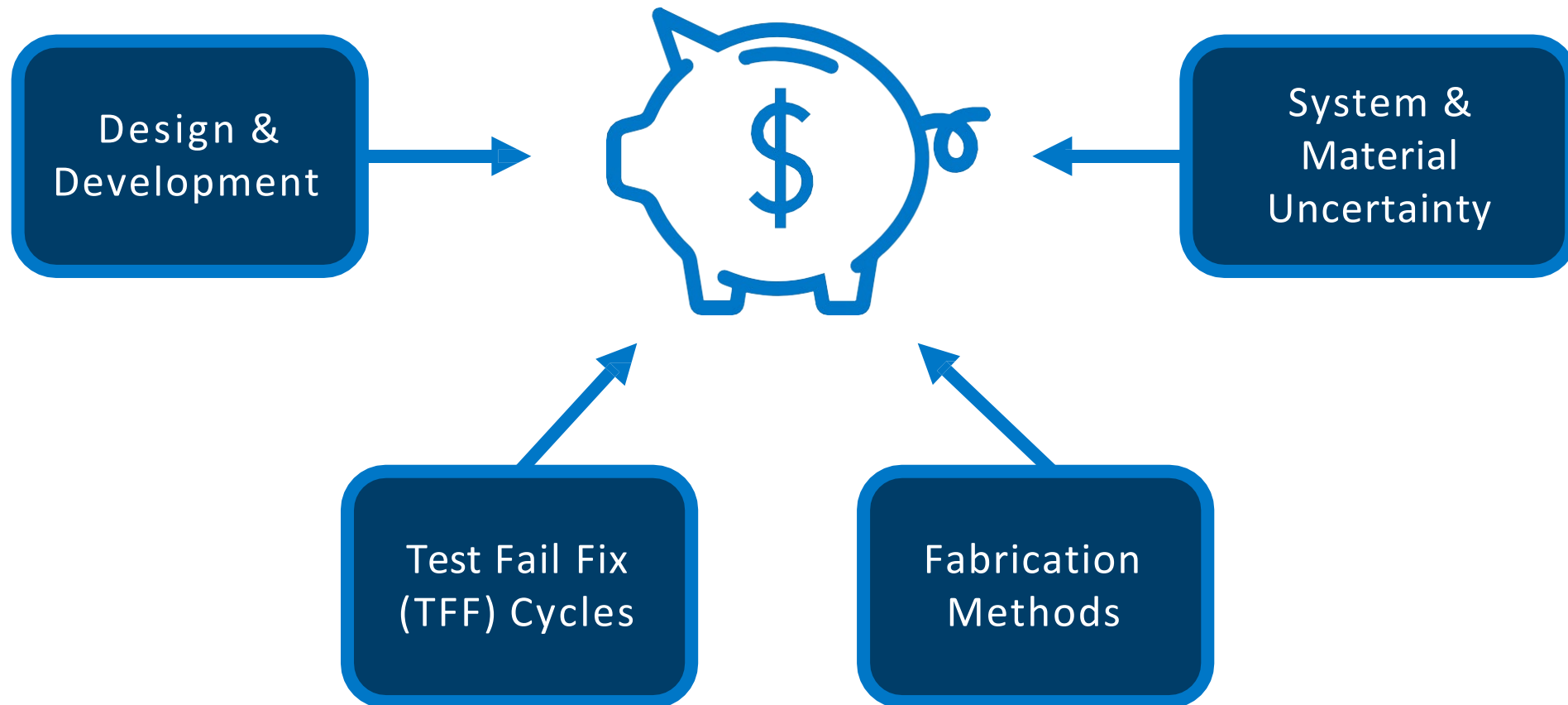
Space Shuttle Main Engine



Space Launch Systems Core Stage Engines

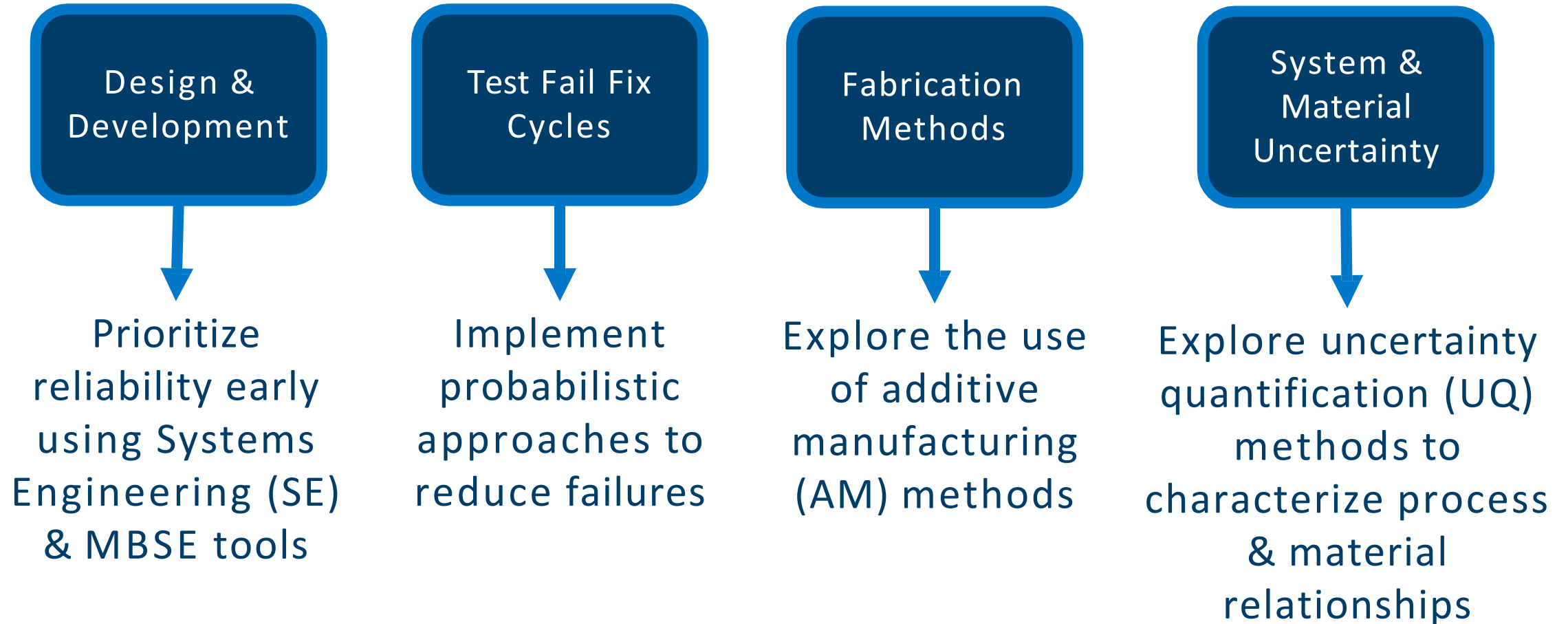
RS-25 Affordability Project Overview

Driving Forces of Cost



RS-25 Affordability Project Overview

Research Strategies to Address Costs

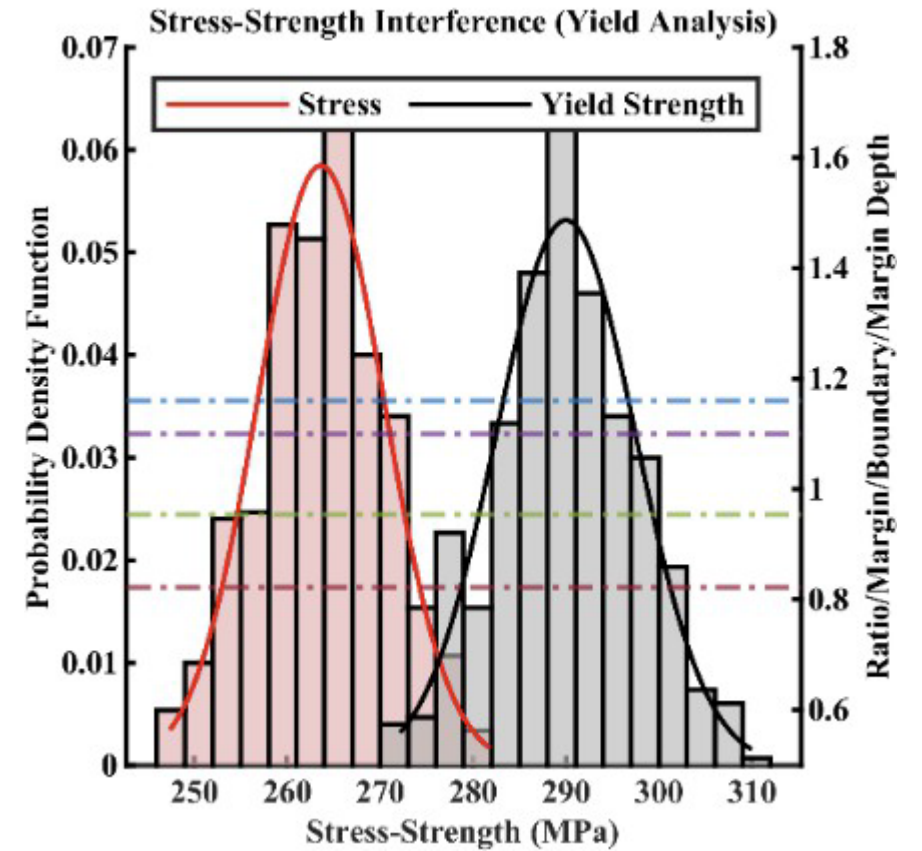
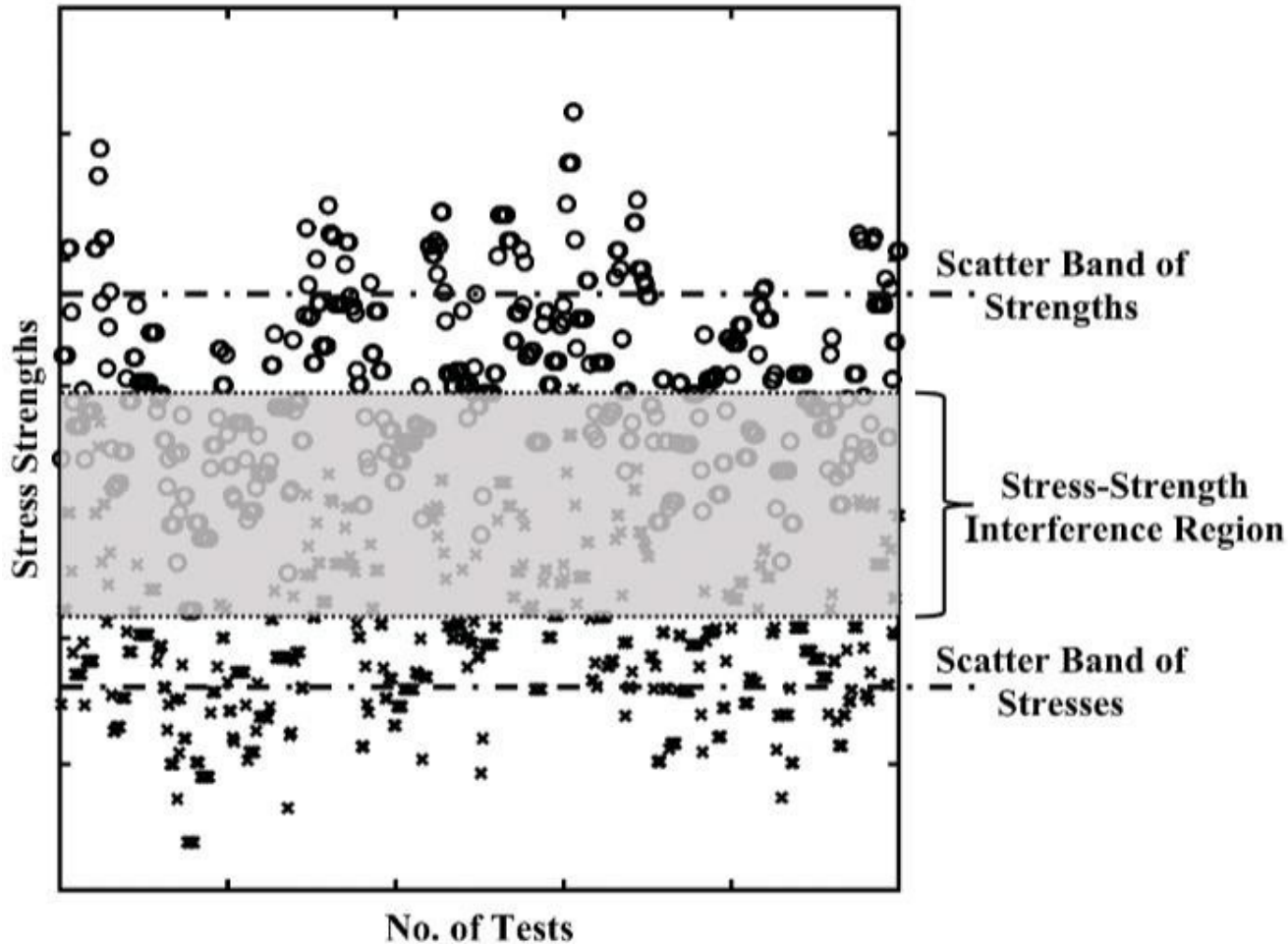


Background

*Foundations of the Structural Margin
Approach*

Defining Failure Potential

Stress-Strength Interference



Deterministic Factors of Safety



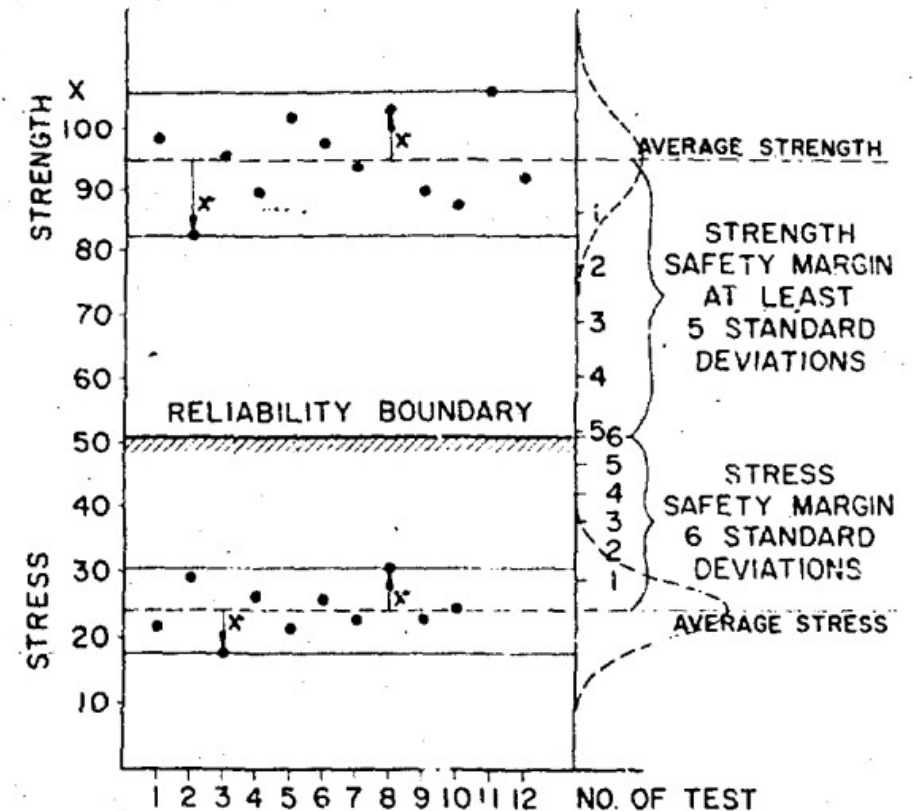
$$\text{Factor of Safety (FOS)} = \frac{\text{Ultimate Tensile Strength}}{\text{Applied Stress}} \text{ OR } \frac{\text{Yield Strength}}{\text{Applied Stress}}$$

*Common values are 1.4 for Ultimate Analysis, and 1.1 for Yield Analysis

Lusser's Method

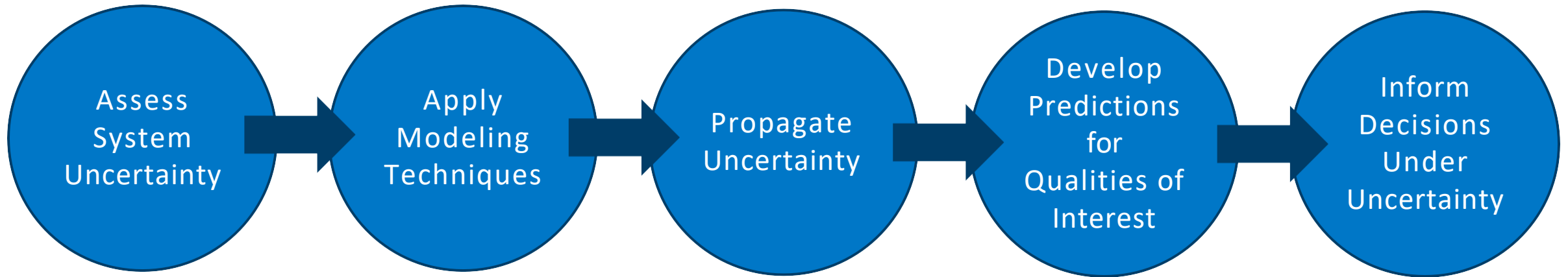
Developing Safety Margins

- Accounts for inherent variation relative to material strength and applied stress measurements
- Focusses on developing suitable safety margins based on standard deviations



How Scatterbands of Stress and Strength Shall be Separated by a Reliability Boundary [1]

Uncertainty Quantification for AM

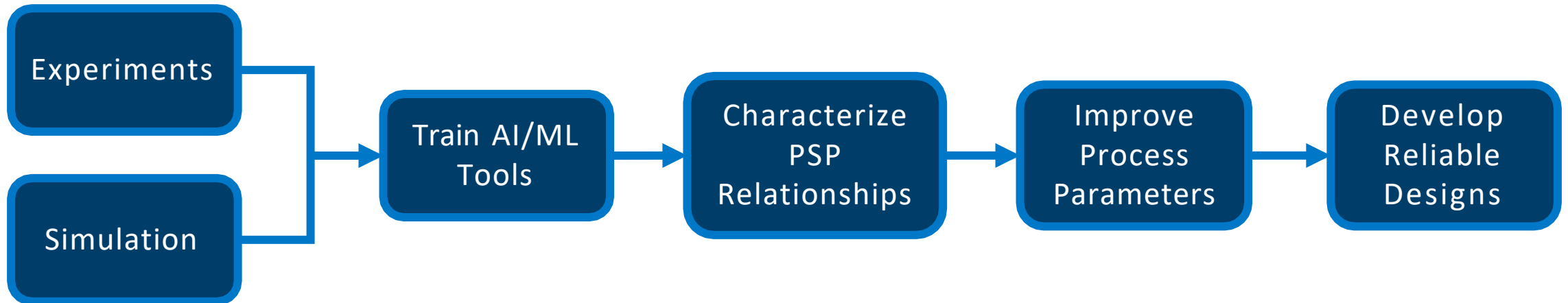


- Uncertainty quantification (UQ) is a tool used to characterize and evaluate uncertainties present in both physical systems and the computational tools used to model them
- Extensively used to understand Process-Structure-Property relationships for AM materials

Uncertainty Quantification

Usage of AI & Machine Learning Techniques

- Opportunity to learn more from experimental and training data
 - Possible integration with digital engineering tools and AM part production
 - Potential approach to enhancing performance and reliability

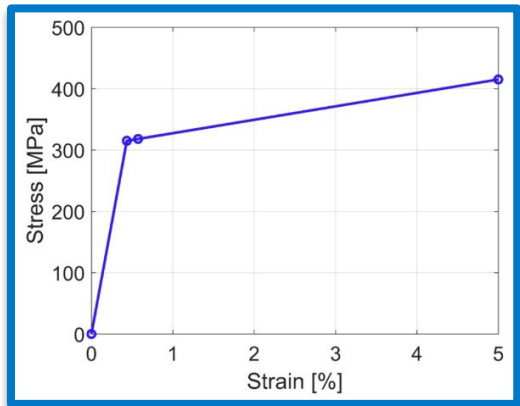


Methods

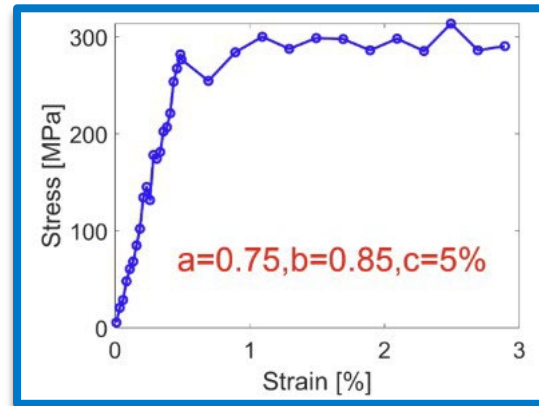
*Probabilistic Approaches to Rocket Engine
Development*

Structural Margin Approach

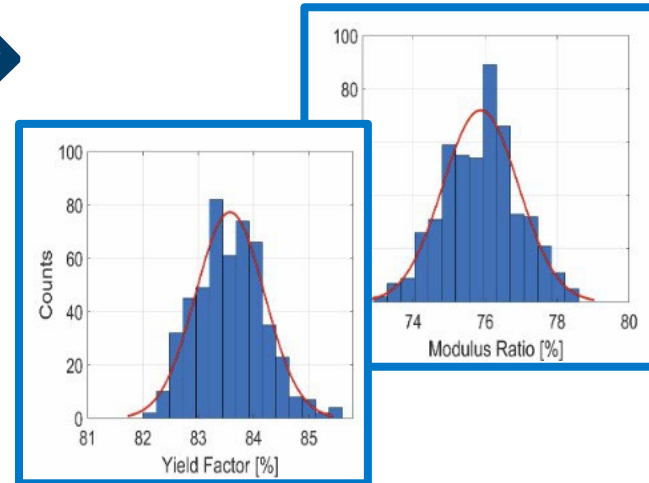
Overview



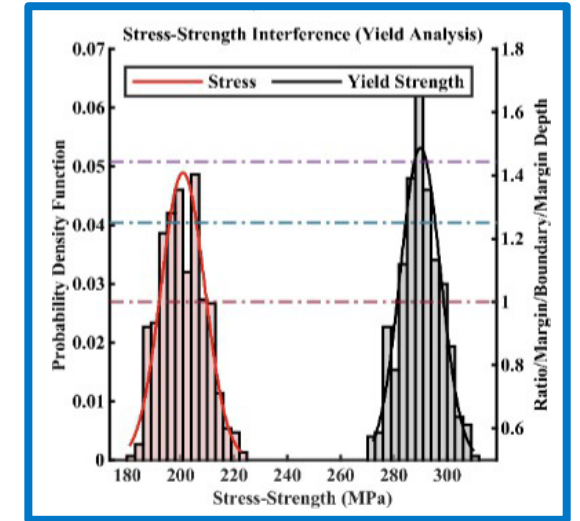
Develop Bilinear Stress-Strain Curves



Apply Noise & AM Reduction Effects



Develop Material Property Distributions



Create Stress-Strength Interference Plots

Obtaining Stress Strain Curves

Finite Element Analysis

- A simple, 1D rod was selected as the finite element
- Incremental load application formulas are employed to obtain uniaxial tensile load responses
- **Assumptions:**
 - Bilinear constitutive stress-strain relationship
 - Constant fracture strain

Nominal Material

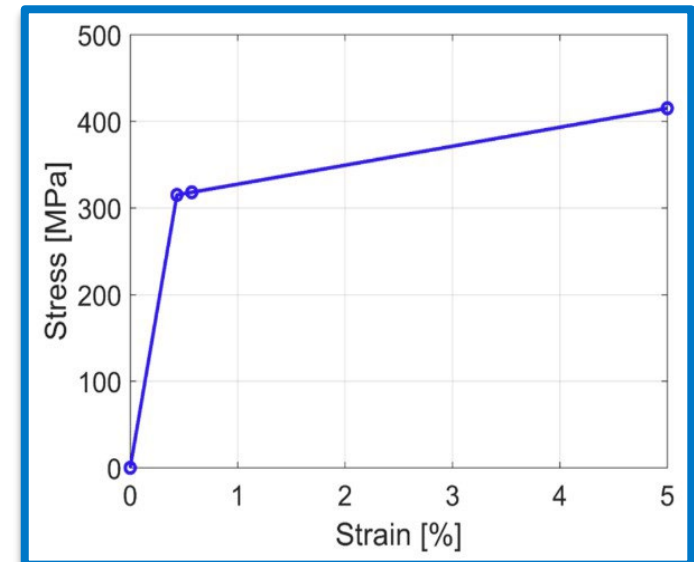
Properties:

Young's Modulus (E)

Yield Stress (S_y)

Strength (S_u)

Fracture Strain (ϵ_u)

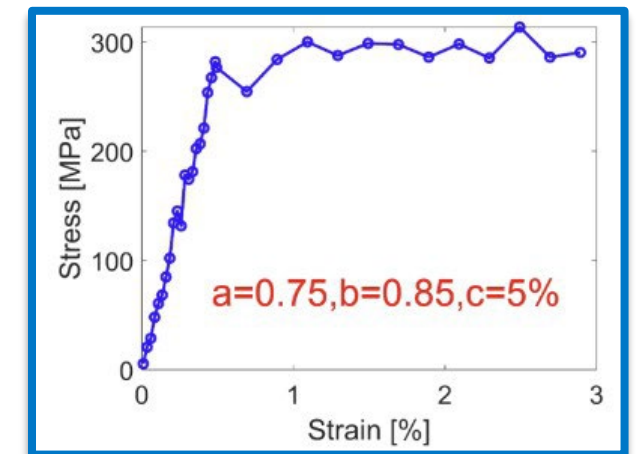
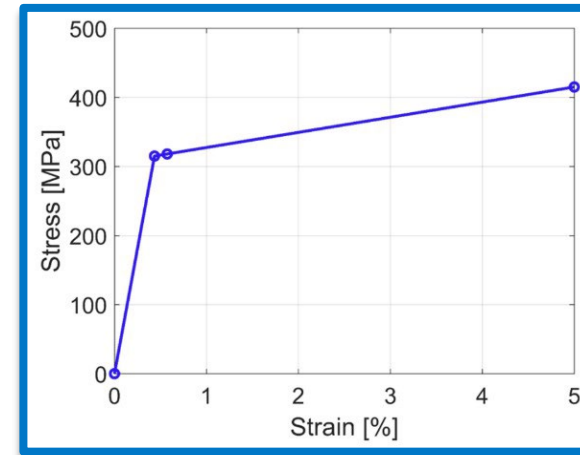


Introducing Uncertainty

AM Reduction Parameters & The Noise Factor

- Random noise is applied to simulate realistic data variation
- Reduction parameters are employed to simulate material property degradation due to AM

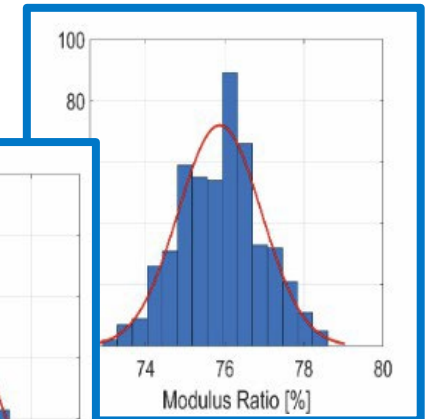
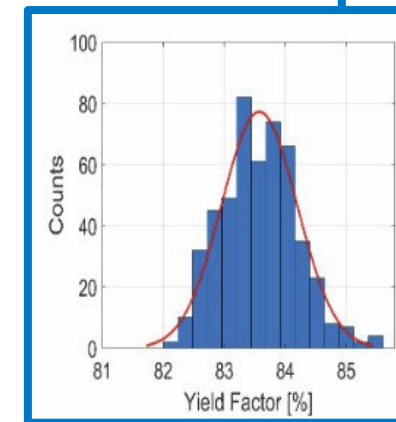
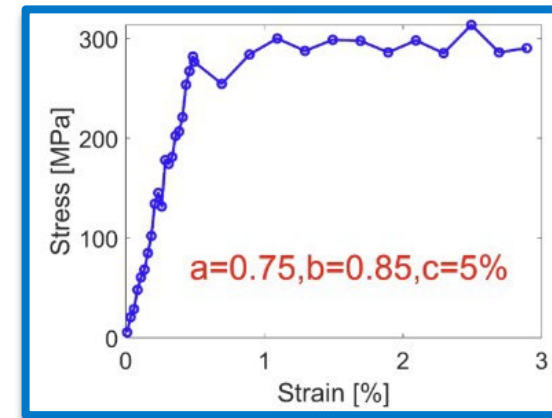
- $E = aE$
- $S_y = bS_y$



The Bayesian Inference Module

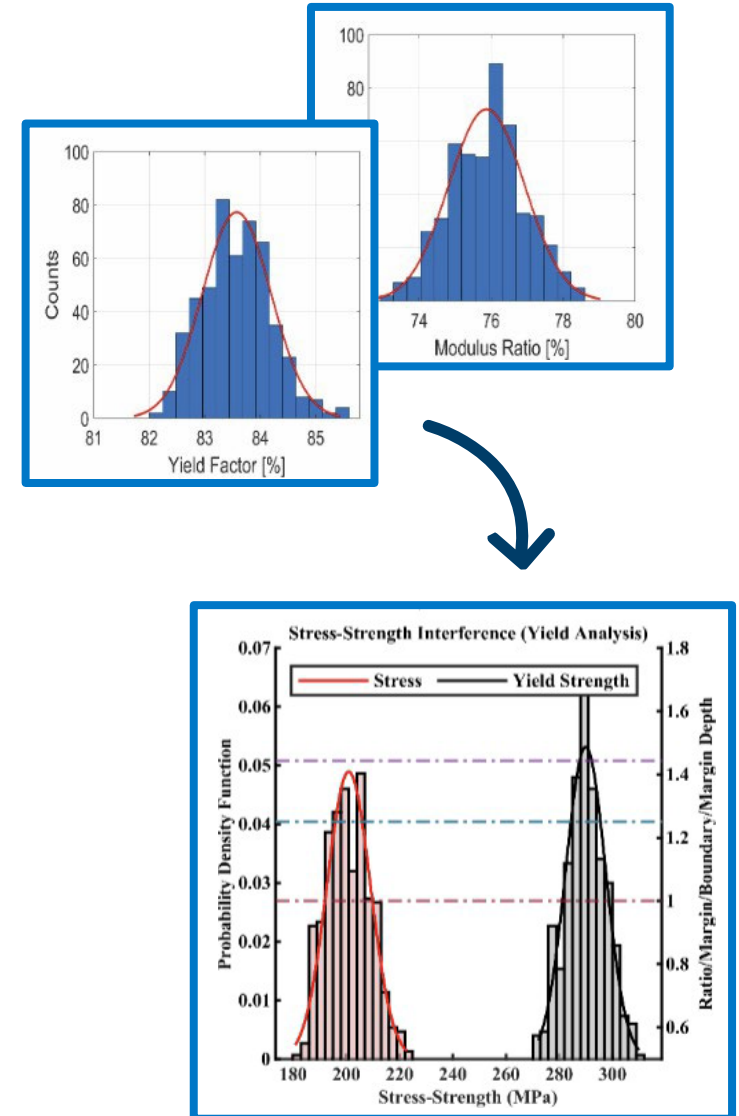
Markov Chain Monte Carlo (MCMC) Simulation

- Data with applied uncertainty is treated as a prior distribution and used to initiate an MCMC using the Random Walk Metropolis Algorithm
- The MCMC relies on the principles of Bayes Theorem to develop posterior distributions for the Yield Factor and Modulus Ratio from this data



Developing Stress Strength Interference Plots

- MCMC data is used to calculate:
 - A Yield Margin
 - A Reliability Boundary
 - Strength Distributions
 - Allowable stress distributions are calculated based on traditional FOS values and formulations from Lusser's method



Removing the Potential for Failure

Important Formulations

Derived From Lusser's Method

- **Reliability Boundary** \equiv $RB = S + d(6 \text{ STD}_S)$
- **Yield Margin** $= YM = S_y - d(5 \text{ STD}_{S_y})$
 - $d =$ correction factor based on sample size

Introduced in the Structural Margin Work

- **Margin Depth Variable** $= MD = YM/RB$
 - Used to assess potential failures
 - When **MD** < 1 , a failure due to stress-strength overlap is expected
 - When **MD** $= 1$, no overlap is observed

Results & Discussion

Application to Aerospace Materials

Application of the Structural Margin Approach

Material System Studied: Al-2024T6

- Al-Cu alloy, desirable for its high strength-to-weight ratio
- Common material used in the aerospace industry
- Suitable material for research in AM

Reduction Factors & Noise

- 64 possible combinations
- Example Shown: $a=75\%$, $b=90\%$, $c=15\%$

FOS Based Stress Strength Interference Plot

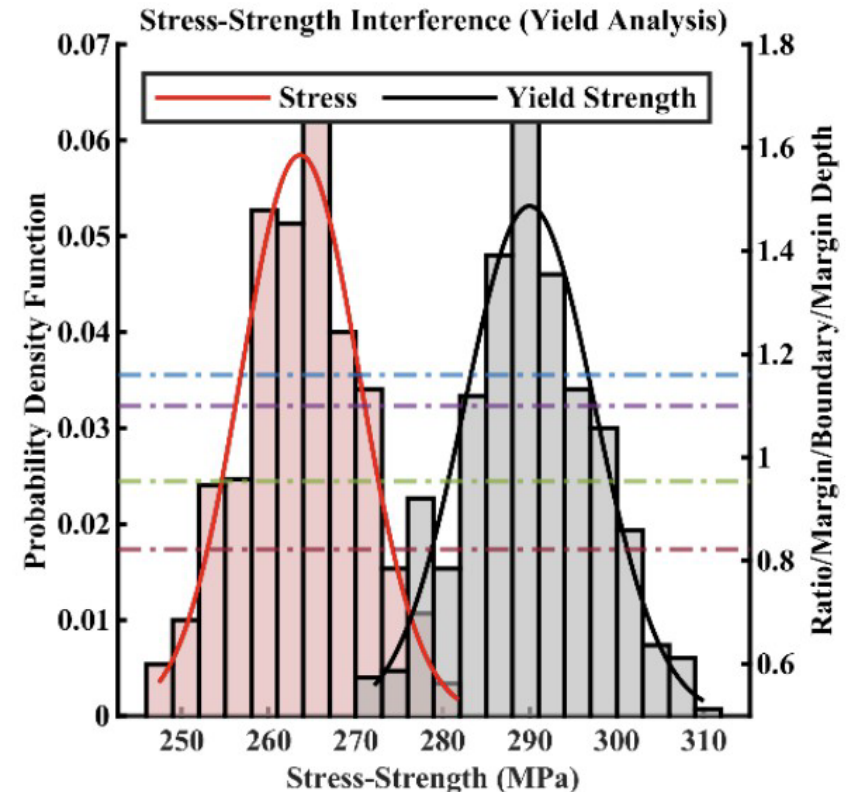
Applied Stress:

$$S = \frac{S_y}{FOS}$$

Strength:

$$S_u = S_y + 0.03E \left(\epsilon_u - \frac{S_y}{E} \right)$$

Ex: a=75%, b=90%, c=15%



(RB = 1.16, YSR = 1.1, YM = 0.9533, MD = 0.8219)

Structural Margin Based Stress Strength Interference Plot

Sets YM=RB → MD is always 1

Ex: a=75%, b=90%, c=15%

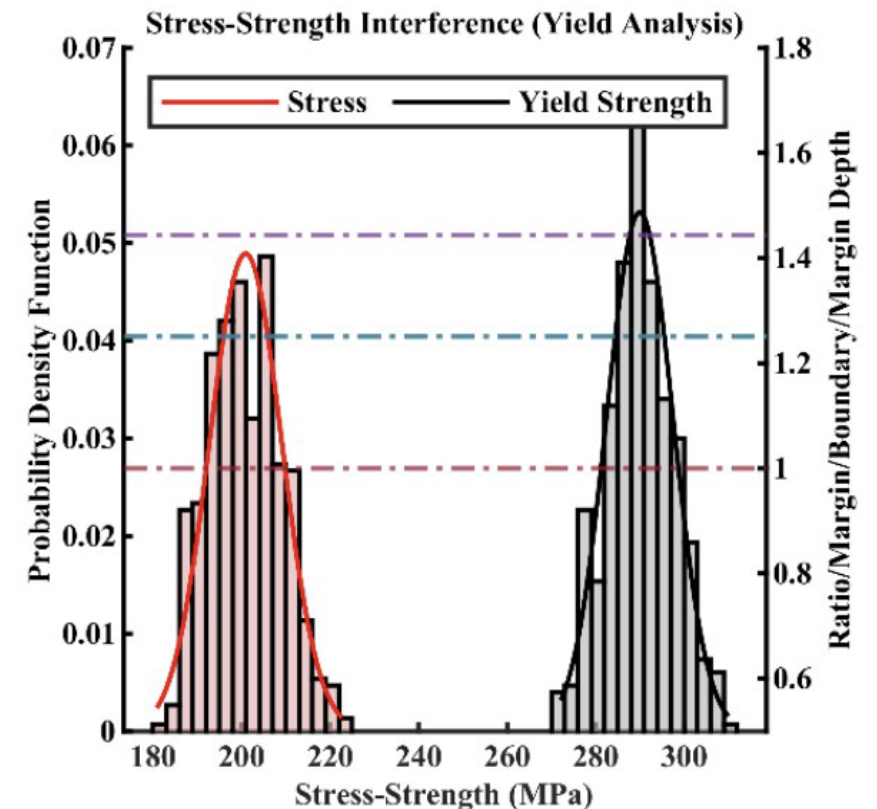
Applied Stress

$$\varepsilon = \frac{\overline{S_y} - d(5 \text{ STD}_{S_y})}{\overline{E} + d(6 \text{ STD}_E)}$$

$$S = E\varepsilon$$

Strength

$$S_u = S_y + 0.03E\left(\varepsilon_u - \frac{S_y}{E}\right)$$



(RB = 1.2509, YSR = 1.4433, YM = 1.2509, MD = 1.0)

Results

FOS Approach

Of the 64 cases studied, **50 resulted** in stress strength interference

- Higher failure potential
- Highlights inefficiencies of the FOS approach

Structural Margin Approach

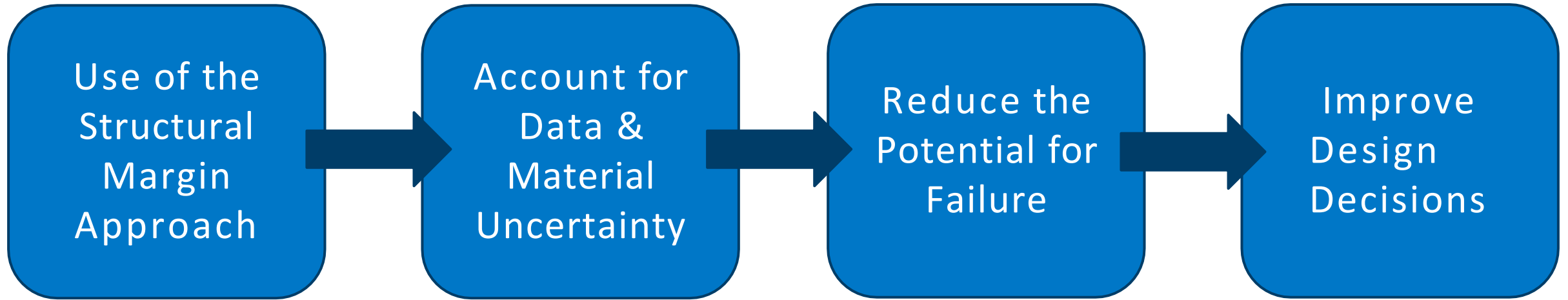
For all 64 cases, **no stress strength interference** was observed

- No failures due to the constrained MD value

Conclusion

Summary & Future Research

Impact



Our research supports the integration of digital engineering tools early in the design cycle to better inform decisions and promote system reliability

Future Research Efforts

AM Considerations

- Introduce more complex geometries
- Study the impact of fatigue and residual stress on engine performance

Additional Materials

- Expand the application of the Structural Margin Approach to promising materials
 - Inconel 718, Inconel 625, NASA-HR1, JBK-75
- Address material properties at elevated temperature conditions

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- Thank you to our advisor, Dr. L. Dale Thomas, and Dr. Gang Wang for their support and guidance!



(Credits: NASA)

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Thank You!

Any Questions?



Dani McDowell
Industrial & Systems
Engineering
Graduate Research
Assistant (GRA)
djm0052@uah.edu



**Shreyas
Lakshmi
Raghu**
Aerospace Systems
Engineering
GRA/Ph.D. Candidate
sr0101@uah.edu



L. Dale Thomas, Ph.D.
CSIL Director,
Professor & Eminent
Scholar of Systems
Engineering
dale.thomas@uah.edu



Gang Wang, Ph.D.
Mechanical &
Aerospace Engineering
Associate
Professor
gang.wang@uah.edu